TEXTBOOK OF LIGHT

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LONDON

UNIVERSITY TUTORIAL PRESS LTD.

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Eighth Edition 1947

Dear Student.

This book is of no value for the year '75, It is very outdated and contains no matter for B.Sc. Students, so take my advice never getting this book issued by mistake

N.K.L.

PRINTED IN GREAT BRITAIN BY UNIVERSITY TUTORIAL PRESS LTD., FOXTON

PREFACE

THIS *Textbook of Light* is intended primarily for the use of students preparing for the Intermediate examinations in Science and Engineering of the University of London and other examinations of a similar standard. It contains a treatment of Geometrical Optics of as wide a scope as is possible without the use of advanced mathematics and is based largely on previous editions of the well-known book by Professor J. Satterly and the late Dr. R. Wallace Stewart, published by University Tutorial Press Ltd.

The seventh edition was thoroughly revised by the late C. T. Archer, Assistant Professor of Physics at the Royal College of Science, and Lecturer in Physics in the University of London, whose connection with the text was so unfortunately terminated by his untimely death through enemy action.

In the present eighth edition the sign convention used in previous editions has been retained in the main text and many of the diagrams have been re-drawn to assist the student in applying it. At the same time it has been thought advisable to make provision for the increasing number of students already familiar with the Real is Positive sign convention by means of appendices at the ends of the chapters concerned. Those portions of the text for which there are alternative passages in these appendices are indicated by asterisks at beginning and end.

The chapter on Interference and Diffraction has been re-written to include an account of the more modern instruments and considerable alteration has been made to the chapter on the Theory of Light. A new section on electrical methods has been added to the chapter on Photometry and some further worked examples have been included in the various chapters. The pagination and diagram numbering of previous editions has been retained as far as possible.

The examination questions at the end of the book have been selected from examination papers set by the University of London for the Intermediate examinations in Science and Engineering, and the permission to reproduce such questions, granted by the Senate of the University, is hereby gratefully acknowledged.

CONTENTS

TEXTBOOK OF LIGHT

CHAPTER 1

INTRODUCTORY

LIGHT is the external physical cause of the sensation called sight. The earliest investigations as to the nature and behaviour of light were most probably made by the ancient Egyptians. Their work was followed by that of the ancient Greeks. *Pythagoras* and his followers (540 B.C.) formulated the theory that images formed in the eye were emanations from objects. The opposite view was taken by *Empedocles* and his followers (444 B.C.), who considered that the eye was endowed with the qualities of an octopus; the eye was supposed to project large numbers of tentacles which seized an object and caused it to be illuminated. *Plato* (430 B.C.) tried to combine the two theories, and explained light as the phenomenon caused by the collision and "neutralisation" of emanations from both the eye and the object. It is believed that Plato's school enunciated two of the fundamental laws of light, (1) that light travels in straight lines (Chapter II.), (2) that light is reflected in such a way that the angle of incidence is equal to the angle of reflection (Chapter III.). *Aristotle* and his disciples (384 B.C.), however, maintained that light was not a material emission from any source but a mere quality of a medium called the "pellucid." *Archimedes* (287 B.C.) is said to have constructed huge mirrors with which he used the sun's rays to set fire to the Roman fleet at Syracuse (212 B.C.). The first mention of refraction (Chapter V.), that is the bending of light when it leaves one transparent medium and enters another, was due to *Ptolemy* (A.D. 100).

Little progress in the science of light was made for some hundreds of years, when Astronomy rendered necessary a study of the subject. *Roger Bacon* (1214) was the first British scientist who took up the work, and a book published by him shows that he had a clear knowledge of the properties of lenses and mirrors. Then came *Vitellio* (1270), *Copernicus* (1473), *Galileo* (1564), and *Kepler* (1611) who made many contributions to the science, including the construction and use of telescopes (Chapter XI.). *Snell* (1621) and *Descartes* both claimed to have first formulated the laws of refraction

(Chapter V.), and *Roemer* (1676) succeeded in calculating the velocity of light (Chapter XII.) from astronomical observations. The greatest name in the history of the subject at this period is that of *Sir Isaac Newton* (1642-1727), who carried out much experimental work and gave a great deal of thought to the theoretical explanation of optical phenomena (Chapter IX. and Chapter XIV.). About the same time, *Huyghens* (1690) made very important contributions to the theory of light. Early in the nineteenth century *Young* and *Fresnel* introduced the wave theory of light, the theory being confirmed by actual experiment, and since then many workers, including *Clerk Maxwell*, *Einstein*, and others, have made important contributions towards the ultimate truth concerning the nature of light.

1. The Cause of Light

Light is generally believed to have its origin in the vibration of the luminous body—just as sound originates in the vibration of a sounding body—and to be transmitted to the eye by means of undulations or waves in the intervening medium, just as sound is transmitted to the ear.

There are, however, important differences between the two phenomena. First, that whereas a sounding body vibrates as a whole, it is essentially the *molecules* and their fundamentals, *electrons*, of the luminous body which vibrate. Secondly, that while the frequency of sound vibrations is only a few hundreds or thousands per second, the frequency of luminous vibrations is of the order of 400 to 800 billions per second. Thirdly, that while the wave-length of sound vibrations varies from about 8 mm. to 1100 cm., the wave-length of light vibrations is *extremely short*, varying from about 0·00004 cm. to 0·00008 cm. Fourthly, that while sound waves will not pass through a vacuum but require a material medium—such as air—for their propagation, light waves will pass through a vacuum, for the waves are propagated through the ether, an imaginary medium which pervades all matter and all space (see Art. 3). All these points will be dealt with more fully later.

Bodies which of themselves produce light are said to be *self-luminous*. Many bodies are able to reflect light from self-luminous objects, and thus become luminous themselves (see page 14). Thus the sun is self-luminous, but the moon is rendered luminous only by reflecting the sun's light.

2. The Transmission of Light

As stated above (Art. 1), an important point of difference between light and sound is found in the medium by which they are propagated.

A properly isolated bell ringing in a good vacuum is quite inaudible, but it is no less visible than before the removal of the air from the vacuum chamber. And it is well known that light from the stars reaches the eye after traversing millions of miles of "empty" space. As the senses give no evidence of anything in these "vacuous" spaces, it was found convenient by the early scientists to postulate the existence of an imaginary medium, called the **luminiferous ether.** This ether was supposed not only to occupy all space, but to interpenetrate all matter and to lie between the molecules of even the densest solid, and to be capable of carrying energy from one body to another. While it is not possible to demonstrate by experiment the existence of this medium—and many scientists now ignore it—the assumption of its existence provides (for beginners in particular) a means of visualising the transmission of energy through space. The ether is quite unlike any known form of matter, but the mathematical theory and the properties have been investigated fully. It must be taken probably as devoid of weight, and perfectly elastic; but the undulations it transmits are known to be transverse, and such therefore as no gas, however rarefied, by reason of its being devoid of cohesion, is competent to transmit. V. P. Singh

It may now be possible to picture the vibrations of the molecules, etc., of the luminous body setting up undulations or waves in the ether which travel with great speed in all directions. As a matter of fact, this speed is about 186,000 miles per second, a speed which would carry it about $7\frac{1}{2}$ times round the earth in a second. Some of these undulations, falling on the eye, set up changes in the optic nerve, which, when transmitted to the brain, produce the sensations of light.

But, whatever may be the nature of light and its mode of transmission, there are some fundamental properties, established by experiment, which may be studied quite independently of any hypotheses on these points. Some of the more important of these fundamental properties will be studied in the chapters which follow; but, as the undulatory or wave theory of light is now completely established, reference will be made to it whenever it seems advisable.

3. Light is Invisible

If it is remembered that light simply consists of undulations in an invisible medium, the statement that light is invisible ought to cause little surprise. When a beam of sunlight, entering through a small window into an otherwise dark room, is apparently seen, what is really seen is not the light itself but a number of floating particles in the air illuminated by the beam. Many of these are so large as to be easily visible separately as dancing motes.

If a lighted Bunsen burner is brought below the beam so as to burn or volatilise these particles, the luminous track will be interrupted by what appears to be black smoke rising from the flame. The Bunsen flame, however, is perfectly smokeless; the black space is full of dust-free air, so that there is nothing in this part of the beam to reflect the light, and consequently the air space is dark.

A further illustration is afforded by comparing the modern searchlight with the projector in a cinema. The powerful beam from the searchlight has large gaps which appear to possess no light, and it is only when an object such as an aeroplane enters such a gap that it is brightly illuminated. The beam from the much weaker source in the cinema, however, appears to be very intense owing to the large number of smoke and dust particles present in the atmosphere.

4. Medium

A *medium* is the name given to any substance through which light passes. A transparent medium is one which transmits the light which enters it more or less completely. Possibly no medium except the ether (Art. 2) allows all the light which enters it to pass through; in other media a portion of the light is *reflected* or *absorbed* by the medium, and that which emerges is consequently less bright. However, a medium which transmits the greater portion of the light entering it is called *transparent*, *e.g.* water, glass, mica.

Media which permit little or none of the light which falls on them to pass through are said to be opaque, *e.g.* wood, iron, lamp-black.

Media which transmit light to some extent but do not enable one to see clearly through them are called translucent, *e.g.* wax, ground glass, china.

Bodies which in their ordinary state appear perfectly opaque, really transmit a very considerable amount of light when obtained in sufficiently thin laminae. Thus, a gold-leaf, supported between

two pieces of glass, will appear to be semi-transparent and bright green when held to the light. A stone may be ground sufficiently thin to become transparent. A piece of cardboard, which under ordinary conditions appears perfectly opaque, transmits much light when held close to an electric arc; and if the hand be similarly held, it is possible to see the bones through the semi-transparent flesh. On the other hand, water is transparent, but a deep layer absorbs so much light that the sea-bottom at a depth of a few hundred fathoms is perfectly dark.

Thus, the terms *transparent*, *translucent*, and *opaque* refer to a difference in degree more than in kind, and for this reason it is perhaps more correct to refer to transparent, translucent, and opaque bodies than to apply these terms to substances.

A medium is said to be **homogeneous** when it is uniform throughout in composition, structure, and properties. A medium which is not uniform is said to be **heterogeneous.**

5. Ray, Beam, Pencil

These terms are of such frequent use in connection with light that it becomes necessary to define them.

The term **ray** is applied to the rectilinear path along which light travels, in any direction, from a point in a luminous object. Since light consists of very small waves, a ray has no real physical existence, but the conception of a ray is useful because it simplifies the consideration of phenomena in light. The direction of the ray at a point may be defined as the straight line joining the centre of a small spherical obstacle, situated at that point, to the centre of the shadow produced by it on a screen, an infinitely small distance beyond it in the direction in which the light is travelling.

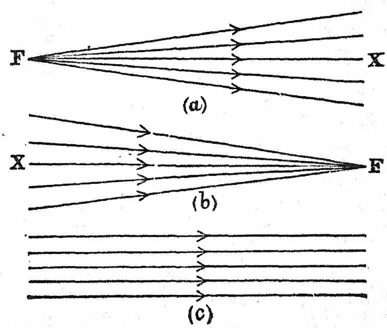


FIG. 1.

(*a*) Beam diverging from focus F. (*b*) Beam converging to focus F. (*c*) Parallel beam: focus at infinity.

A **beam** of light is a collection of adjacent rays of light, and may be *divergent*, *convergent*, or *parallel*—that is, the component rays may diverge from a point, or converge to a point, or run parallel [Fig. 1 (*a*), (*b*), (*c*)]. A divergent or convergent beam has the shape of a cone of small finite angle, while a parallel beam is a cylinder of small cross-section.

The *axis* of a beam, FX (Fig. 1), is the central ray passing along the geometrical axis of the figure of the beam, and the point F, from or to which the rays of a beam diverge or converge, is called its **focus**. The focus of a parallel beam is at infinity.

Very narrow beams are termed **pencils**, and, like beams, may be divergent, convergent, or parallel. When a pencil of light comes from a point on a very distant luminous source such as the sun, moon, or a star, it is considered to be parallel, although, strictly speaking, it is very slightly divergent.

Any ordinary source of light, such as a lamp filament, produces a *divergent* beam of light. If the source is a great distance away the rays become nearly *parallel*. By the use of lenses and mirrors it is possible to collect rays diverging from a source and make them parallel or *convergent*. The beam of a searchlight is made parallel to avoid wasting light, which would normally diverge from the source in many directions. Most optical instruments collect rays diverging from a source and converge them to a focus, as will be discussed in later chapters. A *convergent* beam of light cannot be produced without a mirror or lens.

CHAPTER II

RECTILINEAR PROPAGATION OF LIGHT

MANY familiar phenomena point to the fact that light travels through the same homogeneous medium in straight lines. It is impossible, in general, for light to travel round corners. The beam of light from the headlight of a car is bounded by straight lines. Also, any point on the edge of the shadow of an obstacle produced by a luminous source is always in a straight line with the obstacle and the source. Thus it may be assumed as a fundamental fact that the propagation of light is rectilinear.

1. Light Travels in Straight Lines through a Homogeneous Medium

If three screens be each pierced with a small pinhole and then held one in front of the other in such a position (Fig. 2) that the

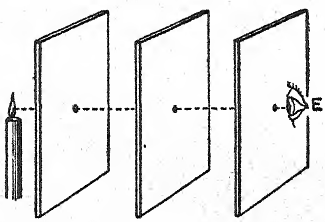


FIG. 2.

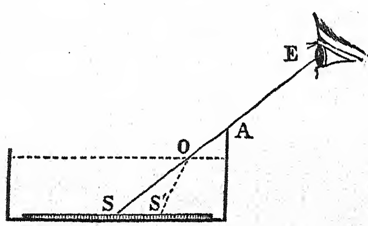


FIG. 3.

three holes and a candle flame are in the same straight line, a ray of light can pass from the candle through the holes to an eye placed in the same straight line beyond the screens. If, however, any one of the screens is displaced slightly in its own plane, the candle flame becomes invisible.

Similarly, if a scale, S, be laid on the bottom of a vessel, and looked at over the edge, A, of the vessel in the way indicated (Fig. 3), it will be found that the line, EAS, is a straight line.

In both these experiments the same medium, air, extends between the eye and the object seen. If, however, in the latter example, water be poured into the vessel, it will be found that a point, S′, on the scale can be seen, and that EAOS′ is not a straight line. Hence, when light passes from one medium to another, it is,

in general, bent from its direct rectilinear path, and it is evident that the bending must take place at the surface of separation of the two media. When, however, a ray of light travels through a non-homogeneous medium, it may suffer gradual and continuous change of direction, if the properties of the medium also undergo gradual and continuous change along its path; it is only on passing from one homogeneous medium to another that a sudden change of direction takes place. The magnitude and direction of this change depend on conditions which will be considered fully in a later chapter (Chapter V.).

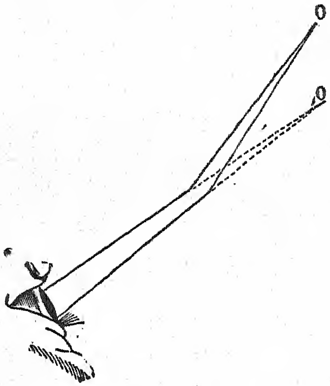


FIG. 4.

In connexion with this point, it should be noticed that the eye takes no cognisance of change of direction in a ray of light. Every object is seen in the direction taken by the axis of the pencil of light from the object which enters the eye. For example, if a pencil of light, starting from O (Fig. 4), be bent, as indicated, then O appears to the eye to be at O′. The point, O′ is the virtual focus of the pencil entering the eye—called *virtual* because the rays do not reallydiverge from O′, but onlyappear to do so.

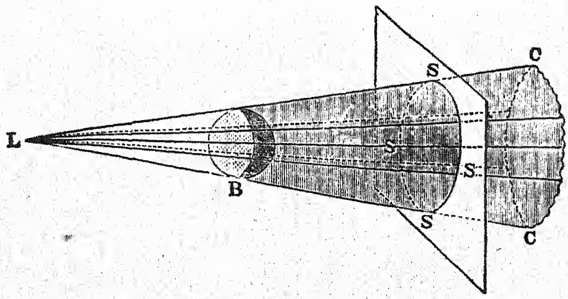


FIG. 5.

2. Shadows

The formation of shadows is a natural and direct consequence of the rectilinear propagation of light.

If an opaque body, B (Fig. 5), be placed so as to intercept a

portion of the light emitted by a luminous point source, L, the cone of light incident on the surface of the body is stopped, and the space beyond B, enclosed by the geometrical continuation of this cone, is screened from the rays diverging from L. The cone here considered, LCC, is called the *shadow cone,* and its trace, SS, on any surface beyond B and intersecting it, outlines the *shadow* cast on this surface.

When, however, the source of light is not a luminous point, but a luminous body, the case is somewhat more complicated. A luminous body may be considered as made up of a great many luminous point sources; then it will be necessary to examine only the rays from the two extreme points of the body.

Let SS′ (Fig. 6) represent a spherical source of light, and OO′ an opaque sphere placed near it. Consider the single cone, SS′UU′,

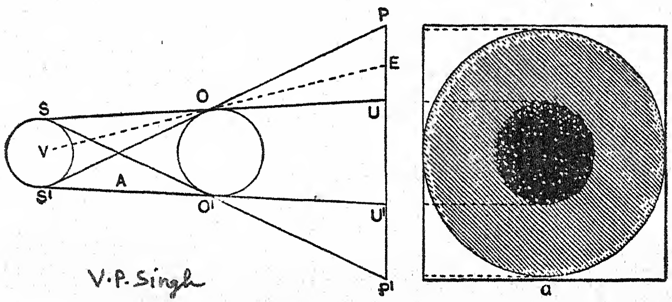


FIG. 6.

which encloses SS′ and OO′. It is evident from the figure that *no* light from SS′ falls within the portion of this cone lying beyond OO′, and for this reason it has been called the cone of *total shadow,* or the cone of the umbra. The portion of it just referred to as being completely screened from the light is called the *umbra.*

Now consider the double cone, SS′APP′, enclosing SS′ and OO′, and having its apex at A. This is the cone of *partial shadow,* or the cone of the penumbra, and the portion of it beyond OO′, and surrounding the umbra, is known as the *penumbra.* From any point in the cone, not within the total shadow, a portion of the source of light can be seen, and for this reason the shadow is only *partial.* The depth of shadow at any point depends on the extent of the source invisible from that point. Thus, to an eye placed at E, all below EV is invisible, while all above is visible; hence, at points near the outer boundary of the penumbral cone, the shadow

is very light, but gradually deepens as the outer boundary of the umbral cone is approached.

The penumbral cone always has the form of a cone diverging from a point, A, lying between SS′ and OO′, but the form of the umbral cone depends on the relative sizes of the source of light and the opaque body. When the latter is the greater, the cone diverges from a point beyond the source (Fig. 6), and when smaller the cone converges to a point beyond the opaque body (Fig. 7).

If the shadow of the opaque body be cast upon a screen suitably placed (Fig. 6), it will be found to consist of a central region of total shadow, the *umbra,* surrounded by a zone of partial shadow, the *penumbra.* The former is of uniform depth all over, but the latter passes gradually from the total shadow of the umbra to a

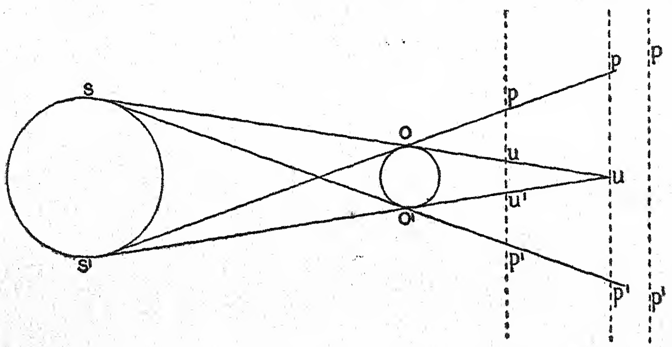


FIG. 7.

complete absence of shadow at its outer boundary, and consequently neither its outer nor its inner edge is sharply defined. The relative sizes of the umbra and penumbra in any particular case depend on the conditions illustrated (Figs. 6 and 7) and on the position of the screen. Thus, in Fig. 7, the screen on the left shows an umbra surrounded by a penumbra, on the middle screen the umbra is just disappearing, having become a point, while on the screen on the right there is no umbra.

Example.—*A spherical uniform source of light, 2 in. in diameter, is placed at a distance of 10 ft. from a sphere 2 in. in diameter. Calculate, approximately, the diameter of the umbra and penumbra cast on a screen 5 ft. beyond the sphere.*

Referring to Fig. 6, and assuming that SS′ and OO′ are the same size, then SS′ = 2 in., OO′ = 2 in., SO = 10 ft., and OU = 5 ft.

$$\text{Diameter of umbra} = UU' = OO' = 2 \text{ in.}$$

$$\text{External diameter of penumbra} = PP'.$$

From the triangles, OUP and OSS′,

$$\frac{UP}{SS'} = \frac{UO}{OS} = \frac{5}{10} = \frac{1}{2}; \quad \text{hence } UP = \frac{SS'}{2} = 1 \text{ in.};$$

$$\therefore PP' = UU' + 2UP = 2 + 2 = 4 \text{ in.}$$

3. Eclipses

These phenomena are the most interesting examples of shadows. They are of two kinds, lunar and solar.

If at full moon the centres of the sun, earth, and moon are nearly in a straight line, the earth, acting as the opaque body, will stop the sun's rays before they reach the moon, and therefore the moon will be either wholly or partially darkened. This phenomenon is called a lunar eclipse, and may be either *total* or *partial*.

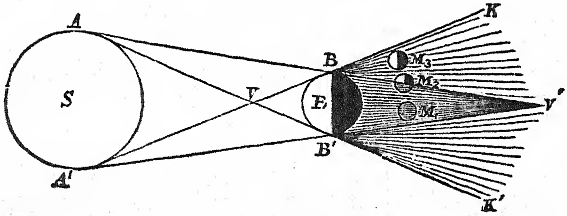


FIG. 8.

On the other hand, if the three centres are nearly in a straight line when the moon is new, the moon, coming between the earth and the sun, will cut off the whole or a portion of the sun's rays from certain parts of the earth's surface. In such parts the earth will be darkened, and the sun will be either wholly or partly hidden. This phenomenon is called a **solar eclipse**, and may be either *total*, *annular*, or *partial*. It is said to be annular when the moon is too near the sun to hide it completely, but leaves the rim of the sun's disc visible, like a ring of light round its own dark body.

Thus, in a lunar eclipse, the earth passes between the sun and the moon. The two types of lunar eclipse mentioned above will be seen from Fig. 8, in which S is the sun, E the earth, and M the moon. If the moon is entirely in the cone of the umbra, BVB′, as at M_1, the whole of it is in darkness and there is a *total* eclipse of the moon. If the moon is partly within and partly without the umbral cone, as at M_2, the part within receives no

light from the sun, while the remaining part receives light from the upper portion of the sun, and there is a *partial* eclipse of the moon. If the moon is entirely in the penumbral cone, as at M_3, it receives light from the upper portion of the sun and there is no true eclipse, but only a decrease in brightness, sometimes called a *penumbral eclipse*.

Again, in a solar eclipse or eclipse of the sun, the moon passes between the earth and the sun. Thus, in Fig. 7, let SS′ represent the sun, OO′ the moon, and the screen, PUU′P′, the earth. Then, from a position in the portion, UU′, a *total* eclipse of the sun will be seen. A person viewing the sun from a position in the penumbra will see a *partial* eclipse. If the earth is in the position of the screen, PP′, on the extreme right, a person just beyond U, within the angle formed by producing OU and O′U will see an *annular* eclipse, a narrow ring of the sun's disc all round the moon, the central portion being dark.

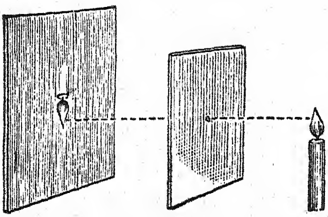


FIG. 9.

It is evident that eclipses occur when the earth, moon, and sun are in a straight line, or nearly so. At full moon, the earth is in the middle and lunar eclipses are possible. At new moon, the moon is in the middle and solar eclipses are possible. Eclipses do not occur at every new and full moon, because the moon does not move in the *plane of the ecliptic*, and the three are not in a straight line, therefore, at every new and full moon.

4. The Pinhole Camera

This is another interesting example of the rectilinear propagation of light.

If a sheet of cardboard, pierced at its centre with a large pinhole, be placed between a burning candle and a thin paper screen, shaded from external light, a more or less distinct representation of the candle flame, in an inverted position, will be seen on the screen (Fig. 9). If the cardboard forms the front, and the screen the back, of a closed box, the representation can be seen very distinctly from behind through the paper, or better ground-glass, screen.

The explanation of this is simple. Let AB (Fig. 10) represent the candle flame, or other brightly illuminated object, O the hole in the cardboard, and S′ the screen. From every point on AB

rays of light are emitted in all directions, and consequently from every point of AB a small pencil of rays passes through O, and forms a small circular or elliptical spot of light on the screen. The result of this is that there appears on the screen an assemblage of nearly circular spots of light which, owing to the crossing of the rays at O, define an *inverted* representation, A′B′, of the object AB. A practical form of pinhole camera is shown in Fig. 10*b*.

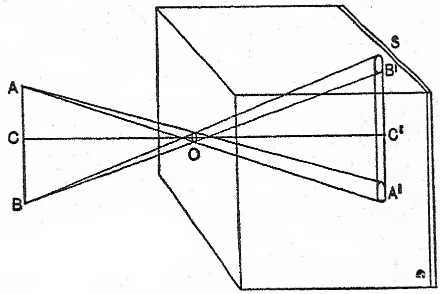


FIG. 10.

Example.—*In a pinhole camera the distance from the aperture in front to the screen at the back is* 18 *in. Find the relative dimensions of the representation on the screen of an object placed* 6 *ft. in front of the camera.*

Referring to Fig. 10, and treating the pencils from A, B to A′, B′ respectively as lines, it is evident that the triangles, AOB, A′OB′, are equiangular and therefore similar.

$$\text{Thus, } \frac{AB}{A'B'} = \frac{CO}{OC'} = \frac{6}{1 \cdot 5} = 4.$$

$$\text{Hence } AB = 4A'B'.$$

If these spots are large, they overlap one another, and the representation is blurred and indistinct; this is called *lack of definition*. Hence, in order to obtain a well-defined picture of AB on the screen, the aperture at O must be very small, for the size of any spot on the screen depends, for given positions of AB and SS, upon the size of the aperture.

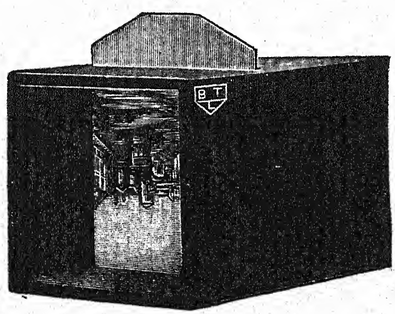


FIG. 10*b*.

If a photographic plate is substituted for the screen, a photograph of the object can be obtained. Such a camera has the advantage that no *focusing* is required if the pinhole is small, but has the serious disadvantage that a long exposure is necessary.

The circular and elliptical spots of light seen on the ground beneath trees when the sun is shining are images of the sun formed by rays passing through the small apertures between the leaves.

5. Self-Luminous and Non-Luminous Bodies

Every point of a self-luminous body emits light in all directions. The eye intercepts a pencil of rays from each visible point, and the light received is registered by the nerves of the eye. The nerves convey to the brain the sensation of sight.

When no self-luminous body is present, as in a darkened room, no object can be seen. It is inferred from this that, if a body is visible, light must be coming from it to the eye. If a body is non-luminous, it may possess the power of transferring light, which falls on it, to other objects, and this light on entering the eye renders the body visible.

A candle flame, when introduced into a darkened room, will not only itself be visible, but will render the walls of the room and objects in the room visible also. Since, in the absence of the candle flame, the walls and objects could not be seen, the latter are not self-luminous; when visible, they are rendered visible by light derived from some self-luminous body.

The sun is the most familiar self-luminous body; the moon is non-luminous, but is rendered self-luminous by light received from the sun.

CHAPTER III

REFLECTION AT PLANE SURFACES

When light travelling in one medium, A, is incident on the surface of another medium, B, it is, in general, divided into three parts—

(1) A portion which is reflected from the surface of B, back into A, according to certain laws. This portion is said to be reflected at the surface of B in accordance with the laws of reflection (Art. 3).

(2) A second portion passes into B, and travels through that medium in a direction determined by another law. This portion is said to be refracted into the medium B, in accordance with the laws of refraction (page 59).

(3) A third portion passes into B, and is absorbed by that medium; that is, it is converted into some other form of energy within that medium.

The amounts of each vary considerably with the nature of the surface on which the light falls, and the greater portion of the light may be scattered or diffused by the surface into both media in an irregular manner. The light thus scattered renders the surface luminous, and it is because of this scattering of light by the surfaces of non-luminous bodies that they become luminous in the presence of a self-luminous body (pages 14 and 32).

When light is incident on an opaque body no portion of the light is refracted or diffused into it, and the ratio of the quantities reflected or diffused back depends on the nature of the surface of the body and on the angle at which the light falls on the surface. A rough, uneven surface *scatters* the greater portion of the light falling on it, but a smooth, highly polished surface *reflects* nearly all the incident light. Also, the more obliquely the light falls upon any reflecting surface, the greater is the proportion of reflected light. Since a surface is rendered visible by scattering the light incident upon it, it follows that a perfectly reflecting surface would be invisible.

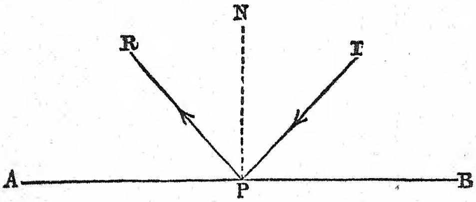
1. Mirrors

Any good reflecting surface is a mirror. The term is usually confined, however, to polished surfaces of a definite geometrical form—*e.g.* plane, spherical, cylindrical, etc. The earliest mirrors

were of polished metal, and this form of reflector is much used now for optical purposes.

The ordinary plane mirror consists of a sheet of plate glass backed by a thin layer of silver, which forms the reflecting surface. Up to 1840, all glass mirrors were backed with an amalgam of mercury and tin. This process has since been superseded almost entirely by the silver process. The silver is deposited on the glass from a solution of silver nitrate, either by the use of tartaric acid (hot process), or by sugar and acetic acid (cold process). When dry the surface is brushed over with a dilute solution of mercuric cyanide, and then coated with red-lead paint to preserve it.

More recently, for scientific purposes, silver *specula* have been employed as mirrors. These are formed of glass surfaces, of the required geometrical form, coated on the front surface with a thin layer of silver which is very highly polished.



AB, Reflecting surface. IP, Incident ray.
PN, Normal. PR, Reflected ray.
IPN, Angle of incidence. NPR, Angle of reflection.

FIG. 11.

2. Definitions

The *normal* to a reflecting surface at any point is a line drawn at right angles to the tangent plane to the surface at that point. If the surface is plane, then the normal at any point is at right angles to the surface, and if spherical it is coincident in direction with the radius drawn at that point.

The angle of incidence of a ray of light falling on the surface of a medium is the angle between the direction of the ray and the normal to the surface at the point of incidence. The *plane of incidence* is the plane containing the normal and the incident ray.

The angle of reflection is the angle made by the reflected ray with the normal at the point of incidence. The *plane of reflection* is the plane containing the normal and the reflected ray.

At this point may be mentioned what is called the reversibility of light. It may be stated thus: if by any means light is able to travel from a source at A to a point B, then, if the source is placed at B, light will travel back by the same means to the point, A, by the same path.

3. Laws of Reflection

When a ray of light is incident on a reflecting surface, it is reflected in accordance with two laws, which may be stated thus:

(1) *The angle of incidence is equal to the angle of reflection.* Thus, if AB is the reflecting surface (Fig. 11), IP an incident ray, PN the normal at the point of incidence, and PR the reflected ray, then the angle of incidence, IPN, is equal to the angle of reflection, NPR.

(2) *The plane of incidence and the plane of reflection are coincident.*

These may be expressed as one law thus: *the angles of incidence and reflection are in the same plane, and are equal to one another.*

This law is established by experiment, and may be verified directly by means of the apparatus shown (Fig. 12). A graduated circle, fixed in a vertical plane, has a small plane mirror, *m*, attached horizontally at its centre, and carries two sighting tubes, T and T′, having their axes parallel to the plane of the circle and directed towards the centre. These tubes travel round the circumference of the circle, and the positions of their axes relative to the graduations are shown by marks on the slides to which they are attached. The zero of the graduations is placed at the point where the normal to the central portion of the mirror cuts the graduated circle.

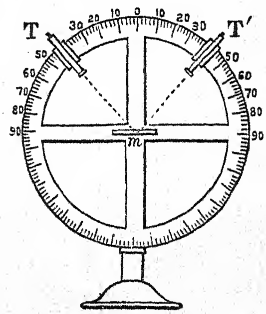


FIG. 12.

A source of light is placed so as to send a beam of light through one of the tubes to the mirror. The other tube is then moved round until, on looking through it, the source of light can be seen reflected in the mirror. It is then found that each of the tubes is at the same angular distance from the zero of the scale—that is, the angle of incidence is equal to the angle of reflection. Moreover, the planes of incidence and reflection are coincident, for both are parallel to, and at the same distance in front of, the plane of the graduated circle.

Other experimental proofs of the laws are described below (Art. 5). The strongest proof of them, however, lies in the fact that in numerous experiments the laws are assumed and the assumption has led invariably to an accurate result.

The laws hold for any polished surface, whether plane or curved. If curved, a small area around the point of incidence will be coincident with the tangent plane at that point, and the normal can be drawn perpendicular to this plane.

4. Images

When a luminous body is viewed directly, pencils of rays of light from every point on the body enter the eye, and thus the body is seen and its shape or form defined. If, however, from any cause these pencils suffer change of direction, so that they actually come from, or appear to come from, an assemblage of luminous points other than the surface of the luminous body, this assemblage of luminous points is called the **image** of the luminous body.

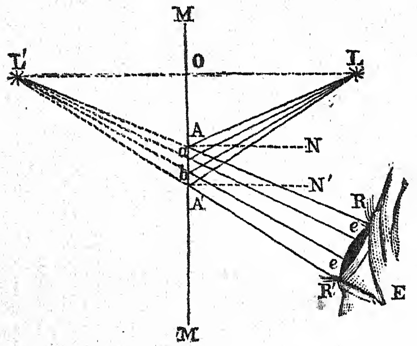


FIG. 13.

An image may be either **real** or **virtual**. In a real image *the rays do pass actually through the points of the image,* but in a virtual image *the rays only appear to pass through the points of the image.* Or, perhaps it is better to say that a virtual image is such that the rays are straight lines whose directions would pass through the image if produced backwards. A real image differs from a luminous body in the fact that the latter emits light in all directions, whereas the former transmits light only in the direction taken by the rays involved in its formation.

A real image may be formed on a screen, while a virtual image cannot. This will be understood later.

5. Reflection of a Divergent Pencil by a Plane Mirror

Let MM (Fig. 13) denote the position of the mirror, and L that of a luminous point. Consider the reflection of any ray, LA. Draw the normal, AN, at A. Then, according to the laws of reflection, the reflected ray will lie in the plane, LAN, and its direction, AR, will be such that the angle, LAN, equals the angle, RAN. Similarly, for the ray, LA′, the reflected ray takes the direction, A′R′, so that the angle, LA′N′, equals the angle, R′A′N′.

Now, to an eye placed at RR′, the pencil of rays reflected from the portion, AA′, of the mirror will appear to come from a point, L′, at the intersection of RA and R′A′. It can be shown that this point, L′, lies on the normal to the mirror passing through L, and at the same distance behind the mirror as L is in front of it.

Through L draw the normal, LOL′, and let RA produced cut it at L′. Then,

Angle LAN = angle OLA, and angle RAN = angle OL′A.

But, in accordance with the laws of reflection,

Angle LAN = angle RAN; hence, angle OLA = angle OL′A.

Therefore, in the triangles, AOL and AOL′,

Angle AOL = angle AOL′; angle OLA = angle OL′A,

And the side, OA, is common to both;

$\therefore$ OL = OL′.

Similarly, it can be shown that any other reflected ray, if produced backwards, passes through L′, and therefore, to an eye in front of the mirror, a *virtual* image of L is seen at L′. The image is virtual because the rays by which it is seen do not actually come from L′, but only appear to do so, owing to the change of direction resulting from the reflection at the surface of the mirror.

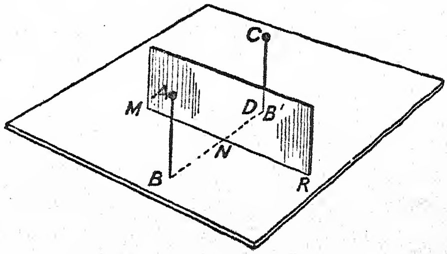


FIG. 14.

It has been proved above, by assuming the truth of the laws of reflection, that the image of a luminous point is at the same distance behind the mirror as the point itself is in front of it. Hence, if this can be shown to be true experimentally, an indirect experimental proof of the laws of reflection is established.

Experiments. To prove the laws of reflection of light.—(*a*) Take a rectangular plate of thin ordinary mirror, stand it in a vertical position upon a sheet of cartridge paper pinned to a drawing-board, and mark its trace, MR (Fig. 14). Stick a pin, AB, into the paper about four inches in front of the mirror. An image, CB′, formed by the reflecting surface, will be seen clearly, apparently behind

the mirror. Place another pin, CD, behind the mirror so as to coincide in position with this image for all positions of the eye—that is, until *parallax*, or side-shifting between image and object when the eye is moved from side to side, is eliminated, remembering that the further of two objects will appear to move, relatively to the nearer, in the same direction as the eye of the observer. Remove mirror and pins, join BD, and, by direct measurement with a pair of compasses or a scale, show that the distances of the image and the object from the reflecting surface are equal, and that the line joining them is perpendicular to the reflecting surface.

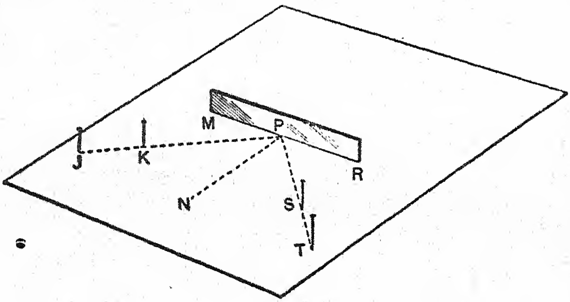


FIG. 15.

(*b*) Mount the thin glass mirror as before. Place two pins, J and K (Fig. 15), almost anywhere in front of the mirror, and, looking into the mirror, move the eye about until the images are in line. Place two more pins, S and T, so as to be in line with these images. Remove mirror and pins. Join JK and ST, and produce them. They will intersect at a point, P, on MR. Draw the normal, PN, at P, and, by direct measurement, prove the equality of the angles, JPN, TPN.

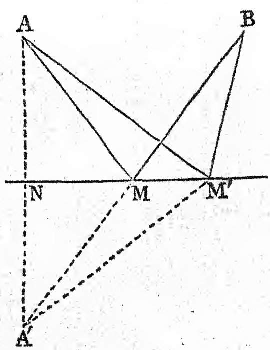


FIG. 16.

Example.—*A ray of light starts from* A, *meets a plane reflecting surface at* M, *and is reflected to* B. *Prove that* AMB *is the shortest possible path from* A *to* B *by way of the mirror* (Fig. 16).

If AMB is not the shortest path, let *any* other path, AM′B, be shorter. Draw ANA′ normal to the mirror, and produce BM to meet AN in A′. Then:—

$$\text{Since } AN = A'N, \text{ we have } AM = A'M \text{ and } AM' = A'M'.$$

$$\text{But, } A'M' + M'B > A'B > A'M + MB.$$

$$\text{Therefore, } AM' + M'B > AM + MB.$$

6. Reflection of a Convergent Pencil by a Plane Mirror

The preceding article deals with the reflection of a divergent pencil of light, and shows that, after reflection, such a pencil appears to diverge from a point at the same distance behind the mirror as that from which it diverges originally is in front of it.

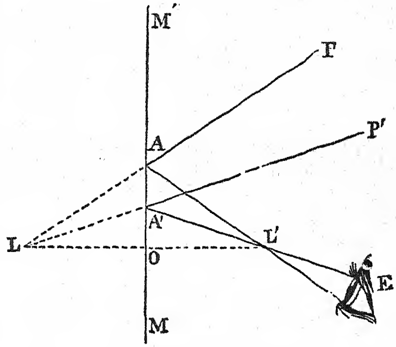


FIG. 17.

Similarly, if a convergent pencil, PLP′ (Fig. 17), converging to a point, L, behind the mirror, be incident at AA′, it is reflected so as to converge to a point, L′, such that LOL′ is normal to the mirror, and OL = OL′. This can be proved in exactly the same way as in Art. 5. An eye placed at E sees a *real* image of L at L′. V.P. Singh

7. Image of a Luminous Object formed by a Plane Mirror

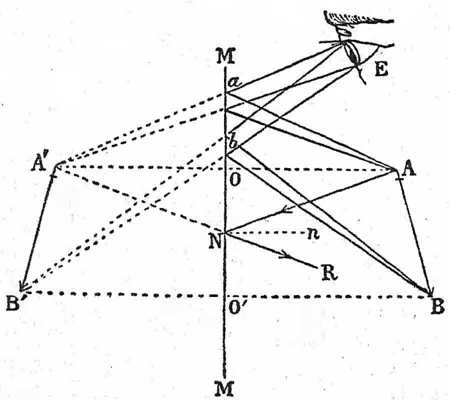


FIG. 18.

Let AB (Fig. 18) represent a luminous object placed in front of the mirror, MM. As in Art. 5, the image of A is formed at A′, such that AOA′ is normal to MM, and AO = OA′. Similarly, the image of B is formed at B′, such that BO′B′ is normal to MM, and BO′ = O′B′. For all points of the object intermediate between A and B, images are formed at corresponding points between A′ and B′, and thus a complete image of the object is formed, A′B′.

A more elaborate construction is used sometimes for determining the position of the image of an object formed by a plane mirror,

and, as the method is general and applicable to spherical mirrors (page 47), it will be given here. It is based on the fact that the intersection of any two reflected rays determines the point on the image from which they diverge, or appear to diverge.

For plane mirrors, the two rays chosen are AO (Fig. 18), incident normally to the mirror, and any other ray, AN, incident in any other direction. The ray, AO, is reflected back along its original path, and AN is reflected along NR, making the angle of incidence, AN*n*, equal to the angle of reflection, RN*n*. The image of A is formed at A′, the *virtual* focus of the reflected rays, OA and NR.

Similarly, the image of any other point, B, is obtained, and the images of intermediate points are assumed to lie on the line, A′B′, and hence A′B′ is said to be the image of AB. When the form of the object is more complex than that considered here, the images of a number of points, sufficient to determine the complex image, must be obtained.

An eye placed at any point, E (Fig. 18), in front of the mirror, sees the image by light reflected from the portion, *ab*, of the surface of the mirror, and the actual paths of the *extreme* rays are shown by the lines, A*a*E, B*b*E. It is evident from this that, in order that any point of an image may be seen, the line joining this point to the eye must cut the surface of the mirror, and that the portion of the surface, at which an image is seen by reflection, is that portion which is intercepted by the cone of light having the eye at its base and the image at its apex.

8. Path of Rays by which an Image is Seen

Let L′ (Fig. 13) represent the image of a luminous point, L, formed by the mirror, MM, and imagine an eye placed at E. Draw lines joining L′ to the extremities, *ee*, of the aperture of the eye, and cutting the mirror at *a* and *b*. Join L to *a* and *b*. Then, the lines, L*ae* and L*be*, define the pencil of light by which L′ is seen (cf. Art. 7). Each

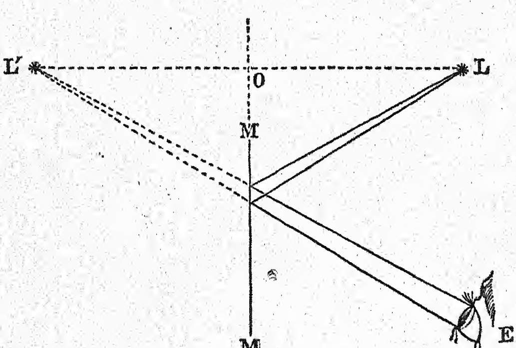


FIG. 19.

point of the image of a luminous object is seen in this way, and the extreme rays bounding the collection of pencils of light reaching the eye are determined as indicated by A*a*E, B*b*E (Fig. 18).

FIG. 19*a*.

In connexion with this question, it is important to notice what must be the position of an object relative to a mirror in order that an image may be formed by that mirror.

Let MM (Fig. 19) represent a plane mirror. Then, if an object, L, be placed anywhere in front of the plane passing through MM, an image of that object will be formed behind this plane at a point, L′, such that LOL′ is normal to the plane and LO = OL′. This can be proved in the same way as in Art. 5. The figure, which corresponds to Fig. 13, shows the necessary construction, and also the paths of the rays by which an eye placed at E is able to see the image, L′.

9. Lateral Inversion

When the image of the face is seen in an ordinary plane mirror, the image of the right eye forms the left eye of the reflected face, while the image of the left eye forms its right eye, and so on.

This is a particular instance of the result of reflection known as *lateral inversion*. It does not affect the appearance of objects which are symmetrical bi-laterally, but with non-symmetrical objects, such as printed or written characters, the effect is sufficiently evident and well-known (Fig. 19 (*a*)).

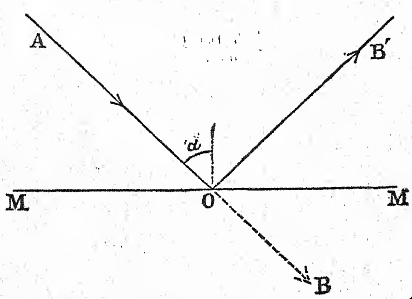


FIG. 20.

10. Deviation

When a ray of light is turned out of its original course, it is said to suffer deviation, and the angle between its initial and final directions determines the amount of this deviation.

The deviation due to a single reflection at a plane surface is determined easily.

Let AO (Fig. 20) be incident on the reflecting surface, MM, at an angle, α, to the normal. Then, since the initial direction of the ray is represented by AB, and its final direction by OB′, the deviation is given by the angle, BOB′.

$$\text{But, angle BOB}' = 180° - \text{AOB}' = 180° - 2\alpha,$$

$$\textit{i.e.}\ \text{Deviation} = 180° - 2\alpha.$$

11. Reflection from a Rotating Mirror

Let NA (Fig. 21) represent a ray of light incident normally on the mirror, MM. If the position of the mirror remain unchanged, then NA will be reflected back along AN. If, however, MM be rotated in the direction shown by the arrows about an axis at A, into the position, M′M′, then NA will be reflected along AN′ according to the laws of reflection. Now:—

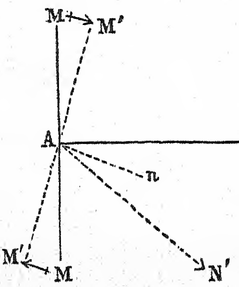


FIG. 21.

$$\text{Angle NA}n = \text{angle MAM}'.$$

Hence, angle NAN′

$$= 2 \text{ angle NA}n = 2 \text{ angle MAM}'.$$

But, NAN′ is the angle through which the reflected ray has been rotated on account of the rotation of the mirror through the angle, MAM′. Hence, *if a mirror be rotated through an angle, α, the reflected ray is rotated through an angle, 2α.* Or, the reflected ray rotates twice as rapidly as the mirror from which it is reflected.

This result finds important application in the measurement of small angular deflections. The angle is too small to be measured directly by pointer and graduated circle, hence a mirror, MM (Fig. 22), is fixed to the rotating system or suspension wires, and the angle, NAN′, is measured instead.

In practice, a telescope and scale are employed. When the mirror is perpendicular to AN, the scale divisions at N, which is just below the telescope, are in the field of view of T. As MM rotates, the scale appears to travel across the field of view, and when MM has reached the position, M′M′, the division, N′, is seen on the cross-wire of the telescope. Now:—

$$\text{Angle NAN}' = 2 \text{ angle MAM}'.$$

$$\text{Hence, angle MAM}' = \tfrac{1}{2} \text{ angle NAN}' = \tfrac{1}{2} \tan^{-1}\frac{\text{NN}'}{\text{AN}}.$$

If the angle is small, the tangent is equal to its circular measure;

$$\therefore \text{ angle MAM}' = \frac{NN'}{2AN}.$$

Since NN′ and AN can be measured accurately, the angle, MAM′, is determined easily.

The method just described is sometimes called Poggendorf's method, or the *subjective* method, and is used largely on the Continent. The most usual practice in England, however, is to use a concave spherical mirror (page 51), by which a real image of a spot of light is focused on the scale. This is an *objective* method.

Another application of this principle is made in the sextant (page 175), an instrument employed for measuring angular elevations.

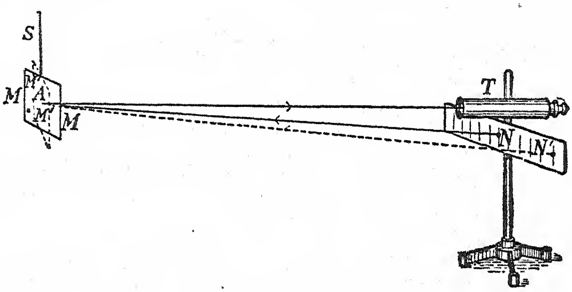


FIG. 22.

12. Reflections at Plane Surfaces Inclined to Each Other

Before considering particular cases of multiple reflections of special interest, it will simplify matters greatly to notice the general principles applicable to all cases.

Imagine an object, A, placed between two plane mirrors, M_1 and M_2, inclined to each other at any angle. An image of A will be formed by each mirror. If the image formed by M_1 lies in front of M_2—that is, if it is anywhere in front of the plane in which this mirror lies (Art. 8)—then an image of this image will be formed by M_2. Similarly, if the image formed by M_2 lies in front of M_1, then an image of this image will be formed by M_1. Such images are said to be images of the second order. In precisely the same way, if this second pair of images are suitably placed, a third pair, of the third order, may be formed, and so on. This multiplication of images stops when a pair is formed in the space behind both mirrors—that is, within the angle vertically opposite that in which the object is placed. A few special cases will now be considered.

(1) PARALLEL MIRRORS.—Let M_1 and M_2 (Fig. 23) represent two parallel plane mirrors, and A an object between them. It is evident that, since the mirrors are parallel, no image can be formed behind both, and hence every pair of images gives rise to another pair. Thus, theoretically, an infinite series of images may be formed.

Through A draw N_1N_2 normal to both mirrors, and produce it indefinitely on both sides. In obedience to the laws of reflection, all the images must lie on this line, and their positions will depend on the position of A between M_1 and M_2, and on the distance between the mirrors.

Consider first the reflection from M_1. An image of A is formed at A_1' on the normal through A, and so that $A_1'N_1 = AN_1$. Similarly, an image of A_1' is formed by M_2 at A_1'' in such a position that $A_1'N_2 = A_1''N_2$. A_1'' in turn gives rise to A_1''' by reflection at M_1, and so on. In the same way, beginning with the first reflection at M_2, the images A_2', A_2'', etc., are formed by successive reflections at M_2 and M_1.

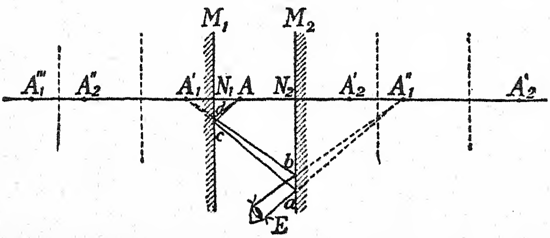


FIG. 23.

In Fig. 23, the positions of the images up to the third order are shown, and to distinguish them the suffix attached to A denotes the mirror at which the *first* reflection took place and the dash indicates the order of the image. Thus the series A_1', A_1'', A_1''' . . . is formed by successive reflections from M_1 and M_2, beginning with M_1. Similarly, the series A_2', A_2'', A_2''' . . . is formed by successive reflections from M_2 and M_1, beginning with M_2. The members of each series are so related that any one may be considered as the image of the one preceding it in the series. For example, A_1''' may be considered as the image of A_1'' formed by M_1, and consequently $A_1'''N_1 = A_1''N_1$.

To determine the paths of rays by which any image is seen, the following construction should be employed. Let it be required to find the paths of the rays by which an eye at E sees the image, A_1''. First trace this image back to A; A_1'' is an image of A_1', which is itself an image of A. Now join the extremities of the aperture of the eye to A_1'' by lines cutting M_2 at *a* and *b*, and mark the real parts of these paths which, since the rays cannot penetrate the

mirror, must lie between the eye and *a*, *b*. Next join *a* and *b* to A_1' by lines cutting M_1 in *c* and *d*, and mark *ac*, *bd* as the real portions of these paths. Then finally join *c* and *d* to A, and the twice reflected rays passing from A to E indicate the required pencil.

From this it is evident that an image of the *second* order, A_1'', is seen by *two* reflections, and similarly an image of the *n*th order would be seen by *n* reflections. The mirror from which the last reflection takes place—that is, the mirror in which the image is seen—depends on whether *n* is odd or even. In either series, the *odd* numbers are seen by reflection from the mirror at which the *first* reflection takes place, while the *even* numbers are seen in the other mirror.

At each reflection there is some loss of light, depending on the polish of the reflecting surface, and, as a consequence, the higher the order of any image the fainter it appears, until finally it becomes too faint to be visible.

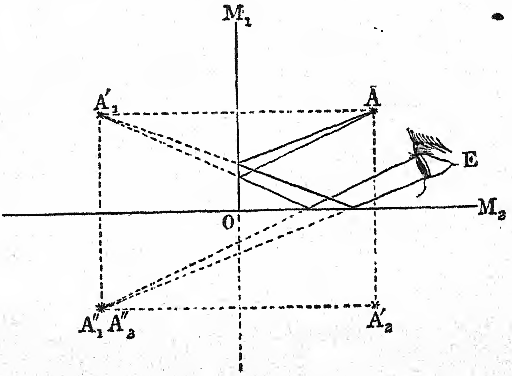


FIG. 24.

(2) MIRRORS INCLINED AT RIGHT ANGLES.—Let OM_1 and OM_2 (Fig. 24) represent two plane mirrors at right angles to each other, and A an object placed between them. Then, an image, A_1', is formed by OM_1, and A_2' by OM_2. But A_1' lies in front of OM_2, and therefore an image, A_1'', is formed by that mirror. Also, A_2' is in front of OM_1, and therefore gives rise to the image, A_2'', which from the geometry of the figure is coincident with A_1''. An eye placed anywhere within the proper limits (Art. 7) sees three images, A_1', A_1'' or A_2'', and A_2', at the three corners of the rectangle, $A_1'A_2'$ *Both* the images, A_1'' and A_2'', cannot be seen at the same time. An eye placed within the angle, M_1OA, sees A_2'', while one placed in the angle, M_2OA, sees A_1''.

The figure shows the paths of the rays by which the image, A_1'' may be seen by an eye placed at E. The method of determining these paths is indicated by the dotted lines, and is similar to that

described above for parallel mirrors. The actual paths of the rays necessarily lie within the angle, M_1OM_2.

(3) MIRRORS INCLINED AT ANY ANGLE.—Let OM_1 and OM_2 (Fig. 25) represent two plane mirrors inclined at the angle, M_1OM_2, and A an object placed between them. With O as centre, describe a circle of radius, OA, cutting OM_1 and OM_2 in N_1 and N_2 respectively. Then all the images of A must lie on the circumference of this circle.

Consider the image, A_1'. According to the laws of reflection, it is so placed that $An_1 = A_1'n_1$, and AA_1' is perpendicular to OM_1. Hence, in the triangles, OAn_1 and $OA_1'n_1$, $An_1 = A_1'n_1$, n_1O is common, and angle An_1O = angle $A_1'n_1O$. Therefore, $OA = OA_1'$, and A_1' lies on the circumference of the circle passing through A. Similarly, any other image lies on this circle.

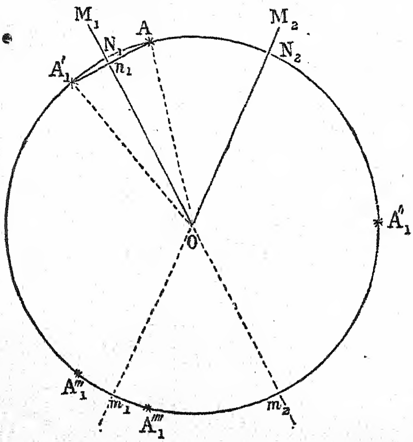


FIG. 25.

The formation of the images is exactly similar to that described above for parallel mirrors—that is, each mirror gives rise to a separate series of images. The number of images which can be formed, however, in this case is limited, the last member in each series being that formed within the angle, m_1Om_2.

The number of images which may be formed is determined as follows: Let the angle, M_1OM_2, between the mirrors be θ, the angle, AOM_1, be α, and the angle, AOM_2, be β. Then:—

(*a*) The number of images in the series, A_1', A_1'', A_1''' . . . is given by the integer next greater than $\frac{\pi - \alpha}{\theta}$, and

(*b*) The number of images in the series, A_2', A_2'', A_2''' . . . is given by the integer next greater than $\frac{\pi - \beta}{\theta}$.

Fig. 25 shows the positions of the members of the series formed by first reflection from OM_1.

If θ be an exact sub-multiple of π—that is, 15°, 30°, 45° . . . ,

then $\frac{\theta}{\pi}$ is an integer, and therefore the number of images in each series will be $\frac{\pi}{\theta}$, for in these cases $\frac{\pi}{\theta}$ is the integer next greater than $\frac{\pi - \alpha}{\theta}$ and $\frac{\pi - \beta}{\theta}$. The total number of images in both series is therefore $\frac{2\pi}{\theta}$. In such cases, however, when θ is an exact sub-multiple of π, *the last images in the two series*, formed on the arc, $m_1 m_2$, *coincide in position*, and therefore the actual total number of images *seen* is not $\frac{2\pi}{\theta}$ but $\frac{2\pi}{\theta} - 1$.

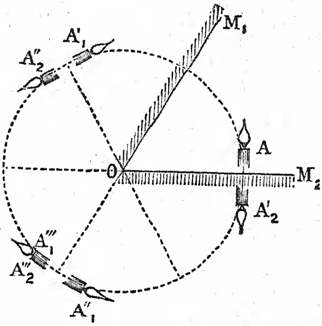


FIG. 26 (*a*).

Thus, in Fig. 24, the angle, θ, between the mirrors is 90°, an exact sub-multiple of π, and the number of images is $\frac{360}{90} - 1 = 3$.

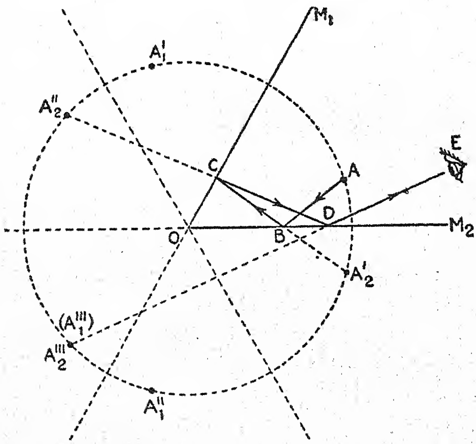


FIG. 26 (*b*).

Similarly, in Fig. 26 (*a*), θ is 60°, and the number of images is

$$\frac{360}{60} - 1 = 5.$$

In fact, a consideration of (*a*) and (*b*) above will show that if $\frac{360}{\theta}$ is an *even* integer, the number of images is $\frac{360}{\theta} - 1$; but if $\frac{360}{\theta}$ is an *odd* integer, the number is $\frac{360}{\theta}$, unless the object is midway between the mirrors, in which case the number is again $\frac{360}{\theta} - 1$.

Experiment. Images formed by inclined mirrors.—Place two mirrors, OM_1 and OM_2 [Fig. 26 (*b*)], on a sheet of cartridge paper at

an angle of 60° with each other, and between them press a pin, A, into the paper so that it stands upright. Looking into the mirrors, a series of images, A_1', A_2'', A_2''', A_1''', A_1'', A_2', will be seen. Locate the positions of these images by other pins. Now remove the mirrors and the pins, and prove that (1) all the images lie on a circle whose centre is O and radius OA, (2) the angles, AOA_1', $A_2''OA_2'''$, $A_2'OA_1''$, are equal, and (3) the angles, AOA_2', $A_1''\,OA_2'''$, $A_2''OA_1'$ are equal also. Show also that if an eye be placed at E, the path of a ray of light apparently coming from the image, A_2''', is ABCDE. The construction is obvious.

Example.—*An object is placed between two mirrors inclined at an angle of 60°. Find how many images are formed, and show that the images formed in the angle vertically opposite that contained by the mirrors are coincident.*

The conditions of this example are represented in Fig. 26 (*b*). Since 60° is an aliquot part of 360°, the number of images formed is given by:—

$$n = \frac{2\pi}{\theta} - 1 = \frac{360}{60} - 1 = 5.$$

Also, A_1''' and A_2''' are the images to be shown coincident. For this purpose it must be proved that the angles, AOA_1''' and AOA_2''', measured in *opposite* directions, together equal 360°. If angle $AOM_1 = \alpha$ and angle $AOM_2 = \beta$, then:—

$$AOA_1''' = 2\alpha + 2 \times 60° = 2\alpha + 120°,$$

$$\text{and } AOA_2''' = 2\beta + 2 \times 60° = 2\beta + 120°;$$

$$\therefore\ AOA_1''' + AOA_2''' = 240° + 2(\alpha + \beta) = 240° + 120° = 360°.$$

The symmetrical distribution of images obtained by repeated reflection between two mirrors, when they are inclined at an angle which is an exact submultiple of 180°, is the principle of the kaleidoscope. Two long narrow plane mirrors, inclined to each other at an angle of 60°, are placed in a slightly longer tube. One end of the tube is closed by a metal disc, pierced at the centre with a hole through which the observer looks. At the other end of the tube a plate of clear glass fits into the tube close up to the mirrors, and a short distance beyond it, at the end of the tube, is a similar plate of ground glass. Between these two plates small pieces of coloured glass are loosely placed, which, with their images, form beautiful and symmetrical patterns visible to an eye placed at the other end of the tube. On rotating the tube, the pieces of glass change position, and thus the pattern seen is changing continually [Fig. 26 (*a*)].

Sometimes three mirrors are employed, the arrangement being such that the cross-section of the three is an equilateral triangle.

Each pair of mirrors acts in the way described above, so that the arrangement gives rise to intricate, though symmetrical, patterns, which are capable of giving material aid to designers.

13. Deviation by Successive Reflection from Two Inclined Mirrors

Let OM_1 and OM_2 (Fig. 27) represent two plane mirrors inclined at an angle, α, represented by M_1OM_2, and let ABCDE . . . represent a ray of light reflected successively from OM_1 and OM_2 at the points, B, C, D, E . . . Also, let ϕ_1, ϕ_2, ϕ_3, . . . denote the angles which the incident and reflected rays make with the reflecting surfaces at the points, B, C, D, E, . . . respectively.

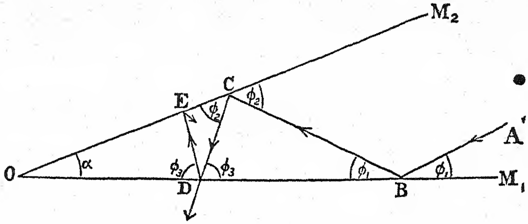


FIG. 27.

Then, from the triangle BCO,

$$\phi_2 = \phi_1 + \alpha,$$

or $\phi_2 - \phi_1 = \alpha$.

Similarly,

$\phi_3 - \phi_2 = \alpha$, and so on. Writing these equations in order:—

$$\left.\begin{array}{r} \phi_2 - \phi_1 = \alpha, \\ \phi_3 - \phi_2 = \alpha, \\ \cdots\cdots\cdots \\ \phi_{n+1} - \phi_n = \alpha \end{array}\right\} \text{By addition } \phi_{n+1} - \phi_1 = n\alpha$$

But if n be *even*, then the angles ϕ_{n+1} and ϕ_1 are measured from the *same surface*, and therefore the difference must give the required deviation, for the ray is inclined to the reflecting surface initially at an angle, ϕ_1, and to the same surface finally at an angle, ϕ_{n+1}. Thus, when $n = 2$, then $\phi_{2+1} = \phi_3$ and ϕ_1 are at the same surface, OM_1 (Fig. 27); further, the original direction of the light is AB, and the final direction, after two reflections, is CD. The deviation is the angle between these two directions, and is clearly $\phi_2 - \phi_1 = 2\alpha$.

Hence, if α is the angle between the mirrors, then in general, when n is even, the deviation produced by n reflections is given by:—

$$\text{Deviation} = n\alpha.$$

Again, when n is odd, the deviation is that due to the $(n - 1)$th *even* reflection and the nth *odd* reflection, and is given by:—

$$\text{Deviation} = (n - 1)\,\alpha - 2\phi_n.$$

The minus sign is necessary because the deviation, ϕ_n, is in a direction opposite to that denoted by $(n - 1)\,\alpha$. Thus, after the *third* reflection (Fig. 27), the deviation is $2\alpha - 2\phi_3$.

When ϕ_n becomes greater than $\frac{\pi}{2}$, or 90°, the ray begins to travel outwards from the intersection of the mirrors, not generally following its original path. But, if $\phi_n = \frac{\pi}{2}$, then the ray travels back along the path by which it came. At each reflection the value of ϕ is increased by α, and hence, in order that ϕ_n may be equal to $\frac{\pi}{2}$, ϕ_1 must be chosen so that α is an exact sub-multiple of $\left(\frac{\pi}{2} - \phi_1\right)$. Thus, if $\alpha = 10°$ and $\phi_1 = 20°$, then the ray will return along its original path after $(90 - 20)/10 = 7$ reflections after the first.

When a ray is reflected twice, as in Hadley's sextant (page 175), the deviation is twice the angle between the mirrors.

Example.—*What must be the angle between two plane mirrors in order that a ray incident parallel to one of them may, after two reflections, be parallel to the other?*

Let α be the angle between the mirrors. Then, after two reflections, the deviation produced $= 2\alpha$. But, the deviation required $= 180° - \alpha$. Hence:—

$$2\alpha = 180° - \alpha;$$

$$\therefore\ 3\alpha = 180°, \text{ or } \alpha = 60°.$$

14. Irregular or Diffuse Reflection

When a parallel beam of light from a bright source in a dark room falls on a piece of white card, the light after incidence is not confined to one course, but is scattered in all directions. From anywhere in front the card is brightly visible, and the room is no longer wholly dark. If a mirror had been used, almost all the light would have been reflected in some definite direction.

At first sight there appears to be a great difference between the two phenomena, and it has been sought to explain the behaviour of the card by comparing it with a mirror with many small facets, which reflect the light quite regularly but in different directions, because the facets are at different angles. This phenomenon does in fact occur when light is reflected from water on the surface of which are ripples, and accounts for the wide luminous path seen on

a lake in sunlight and moonlight. In the case of the card, however, the fact is that diffusion, not reflection, is the fundamental phenomenon. Diffusion therefore cannot be explained by reflection, though the latter is a consequence of the former.

A full explanation of this cannot be given here, but the essential fact is that light consists of a series of waves. Waves striking any obstacle are always scattered in all directions. This scattering, however, produces a regularly reflected wave whenever the obstacles are ranged in a surface whose inequalities are small compared with the wave-length. Thus, the sound waves from the tick of a watch have a wave-length of about one inch, and are regularly reflected by a surface whose inequalities are less than, say, half an inch. Water waves on the sea may have a wave-length of one hundred feet or more, and are reflected regularly by a somewhat irregular coast-line. Sound waves of wave-length six feet are reflected so as to produce a true echo from a hedge or cliff. Light waves have a wave-length of 40 to 75 millionths of a centimetre, and are reflected regularly from polished metal or glass, or the surface of a liquid, in which the inequalities are of this order of magnitude, but not from cardboard, in which they are larger.

By means of the cardboard, however, another consequence of the wave theory of light can be proved—that a ray of light is reflected fairly regularly if it strikes the surface at very oblique incidence, so as to be almost parallel to the surface, even if the surface be rough (see Art. 15).

Twilight is explicable by diffusion. Clouds, dust, and other floating particles in the atmosphere are illuminated by the sun some time after it has set at any particular place. These scatter the light in all directions, some of the scattered rays reaching the earth, illuminating it, some time after sunset. Moreover, some of the scattered light is transmitted to other particles in the atmosphere farther away from the sun, and these scatter the rays a second time, still further increasing the duration of twilight. Twilight is said to end when this scattered light becomes imperceptible. By observation, this has been found to occur when the sun is about 18° below the horizon. (See also p. 83 for effect of refraction.)

If the earth had no atmosphere, surfaces on it exposed to the sun's rays would be dazzlingly bright, whilst other surfaces would be in black shadow, except such parts which might be illuminated by reflection and diffusion from surrounding surfaces. This state actually exists on the moon, where the contrast between light and shade, and the sharpness of shadows, are extremely great.

It is only by means of *scattered* light that all bodies, except self-luminous ones, are made visible. The scattering of the light of the sun by white clouds is the cause of the difference between ordinary daylight, with its soft gradations of light and shade, and direct sunlight, with its intense light and deep shadows. The effect of scattering on the colour of light will be dealt with later (see page 173).

15. Plane Surfaces

It is extremely difficult to make accurately plane surfaces. The simplest test of flatness which can be applied is that of reflection.

Experiment. Make a smooth circular hole in a piece of tinfoil, and place it in front of a bright source of light, L (Fig. 28). Fix a telescope, T, in a stand at a distance from L, and focus the telescope on the hole. On the bench midway between L and T lay the

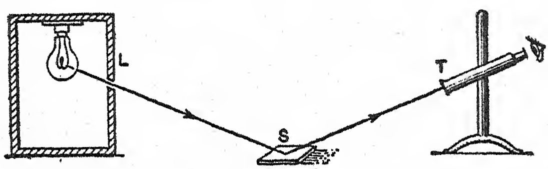


FIG. 28.

surface, S, to be tested, and tilt the telescope until the image of the hole by reflection from the surface is seen.

If the surface is plane, the image will be sharp, if irregular the image will be ill-defined and may spread out into a large and irregular blur similar to the image of the moon in a lake.

Test various kinds of glass, from ordinary window glass to optical plane glass.

The experiment may be varied by holding the surface horizontally in the hand, just below the level of the eye, and viewing the images formed by reflection of the bars of a well-lighted window frame. If the bars appear sharp and straight the surface is plane, but if wavy and crumpled the surface is irregular.

The proportion of the incident light reflected at a surface depends largely on the nature of the media in contact, on the state of polish of the surface, and on the angle of incidence.

By photometrical experiments (see page 270) it has been found that polished silver reflects about 90 per cent., and a clean mercury

surface about 67 per cent., of a direct incident beam in air. Transparent substances reflect much less, a polished glass-air surface reflecting only 4 per cent., and a water-air surface only 2 per cent. of a direct incident beam. When, however, the angle of incidence is 89·5°, both water and mercury reflect the same proportion of the incident beam, 72 per cent.

Even with an optically rough surface, such as that of smoked glass, paper, or an ordinary coin, a very good image of an object can be obtained by reflection at nearly grazing incidence.

V. P. Singh

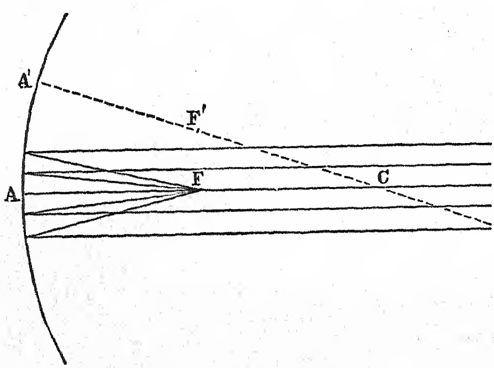
CHAPTER IV

REFLECTION AT SPHERICAL SURFACES

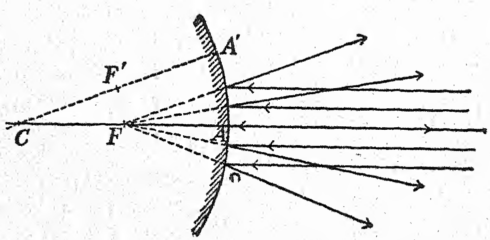
IN the last chapter the laws of reflection as applied to plane reflecting surfaces were dealt with. In optical instruments, however, mirrors of spherical, parabolic, and cylindrical curvature are sometimes used. The discussion in this chapter will be confined to spherical mirrors.

1. Definitions

A spherical mirror, AA′ (Fig. 29), is usually a very small segment of a spherical surface, and may be either *concave* (A) if the inside portion acts as the reflecting surface, or *convex* (B) if the outside portion acts as the reflecting surface. The centre, C, of the spherical surface of which the mirror is a part, is called the **centre of curvature** of the mirror. The line, CA, joining the centre of curvature and the central point, A, of the mirror, is the **principal axis** of the mirror, the point, A, being sometimes called the **pole** of the mirror, or *centre of the face*. Any other line, CA′, drawn through C and cutting the mirror is called a *secondary axis*, and, like the principal axis, is *normal* to the mirror. A section of the mirror by a plane passing through the pole and the centre of curvature is called a *principal section*.



A



B

FIG. 29.

The aperture of the mirror is the angle enclosed by two straight lines drawn from the centre of curvature to opposite points in the edge of the mirror. The discussion will be limited at first to mirrors of small aperture—not exceeding 10°, say—though for the sake of clearness the diagrams will show greater apertures.

When a parallel pencil of light is incident on a spherical mirror, in a direction parallel to the principal axis, the reflected pencil converges to, or *appears* to diverge from, a point, F, on the principal axis. This point is called the **principal focus** of the mirror, and the distance, AF, between the pole and the principal focus of the mirror is termed the **focal length** of the mirror. If the pencil is incident parallel to a secondary axis, the reflected rays are brought, in a similar way, to a focus at a point, F′, on that axis.

In the case of spherical mirrors, it is not strictly true to say that the reflected rays meet accurately at, or appear to diverge accurately from, a point. If the pencil is small this is approximately the case,

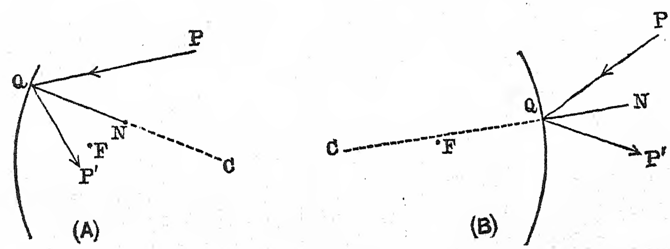


FIG. 30.

but with large pencils the outer rays are reflected to, or appear to diverge from, points nearer the mirror than the others. This irregularity of reflection from a spherical surface is called *spherical aberration* (see Art. 8).

2. Construction for Reflected Ray

Let PQ (Fig. 30) be any ray incident at a point, Q, on a spherical mirror, concave (A) or convex (B). At Q draw the normal, QN, to the reflecting surface, by joining CQ and producing it if necessary. Then, in accordance with the laws of reflection, the reflected ray, QP′, is obtained by drawing QP′ in such a direction that the angle of incidence, PQN, is equal to the angle of reflection, P′QN.

From this it is evident that a ray incident along a normal is reflected back along its original path. Also (Art. 1) a ray incident

parallel to the principal axis is reflected through the principal focus. These two particular cases of reflection should be remembered carefully.

3. Position of Principal Focus

Let PQ (Fig. 31) be a ray incident on the concave mirror, AQ, in a direction parallel to the principal axis, CA. Then, PQ is reflected

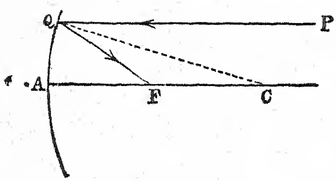


FIG. 31.

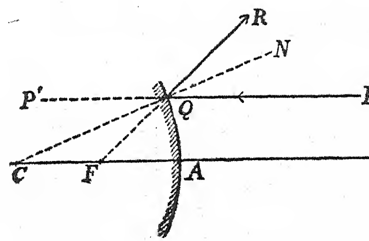


FIG. 32.

through the principal focus, F, and the angle, PQC, is equal to the angle, FQC. But, the angle, PQC, is equal to the angle, FCQ. Therefore, the angle, FQC, equals the angle, FCQ, and FQ = FC.

Now, if AQ is small, FQ is approximately equal to FA, and therefore FC = FA. Or, the principal focus, F, is midway between the pole, A, and the centre of curvature, C; and, if AF be denoted by f, and AC by r, then $f = \frac{r}{2}$. Or, *the focal length of a spherical mirror, for rays incident on a small portion of its surface near the pole, is equal to half the radius of curvature of that mirror.*

An exactly similar proof is applicable in the case of a convex mirror. The principal focus, F (Fig. 32), is in this case on the backward prolongation of the reflected ray, QR, and is, of course, *virtual.*

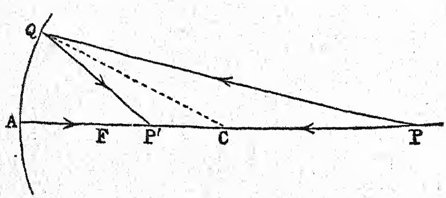


FIG. 33.

4. Conjugate Foci. Concave Mirror

Let P (Fig. 33) represent the position of a luminous point on the principal axis of the concave mirror, AQ. Then the image of P will be formed at the intersection, after reflection, of any two

rays coming from P (see page 22). Consider the rays, PA, incident along the normal to the mirror, and PQ, incident at Q. PA is reflected back along AP, and PQ is reflected along QP′, making the angle of reflection, P′QC, equal to the angle of incidence, PQC. Let the reflected rays, AP and QP′, intersect at P′. Then, P′ is the image of P, and lies on the principal axis of the mirror. Also, since angle P′QC = angle PQC,

$$\frac{QP'}{QP} = \frac{P'C}{CP}.$$

But, if AQ is small, then PQ = PA, and P′Q = P′A, and therefore

$$\frac{AP'}{AP} = \frac{P'C}{CP}.$$

If now AC be denoted by r, AP by u, and AP′ by v, then P′C = AC − AP′ = $r - v$, and CP = AP − AC = $u - r$. Thus, the above proportion becomes

$$\frac{v}{u} = \frac{r-v}{u-r}; \quad \therefore u(r-v) = v(u-r),$$

$$\text{and } ur + vr = 2uv.$$

$$\text{Dividing by } urv, \ \frac{1}{v} + \frac{1}{u} = \frac{2}{r}.$$

$$\text{But (Art. 3), } f = \frac{r}{2}, \text{ hence, } \frac{2}{r} = \frac{1}{f};$$

$$\therefore \frac{1}{v} + \frac{1}{u} = \frac{1}{f} = \frac{2}{r},$$

where u denotes the distance of the luminous point, P, from the pole of the mirror, v denotes the distance of P′, the image of P, from the pole of the mirror, r denotes the radius of curvature of the mirror, and f denotes the focal length of the mirror.

This may be expressed in words by saying that *the algebraic sum of the reciprocals of the distances of the luminous point and its image from the pole of the mirror equals the reciprocal of the distance of the principal focus from the pole.* The relation thus obtained is of great importance.

The points, P and P′, connected by this relation are said to be **conjugate foci,** because of the fact that either point may be considered as the image of the other. From the construction it is evident that the image of a luminous point at P′ would be formed at P, just in the same way as the image of P is formed at P′.

This may be illustrated experimentally by means of a luminous object and a concave mirror. If the luminous object be placed on the principal axis of the mirror at any point beyond C, the centre of curvature, an image of the object will be seen between C and the mirror. The position of this image can be marked by adjusting the position of a needle until it appears to coincide with the image. It will then be found that if the luminous object be placed at the point marked by the needle, the image will be seen at the point originally occupied by the object. This is an instance of the reversibility of light (see page 16).

If the luminous point, P, is not on the principal axis, then its conjugate focus, P′, will be on the secondary axis passing through P, and, distances being measured along this axis, the relation $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$, can be established in the way described above. In fact the two cases are identical, for the geometrical relations of a secondary axis to a spherical mirror are exactly the same as those of the principal axis.

5. Convention of Signs

It will be noticed that in the case considered above (Art. 4) all distances are measured in the same direction from the pole of the mirror, A. This is not the case in general, and we must make some convention as to sign in order to make our formulae sufficiently general. Unfortunately several conventions are in use, and the student should make a practice of stating quite clearly which one he is using.

This is not the place to discuss the merits of the various systems. It is emphasised that all the results of geometrical optics can be obtained using *any one convention throughout*. The results discussed in this book will be obtained in terms of two of the most usually adopted conventions, matter being duplicated wherever this is necessary for clarity. Matter that has been so duplicated will be marked thus * at beginning and end. The reader who wishes to use the first convention to be described below can ignore these signs. The reader using the "Real is Positive" convention should read the alternative passages. The student may therefore adopt whichever of these two conventions he prefers, and may verify that identical results are obtained whichever convention is adopted. The first convention we shall use is the one that lends itself best to illustration by means of diagrams. The diagrams in the book have been so

drawn that distances, radii of curvature, etc., measured to the right from the pole of the mirror are positive, while distances measured to the left are negative, according to the convention. We may state the convention thus: *Distances measured in a direction* opposite *to that in which the incident light is travelling are reckoned* positive; *distances measured in the* same *direction as that of the incident light are reckoned* negative.

If the incident light comes from the *right*, the convention is just the same as that ordinarily used in drawing graphs, and the mirror and lens diagrams in the book are all drawn in this fashion to assist the student. The curvature of reflecting and refracting surfaces has been exaggerated in order to show quite clearly the sign of the radius of curvature in each case. With this convention, concave mirrors have positive radii of curvature and focal lengths, while convex mirrors have negative radii of curvature and focal lengths.

The second convention that we shall use, though it does not lend itself quite so well to graphical illustration as does the one described above, has certain advantages and seems to be gaining in popularity. This alternative convention is usually stated in three words thus: **Real is positive.** It was found above (Art. 4) that a concave mirror usually produces a real image in front of the mirror (positive), and it will be found below (Art. 6) that a convex mirror usually produces a virtual image behind the mirror (negative). Applying this to the case of parallel light, a concave mirror has a positive focal length (real focus), while a convex mirror has a negative focal length (virtual focus). The sign of the radius of curvature of a mirror is always reckoned the same as its focal length. *As far as* mirrors *are concerned, therefore, the two conventions are identical, positive distances being measured in front of the mirror, negative distances behind the mirror.* However, when one is concerned with refracting surfaces and with lenses the two conventions are different. The application of them will be described in Chapter VII.

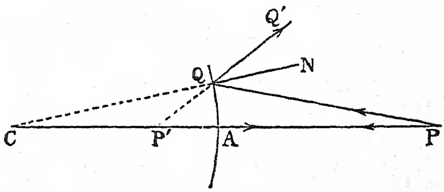


FIG. 34.

It may be verified that the formula $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$, paying proper attention to the signs, is valid for all positions of the object and

image and for convex and concave mirrors. Without the sign convention each type of case discussed below (Art. 7) would have to be considered separately.

6. Conjugate Foci. Convex Mirror

The same relation for conjugate foci, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$, can be proved for a convex reflecting surface. Let P (Fig. 34) represent a luminous point placed in front of the convex mirror, AQ. Then, as in the case of the concave mirror, an image of P is formed at P′ on the axis passing through P. In this case, P′ is a *virtual* image, from which the reflected rays, AP and QQ′, appear to diverge.

To determine the position of P′, from the construction, we have the relation:—

$$\frac{QP'}{QP} = \frac{P'C}{PC}.$$

Also, if AQ be small, this proportion becomes:—

$$\frac{AP'}{AP} = \frac{P'C}{PC} = \frac{AC - AP'}{AC + AP}.$$

*Adopting the first sign convention given above (Art. 5), AP must be considered as *positive*, and AP′ and AC must be considered as negative. Hence, using the same meanings as in Art. 4 for u, v, and r, and substituting in the above proportion, it becomes:—

$$\frac{-v}{u} = \frac{-r-(-v)}{-r+u} = -\frac{r-v}{u-r};$$

$$\therefore\ vu - vr = ur - uv,$$

$$\text{and } ur + vr = 2uv.$$

Dividing this equation by uvr gives:—

$$\frac{1}{v} + \frac{1}{u} = \frac{2}{r} = \frac{1}{f}.$$

In an exactly similar way this relation may be established for the reflection of a convergent pencil at a spherical surface, and the student will find it an instructive exercise to draw the necessary diagrams and deduce the relation.

Examples.—(1) *An object is 20 in. in front of a concave mirror of focal length 5 in.; find the position of the image.*

Here $u = 20$ in., $f = 5$ in., and both are positive.

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f};$$

$$\therefore\ \frac{1}{v} + \frac{1}{20} = \frac{1}{5}, \text{ i.e. } \frac{1}{v} = \frac{1}{5} - \frac{1}{20} = \frac{3}{20}; \text{ i.e. } v = 6\tfrac{2}{3} \text{ in.}$$

As v is positive, the image is formed $6\frac{2}{3}$ in. *in front of* the mirror and is a *real* image (see Fig. 33).

(2) *Find the position of the image if the object is placed* 3 *in. in front of the same concave mirror.*

As before, $\frac{1}{v} + \frac{1}{3} = \frac{1}{5}$, i.e. $\frac{1}{v} = \frac{1}{5} - \frac{1}{3} = -\frac{2}{15}$; $\therefore v = -7\frac{1}{2}$ in.

As v is negative, the image in this instance is formed $7\frac{1}{2}$ in. *behind* the mirror, and is a *virtual* image.

(3) *An object is* 15 *cm. in front of a convex mirror of focal length* 30 *cm.; find the position of the image.*

Here, $u = 15$ cm., and $f = 30$ cm., but u is positive and f is negative. Hence:—

$$\frac{1}{v} + \frac{1}{15} = -\frac{1}{30}; \text{ i.e. } \frac{1}{v} = -\frac{1}{30} - \frac{1}{15} = -\frac{3}{30}; \quad \therefore v = -10 \text{ cm.}$$

As v is negative, the image is formed 10 cm. *behind* the mirror, and is a *virtual* image (see Fig. 34).

(4) *An object* 10 *cm. in front of a spherical mirror produces an image* 20 *cm. behind the mirror; find the focal length.*

Here, $u = 10$ cm., and $v = 20$ cm., but u is positive, and v is negative. Hence:—

$$\frac{1}{f} = -\frac{1}{20} + \frac{1}{10} = \frac{1}{20}; \therefore f = 20 \text{ cm.}$$

As f is *positive*, the mirror is *concave*. *

7. Relative Positions of Conjugate Foci

In Art. 4 and Art. 6 above, it has been shown that the relation, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$, holds good for all cases of reflection at a spherical surface, either concave or convex. By a general discussion of this relation, it is possible to determine the position of the image, P′, for any given position of the object, the luminous point, P.

If u be infinite—that is, if the incident light be parallel

$$\frac{1}{v} + \frac{1}{\infty} = \frac{2}{r}. \text{ Therefore, since } \frac{1}{\infty} = 0,\ v = \frac{r}{2}.$$

This means that if a pencil of parallel rays of light be reflected at a spherical surface, its focus, after reflection, is on the axis parallel to the incident light at a point whose distance from the mirror is equal to half the radius of curvature of the mirror (see Art. 3).

The general application of this method, however, is somewhat troublesome, and the question will be considered, therefore, in another way.

The relation $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ may be written as follows:—

$$uf + vf - uv = 0.$$

Adding f^2 to both sides of this equation gives:—

$$uv - uf - vf + f^2 = f^2;$$
$$\therefore\ u(v-f) - f(v-f) = f^2;$$
$$\text{i.e. } (u-f)(v-f) = f^2.$$

Now, if x and x' denote the distances of P and P respectively from F (Fig. 33), then:—

$$FP = x = (u-f) \text{ and } FP' = x' = (v-f);$$
$$\therefore\ xx' = f^2.$$

Whichever sign convention we use, x is to be reckoned as of the same sign as u if the principal focus F and the pole A are on the same side of the object. Similarly, x' and v are of the same sign if F and A are on the same side of the image.

(*a*) Since f^2 is always positive, being a square, x and x' must always have the same sign—that is, *the conjugate foci,* P and P′, *always lie on the same side of F.*

(*b*) If x is greater than, equal to, or less than f, then x' is less than, equal to, or greater than f—that is, if $x >=< f$, then $x' <=> f$.

In addition to the rules above, the following general rule will be found of great use in determining the motion of the image corresponding to any given motion of the object along an axis of the mirror.

(*c*) *When an image is formed by reflection, any motion of the object in a given direction along an axis of the mirror causes motion of the image in an* opposite *direction along the same axis.* In the case of a plane mirror, any *normal* to its surface may be considered as an axis.

By the application of these rules, the position of P′, as P travels from infinity in front of the mirror up to the mirror, may be traced.

I. CONCAVE MIRROR

Let A (Fig. 35) be the pole of the mirror, F the principal focus, so that AF $= f$, and C the centre of curvature, so that AC $= 2f$, and FC $= f$. Note again that x and x' in the relation, $xx' = f^2$ above, are the distances of the object, P, and image, P′, from F.

When x is infinite, x' is zero, for $x' = \dfrac{f^2}{x} = \dfrac{f^2}{\infty} = 0$; that is, when P is at infinity in front of the mirror, P′ is at the principal focus, F.

As x decreases from $+\infty$ to f, so x' increases from zero to f; that is, when P travels from infinity on the right up to the centre of curvature, C, which is, of course, distant f from F, P′ travels from F to C. When $x = f$, then, since $xx' = f^2$, both x and x' are equal to f; that is, as just indicated, when P is at the centre of curvature, C, P′ is also at C.

As x decreases from f to zero, then x' increases from f to $+\infty$; that is, when P travels from C, where $x = f$, to the principal focus, F, where $x = 0$, P′ travels from C, where $x' = f$, to infinity on the right, where $x' = +\infty$.

As x *decreases* from zero to $-f$, $-f$ being less than zero, x' increases from $-\infty$ to $-f$; that is, as P travels from F to A, the pole of the mirror, P′, after disappearing at infinity on the right, reappears at infinity on the left and travels from infinity behind the mirror to A, where P and P′ again coincide as at C.

Now, if P be a real luminous point, or small luminous object, it can evidently travel no further than A, and hence all possible

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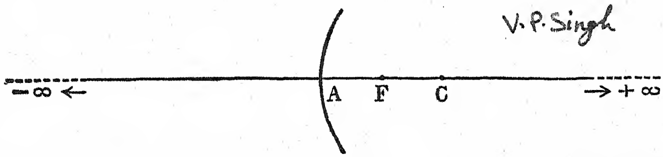


FIG. 35.

positions of the image of a real luminous point, placed anywhere in front of a concave spherical mirror have been traced.

These results may be represented graphically by plotting the distance of an object, and that of its image, as measured from a concave mirror as the distance of the object is varied progressively. Fig. 36 shows a typical graph thus obtained. In this graph, the curve, AB, is a rectangular hyperbola, given by the equation, $xx' = f^2$, where x, x', are the co-ordinates measured from the axes, FX, FX′. Also, the curve, CD, is the rectangular hyperbola, $xx' = f^2$, where both x and x' are negative.

The points of practical importance mentioned above, for a concave mirror, may be usefully summarised:—

(1) Luminous point between $+\infty$ and C. Real image between F and C.

(2) Luminous point at C. Real image at C.

(3) Luminous point between C and F. Real image between C and $+\infty$.

(4) Luminous point between F and A. Virtual image between $-\infty$ and A.

(5) Luminous point at A. Virtual image at A.

Although the case is of little practical importance, as in all ordinary cases the reflection of a divergent pencil is involved, yet the case of a convergent pencil incident on a concave mirror may be treated in the same way. The focus of such a convergent pencil, P, is behind the mirror, and the reflected pencil has its focus, P′, in front of the mirror. P′ may, in this case, be considered as the image of a point, P, behind the mirror. Hence, as x decreases from

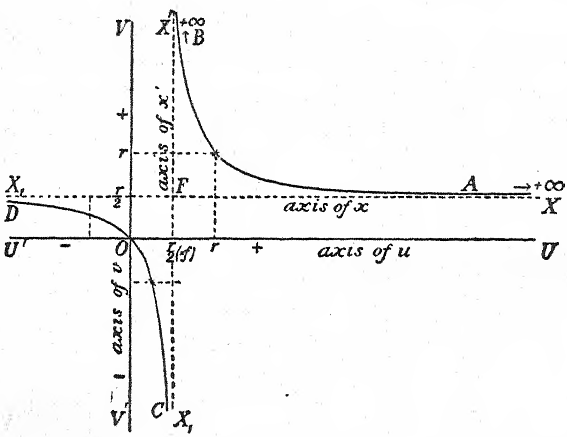


FIG. 36.

$-f$ to $-\infty$, x' increases from $-f$ to zero; that is, as P travels from A to infinity behind the mirror, P′ travels from A to F.

II. CONVEX MIRROR

The principal focus, F (Fig. 37), is here *behind* the mirror. Hence, since P is a real luminous point in front of the mirror, x, the distance of P from F will be greater than f, which is equal to the distance, AF, and therefore x' will be less than f, since $xx' = f^2$. When P is at infinity on the right, P′ is at F. As P travels from infinity in front of the mirror up to the pole A of the mirror, the image, P′, travels from F behind the mirror up to A. This is the only case of practical importance, as will easily be seen, and may be summarised briefly:—

Luminous point between $+\infty$ and A. Virtual image between F and A.

When a convergent pencil of light is incident on a convex mirror, its focus, P, is behind the mirror, and the position of the conjugate focus, P′, corresponding to any given position of P, can be determined as explained above by using the relation, $xx' = f^2$.

It should be noticed particularly that in the cases which are of practical importance, that is, when a real object in the form of a luminous point is used, a *concave* mirror may produce a *real* or a *virtual* image, whilst a *convex* mirror will produce a *virtual* image. It should be remembered also that an object at infinity gives an image at the principal focus.

8. Images of Finite Objects

When a luminous object is placed in front of a spherical mirror, an image is formed which may be *real* or *virtual* according to the

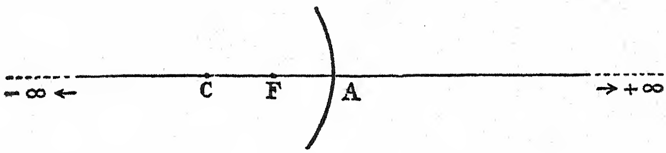


FIG. 37.

circumstances. *If real, the image is formed in front of the mirror and can be received on a screen; but if virtual, it appears to be behind the mirror and cannot be received on a screen.* In the latter case, however, it may be located by means of a pin (see pages 126 and 128), the paths of the rays by which these images are seen being described in Art. 9 below.

A general construction for determining the image of a finite object, formed by a spherical mirror, will now be described.

Let AB (Figs. 38, 40, 41) represent an object placed in front of the mirror, PM. Consider the ray, AM, coming from the point, A, on the object and incident on the mirror normally at M. Its direction is obtained by joining AC, and, if necessary, producing the line to cut the mirror in M. The reflected ray, MA, travels back along the path of the incident ray (Art. 2), and the image of A lies somewhere on this path.

Again, the ray, AP, drawn parallel to the principal axis is reflected along PF (Art. 2), and the image of A lies on this line also.

A third ray, AP′, through F may be drawn also; it is reflected along P′A′ parallel to the principal axis.

Hence, the image of A is found at A′, the intersection of the lines, MA and PF, or MA and P′A′. Similarly, an image of B is formed at B′, and images of points lying between A and B are formed at corresponding points between A′ and B′. Therefore, A′B′ is the complete image of AB.

It should be noted particularly that in drawing the image of an object formed by reflection at a spherical mirror, (1) a ray is drawn from a point on the object through the centre of curvature, C; this, being normal to the surface where it meets the mirror, is reflected back along the path of the incident ray; (2) a ray from the same point of the object is drawn parallel to the principal axis; this, after reflection, passes through the principal focus, F. The point of intersection of the two reflected rays gives the image of the point chosen on the object. Other points on the object are treated similarly.

In connexion with the formation of images, the following four points have to be considered:—

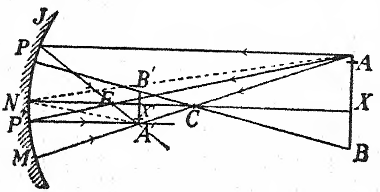


FIG. 38.

(1) *Relative position of object and image.* This has been considered fully above (Art. 7); the reasoning there employed is applicable whether the luminous point, P, be an isolated point or a point on an object of finite size.

(2) *Whether the image is real or virtual.* Whenever the image appears behind the mirror it must necessarily be virtual. Hence, it is necessary only to know the position of an image to decide whether it is real or virtual.

(3) *Whether the image is erect or inverted.* It is evident (see Fig. 38) that when object and image are on opposite sides of the centre of curvature, C, the image is inverted because of the crossing of the rays passing through C. Hence, if the relative positions of object and image are known, this point is decided readily. It may be remarked that the image of a real object is always inverted if real, and erect if virtual.

(4) *Relative size of object and image.* The ratio of the linear dimensions of the image to the corresponding linear dimensions of the object is called the **magnification.** Thus in Fig. 38, the magnification is equal to the ratio of A′B′ to AB.

When the image is erect, the magnification is regarded as positive, and when inverted, it is regarded as negative. The magnification is often written as $\frac{\text{Image}}{\text{Object}}$, or m.

There are four different, but not independent, relations between *magnification* and the quantities u, v, and f used in previous relations, and these will now be deduced, using Fig. 38 for this purpose, since in that diagram these last three quantities are positive.†

(*a*) In the triangles, A′B′C and ABC, we have the relations:—

$$\frac{A'B'}{AB} = \frac{CX'}{CX} = \frac{c'}{c};$$

$$\therefore\ m = \frac{\text{Image}}{\text{Object}} = \frac{c'}{c} = \frac{\text{Distance of image from C}}{\text{Distance of object from C}} = -\frac{r-v}{u-r},$$

the negative sign being given because the image is inverted. The negative sign is not needed before the ratio $\frac{c'}{c}$, because if the image is inverted, c' is of opposite sign to c.

(*b*) A ray, AN, incident at the pole of the mirror is reflected to A′, since A′ is the image of A. Thus, the angles, ANX and A′NX′, are equal, and the triangles, ANX and A′NX′, are similar. Hence:—

$$\frac{A'X'}{AX} = \frac{NX'}{NX}, \text{ and similarly } \frac{B'X'}{BX} = \frac{NX'}{NX}.$$

$$\text{Therefore, by addition, } \frac{A'B'}{AB} = \frac{NX'}{NX} = \frac{v}{u};$$

$$\text{i.e. } m = \frac{\text{Image}}{\text{Object}} = -\frac{v}{u} = \frac{\text{Distance of image from mirror}}{\text{Distance of object from mirror}},$$

the negative sign being applied since the image is inverted.

(*c*) The triangle, AFX, is similar to the approximate triangle, P′FN. Hence:—

$$\frac{P'N}{AX} = \frac{NF}{XF} = \frac{f}{u-f}.$$

$$\text{But, } \frac{P'N}{AX} = \frac{A'X'}{AX} = \frac{A'B'}{AB};\ \therefore\ \frac{A'B'}{AB} = \frac{f}{u-f};$$

† This demonstration holds also if the Real is Positive convention is used, because, if the mirror is concave, f is positive, and the object and image are both real.

$$i.e.\ m = \frac{\text{Image}}{\text{Object}} = -\frac{f}{u-f} = \frac{\text{Focal length of mirror}}{\text{Distance of object from focus}},$$

the negative sign again being applied, since the image is inverted.

(*d*) The triangle, A′FX′, is similar to the approximate triangle, PFN. Hence:—

$$\frac{A'X'}{PN} = \frac{FX'}{NF} = \frac{v-f}{f}. \qquad \text{But PN} = \text{AX};$$

$$\therefore \frac{A'X'}{AX} = \frac{A'B'}{AB} = \frac{FX'}{NF} = \frac{v-f}{f};$$

$$\text{i.e. } m = \frac{\text{Image}}{\text{Object}} = -\frac{v-f}{f} = \frac{\text{Distance of image from focus}}{\text{Focal length of mirror}},$$

the negative sign again being applied, since the image is inverted.

Summarising these four relations, we have:—

$$m = \frac{\text{Image}}{\text{Object}} = \frac{c'}{c} = -\frac{v}{u} = -\frac{f}{u-f} = -\frac{v-f}{f}.$$

That these four expressions are equal can be proved very simply from the general relation, $(1/v) + (1/u) = 1/f$.

Thus, to prove the equality of the second and third, multiply the general relation by u. Then:—

$$\frac{u}{v} + 1 = \frac{u}{f}; \quad \therefore \frac{u}{v} = \frac{u}{f} - 1 = \frac{u-f}{f};$$

$$\text{i.e. } \frac{v}{u} = \frac{f}{u-f}.$$

These expressions apply to linear dimensions only. For the relative areas,

$$\frac{\text{Area of image}}{\text{Area of object}} = \left(\frac{c'}{c}\right)^2 = \left(\frac{v}{u}\right)^2 = \left(\frac{f}{u-f}\right)^2 = \left(\frac{v-f}{f}\right)^2.$$

It thus appears that if the positions of the object and its image are known, the nature of the image can be determined completely. The results of Art. 7, as there summarised, are therefore of great importance, and, for this reason, some cases for a luminous object of finite size will now be given with appropriate figures.

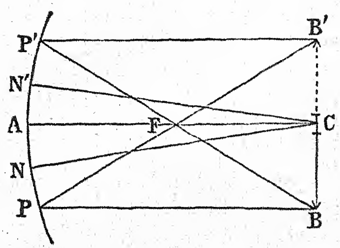


FIG. 39.

I. CONCAVE MIRROR

(1) Object between infinity of mirror and centre of curvature, C (Fig. 38). The image lies between the principal focus, F, and C, and is *real, inverted,* and *diminished.*

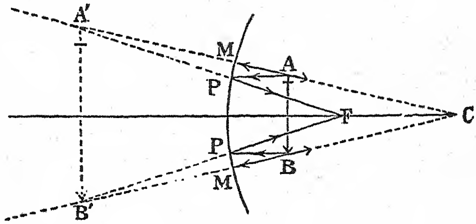


FIG. 40.

(2) Object, BC, at centre of curvature, C (Fig. 39). The image, B′C, at C, is *real, inverted,* and of the *same size* as the object. The construction for this image should be noticed. The image, B′, of the point, B, is found in the usual way. Also, any two rays, CN, CN′, from C are normal to the mirror, and, therefore, on reflection, again intersect at C; that is, the image of C is formed at C.

(3) Object between centre of curvature, C, and principal focus, F. The image lies between C and infinity in front of the mirror, and is *real, inverted,* and *magnified.* This case is illustrated by Fig. 38; if A′B′ is taken to represent the object, then AB represents its image.

(4) Object between the principal focus, F, and the pole of the mirror (Fig. 40). The image lies *behind* the mirror, between infinity and the pole, and is *virtual, erect,* and *magnified.*

In the limit, when the object is at the pole of the mirror, the image is also at the pole, and coincides with the object and is equal in size (Art. 7, I., 5).

II. CONVEX MIRROR

Object in front of mirror between infinity and the pole of the mirror (Fig. 41). The image lies between the principal focus, F, and the pole, and is *virtual, erect,* and *diminished.*

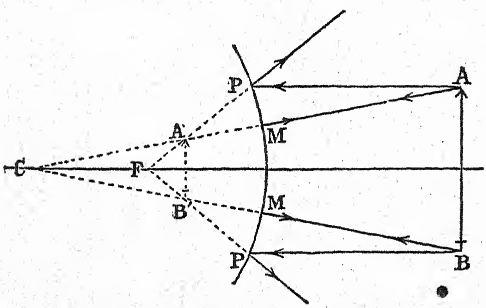


FIG. 41.

To summarise: In the case of a concave mirror, as the object moves from infinity up to the centre of curvature, C, the image is

real and moves from the principal focus, F, to C. As the object moves from C to F, the image is *real* and moves from C to infinity in front of the mirror. As the object moves from F to the mirror, the image reappears at infinity behind the mirror, is therefore *virtual*, and moves up to the mirror.

In the case of a convex mirror, as the object moves from infinity up to the mirror, the image is *virtual*, and moves from the principal focus, F, behind the mirror up to the mirror.

Examples.—(1) *An object is placed 15 cm. in front of a concave mirror of focal length 30 cm. Find the position of the image, and the ratio of its size to that of the object.*

Here, $u = 15$ cm., $f = 30$ cm. Substituting in the general relation,

$$\frac{1}{v} + \frac{1}{15} = \frac{1}{30}; \text{ and } \frac{1}{v} = -\frac{1}{30}; \therefore v = -30 \text{ cm.}$$

†That is, the image is 30 cm. *behind* the mirror, and is therefore *virtual*.

Also, image and object are on the same side of the centre of curvature, and therefore the image is *erect*.

$$\text{Also:—} \quad m = \frac{\text{Image}}{\text{Object}} = -\frac{v}{u} = -\frac{-30}{15} = +2.$$

That is, image is virtual, erect, and twice the size of the object.

This problem may also be solved by application of the relation, $xx' = f^2$ (Art. 7).

From data, $x = -(30 - 15) = -15$ cm., and $f = 30$ cm.

Hence, substituting, $-15x' = 30^2$; $\therefore x' = -60$ cm.

That is, the image is 60 cm. from the principal focus, in the same direction as the mirror, or 30 cm. *behind* the mirror. Also:—

$$m = \frac{\text{Image}}{\text{Object}} = -\frac{f}{u-f} = -\frac{30}{15-30} = +2.$$

That is, the image is *virtual* and twice the size of the object.

(2) *A pencil of rays, converging to a point 20 cm. behind a spherical mirror, is brought to a focus, by reflection from its surface, at a point 10 cm. in front of the mirror. Determine whether the mirror is convex or concave, and find its radius of curvature.*

Here, $u = -20$ cm., and $v = 10$ cm. Hence:—

$$\frac{1}{10} - \frac{1}{20} = \frac{2}{r}; \text{ that is, } \frac{2}{r} = \frac{1}{20}; \therefore r = 40 \text{ cm.}$$

That is, the mirror is *concave*, and its radius of curvature is 40 cm.

† If using the Real is Positive convention, read instead of this line:— "That is, the image is *virtual*, and therefore 30 cm. *behind* the mirror."

(3) *An object, 3 cm. in length, is placed 20 cm. in front of a convex spherical mirror, of focal length 12 cm. Find the nature and position of the image.*

$$\text{Here, } u = 20 \text{ cm., and } f = -12 \text{ cm.}$$

$$\text{Hence, } \frac{1}{v} + \frac{1}{20} = -\frac{1}{12}; \text{ that is, } \frac{1}{v} = -\frac{1}{20} - \frac{1}{12} = -\frac{2}{15};$$

$$\therefore v = -7{\cdot}5 \text{ cm.}$$

† That is, the image is 7·5 cm. *behind* the mirror, and is therefore *virtual.*

$$\text{Also, } m = \frac{\text{Image}}{\text{Object}} = -\frac{v}{u} = +\frac{7{\cdot}5}{20} = +\frac{3}{8}.$$

That is, image is *virtual*, and length of image $= \frac{3}{8} \times 3 = 1{\cdot}125$ cm.

Using the relation, $xx' = f^2$, in this problem, $x = 20 + 12 = 32$ cm., and $f = 12$ cm.;

$$\therefore 32x' = 12^2, \text{ and } x' = \tfrac{144}{32} = 4{\cdot}5 \text{ cm.}$$

That is, image is 4·5 cm. from the principal focus in the positive direction, or 7·5 cm. *behind* the mirror.

$$\text{Also, } m = \frac{\text{Image}}{\text{Object}} = -\frac{f}{u-f} = -\frac{-12}{20-(-12)} = +\frac{12}{32} = +\frac{3}{8},$$

the same result as above.

(4) *A gas flame is placed at a distance of* 8 *ft. from the wall of a room. Find the radius of curvature of a concave spherical mirror, and where it must be placed, in order that it may produce, on the wall, an image of the flame magnified threefold.*

Let x ft. denote the distance of the mirror from the gas flame. Then,

$$u = x \text{ ft., and } v = x + 8 \text{ ft.}$$

$$\text{Also, } m = \frac{\text{Image}}{\text{Object}} = -\frac{v}{u} = -\frac{x+8}{x} = -3; \text{ that is, image is } \textit{inverted}.$$

$$\text{From this, } 3x = x + 8, \text{ and } x = 4 \text{ ft.}$$

$$\text{Again, } m = \frac{\text{Image}}{\text{Object}} = -\frac{f}{u-f} = -\frac{f}{4-f} = -3;$$

$$\therefore 12 - 3f = f, \text{ or } f = 3 \textit{ ft.}, \text{ and } r = 6 \text{ ft.}$$

Or, using the relation, $\frac{1}{v} + \frac{1}{u} = \frac{2}{r}$, we get:—

$$\frac{1}{12} + \frac{1}{4} = \frac{1}{3} = \frac{2}{r}; \quad \therefore r = 6 \text{ ft.}$$

Hence, the mirror must be placed 4 ft. from the gas flame—that is, 12 ft. from the wall—and its radius of curvature should be 6 ft.

† If using the Real is Positive convention, read instead of this line:— "That is, the image is *virtual*, and therefore 7·5 cm. *behind* the mirror."

(5) *A square piece of cardboard of* 1 *in. side is placed at right angles to the principal axis of a concave spherical mirror of focal length* 18 *in. At what distance from the mirror must it be placed in order that an image* 9 *sq. in. in area may be formed?*

$$\frac{\text{Area of image}}{\text{Area of object}} = \left(\frac{f}{u-f}\right);$$

$$\text{That is, } \frac{9}{1} = \left(\frac{18}{u-18}\right)^2, \text{ or } 3 = \pm\frac{18}{u-18};$$

$$\therefore\ u = 24 \text{ or } 12 \text{ in.}$$

Thus, the object may be placed 24 in. or 12 in. in front of the mirror. In the former case the image is *real* and *inverted*; in the latter it is *virtual* and *erect*.

(6) *An object is placed* 16 *in. from the centre of curvature, and* 12 *in. from the principal focus of a convex spherical mirror. Find the nature and position of the image.*

Here, the distance between the principal focus and the centre of curvature $= (16 - 12) = 4$ in. That is, $r = -8$ and $f = -4$ in.

$$\text{Further, } u = 16 - 8 = 8 \text{ in.}$$

$$\text{Hence, } \frac{1}{v} + \frac{1}{8} = \frac{1}{-4}, \text{ or } \frac{1}{v} = -\frac{3}{8};$$

$$\therefore\ v = -2\tfrac{2}{3} \text{ in.}$$

That is, the image is $2\frac{2}{3}$ in. behind the mirror, and is *virtual*, *erect*, and *diminished*.

9. Paths of Rays by which an Image is Seen

So far the most convenient method of *drawing* the image of an object formed by reflection at a spherical mirror has been dealt with. Now, the *actual paths of the rays by which an image is seen* by an observer will be shown.

If the object is near the principal axis of the mirror, the image will be near the axis, and, therefore, also the eye must not be far removed from the axis.

Let MM (Fig. 42) be a concave mirror, AB an object placed in front of it, A′B′ the real image of AB, and E the position of the eye. Now this image is the locus of intersection of reflected rays, and hence is not self-luminous, so that it can be seen only by

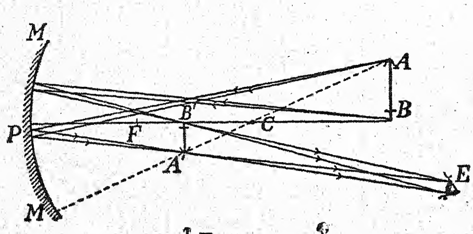


FIG. 42.

those rays which come originally from the object, and, passing through the image, enter the eye.

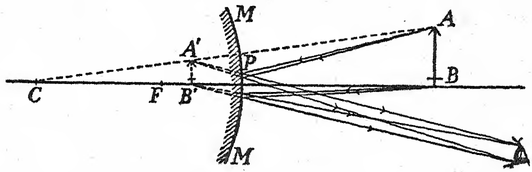


FIG. 43.

Thus, to depict the rays by which E sees A′, draw a pencil of rays diverging from A′ and entering E. Produce the rays backwards to meet the mirror at P, and join the points of intersection to A. The rays by which A′ is seen are included in the incident pencil, AP, and the reflected pencil, PA′E.

The same construction can be applied to other points of the image and object, and a similar construction holds for the rays by which the virtual image of an object placed in front of a convex spherical mirror (Fig. 43) is seen.

V. P. Singh

10. Spherical Aberration. Caustic

In dealing with the laws of reflection from concave spherical mirrors above, the discussion has been limited to mirrors of small aperture, and it has been seen that parallel rays falling on such a mirror are all brought together at one point—the principal focus. It has been seen also that rays diverging from any luminous point are all brought together at one point—the conjugate focus. In order to explain the necessity for this limitation, the case of a concave mirror of a very large aperture, that is, forming a very large segment of a sphere, will now be considered.

Draw a large segment of a circle (Fig. 44). Through its centre draw the diameter, CA, and, parallel with this, a number of equidistant straight lines to represent rays of light in a parallel beam falling on a concave mirror. From C draw radii to every point at which a ray is incident on the mirror. These are the normals at those points. Then from those points, and on the inner sides of the normals, set off angles exactly equal to those on the other sides, and draw straight lines at these angles to represent the reflected rays.

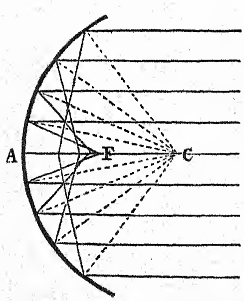


FIG. 44.

It will be seen that the rays nearest the axial ray cut that ray after reflection at a

point, F, as near as possible midway between C and A. A pair of rays a little further from the axis will be found to intersect the axis a trifle behind this point. The next pair of rays cut the axis after reflection considerably behind F, and the next pair still further behind.

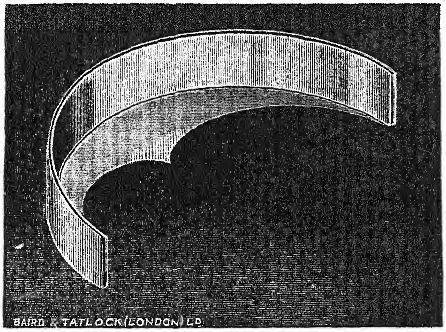


FIG. 44 (*a*).

Such wanderings of the marginal rays from the focus of the central rays is called **aberration,** and this particular case being due to the form (spherical) of curve employed, is called **spherical aberration.**

Fig. 44 shows, and by increasing the number of rays it will be seen more clearly, that all the reflected rays lie within an area bounded at the back by the mirror, and in front by a double curve which touches each reflected ray. This double curve is called a **caustic curve,** and is very bright, especially at the cusp or vertex, which coincides with the principal focus,

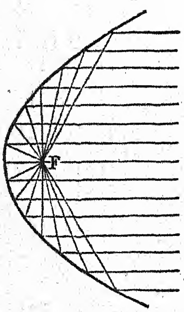


FIG. 45.

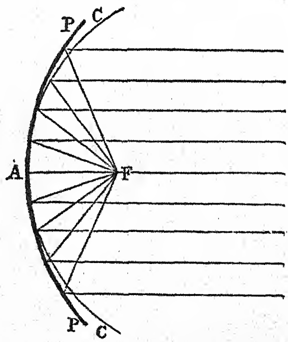


FIG. 46.

F. Such a curve may be seen clearly on the surface of milk, when a tumbler is about three-quarters filled, a source of light being placed so that the inner rim of the glass reflects its light down upon the milk. See Fig. 44(*a*).

11. Paraboloidal Mirrors

For all optical purposes spherical aberration is an important defect. It may be kept within allowable limits by using mirrors of very small aperture, or by screening the margin of the mirror by an opaque plate, called a *diaphragm*, with a central hole in it. The more the mirror is *stopped down*, the sharper becomes the definition, but the loss of light may become very serious. These means do not correct, but simply lessen, the defect, which is inseparable from the spherical form of mirror. A further objection to the use of a diaphragm is explained below (Chapter XV., p. 297). A real remedy, however, may be applied by the substitution of a *parabola*, instead of a circle, as the generating curve of the mirror.

A parabola is a curve formed by a plane section of a cone parallel to its side, and may be defined by the equation $y = x^2$ in coordinate geometry, and a paraboloidal surface is generated by the revolution of this curve around its axis. Portions of two parabolas are shown in Figs. 45 and 46, the former of low, the latter of high angle.

No matter how great the angle, parallel rays falling on a paraboloidal mirror parallel to the axis converge accurately to a point, the focus, F, and, conversely, rays diverging from that point are parallelised accurately. When, as in lightships and railway signal lamps, it is necessary to parallelise rays as perfectly as possible, the form of mirror shown in Fig. 45 is employed. In the specula of large telescopes (see page 206), which have to converge parallel rays as accurately as possible, an attempt is made to give such a figure as that shown in Fig. 46. In this figure, however, the aperture is greatly exaggerated, and the circle is shown for comparison of the curves.

It might appear that spherical aberration could always be avoided by using a properly shaped reflecting or refracting surface. In principle this is true if the object is to be always at the same distance from the lens or mirror, and the determination of the correct shapes in such cases is called *aspherical optics*. In practice, however, the much greater difficulty of producing aspherical surfaces of optical perfection limits the application of the method. The computation involved in such problems is far from simple.

APPENDIX TO CHAPTER IV

ALTERNATIVE TREATMENT IN TERMS OF THE REAL IS POSITIVE CONVENTION

6. Conjugate Foci. Convex Mirror

Adopting the Real is Positive convention (Art. 5), AP [Fig. 46 (*a*)] is the distance to a real object and must be considered positive. AP′ is the distance to a virtual image and must be considered negative. AC is to be considered negative since a convex mirror has a virtual focus. Hence, using the same meanings as in Art. 4, for u, v, and r and substituting in the above proposition it becomes

$$\frac{-v}{u} = \frac{-r-(-v)}{-r+u} = -\frac{r-v}{u-r};$$

$$\therefore \quad vu - vr = ur - uv$$

$$\text{and} \quad ur + vr = 2uv.$$

Dividing this relation by uvr gives

$$\frac{1}{v} + \frac{1}{u} = \frac{2}{r} = \frac{1}{f}.$$

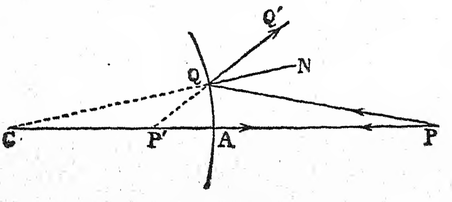


FIG. 46 (*a*).

In an exactly similar way this relation may be established for the reflection of a convergent pencil at a spherical surface, and the student will find it an instructive exercise to draw the necessary diagrams and deduce the relation. The position of the object is to be reckoned as the point to which the pencil would converge if the mirror were not there. The *object* is thus *virtual*, because the rays of light do not actually pass through it, and u, the distance to it has to be reckoned *negative*.

Examples.—(1) *An object is 20 in. in front of a concave mirror of focal length 5 in., find the position of the image.*

Here $u = 20$ in., $f = 5$ in. and both are positive.

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f};$$

$$\therefore \frac{1}{v} + \frac{1}{20} = \frac{1}{5}, \quad i.e. \quad \frac{1}{v} = \frac{1}{5} - \frac{1}{20} = \frac{3}{20},$$

$$v = +6\tfrac{2}{3} \text{ in.}$$

As v is positive, the image is *real* and is therefore formed $6\frac{2}{3}$ in. *in front of* the mirror (see Fig. 33).

(2) *Find the position of the image if the object is placed* 3 *in. in front of the same concave mirror.*

As before $$\frac{1}{v} + \frac{1}{3} = \frac{1}{5}, \quad i.e. \ \frac{1}{v} = \frac{1}{5} - \frac{1}{3} = -\frac{2}{15};$$

$$\therefore \quad v = -7\tfrac{1}{2} \text{ in.}$$

As v is negative, the image in this instance is *virtual*, and is therefore $7\frac{1}{2}$ in. *behind* the mirror [see Fig. 46 (*a*)].

(3) *An object is* 15 *in. in front of a convex mirror of focal length* 30 *cm., find the position of the image.*

Here, $u = 15$ cm. and $f = 30$ cm., but u is positive and f is negative.

Hence $$\frac{1}{v} + \frac{1}{15} = -\frac{1}{30}, \quad i.e. \ \frac{1}{v} = -\frac{1}{30} - \frac{1}{15} = -\frac{3}{30};$$

$$\therefore \quad v = -10 \text{ cm.}$$

As v is negative, the image is *virtual*, and is thus 10 cm. *behind* the mirror.

(4) *An object* 10 *cm. in front of a spherical mirror produces an image* 20 *cm. behind the mirror; find the focal length.*

Here, $u = 10$ cm. and $v = 20$ cm., but u is positive and v is negative (since an image behind the mirror must be virtual).

Hence $$\frac{1}{f} = -\frac{1}{20} + \frac{1}{10} = \frac{1}{20};$$

$$\therefore \quad f = 20 \text{ cm.}$$

As f is *positive*, the mirror is *concave*.

CHAPTER V

REFRACTION AT PLANE SURFACES

IT has been seen that a ray of light travels in a straight line so long as its course lies in the same homogeneous medium, but when it passes from one medium into another, it undergoes a change of direction at the surface of separation of the two media. This change of direction is called **refraction.** This phenomenon is illustrated clearly by the following simple experiments:—

(1) When a piece of stick is partly immersed in water in an oblique position, it appears to be bent at the surface of the water (Fig. 47). This is due to the refraction of the rays of light coming from points of the stick below the surface of the water. For example, rays coming from O are refracted at S in passing from the water to the air, and appear to come from O′. Similarly, other points between O and A appear, to an eye at E, to lie between O′ and A, and thus the portion, OA, of the stick appears bent into the position, O′A.

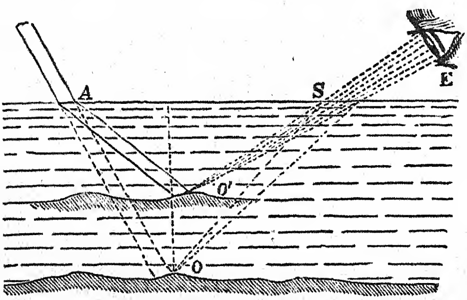
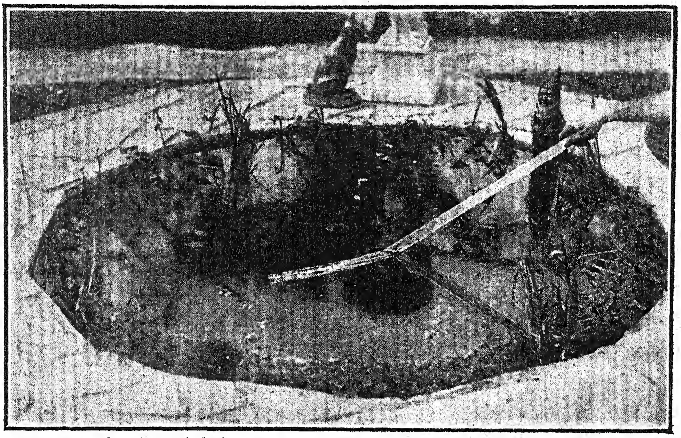


FIG. 47.

(2) If a coin be placed at the bottom of a vessel with opaque sides, in such a position that it is just out of the range of vision of an observer stationed a short distance away, it will be found that, on pouring water into the vessel, the coin soon becomes visible. Thus, if the coin be placed at S′ (Fig. 3), it will be invisible to an eye situated at E, until, on pouring a sufficient quantity of water into the vessel, a small pencil of rays of light coming from S′ and refracted at O, in passing from the water into the air, reaches the eye by the path, S′OAE.

For similar reasons a pool of water appears to be shallower than it really is, and small air bubbles in solid glass objects appear to be nearer to the surface than they actually are.



Photograph of a stick in water. Note refracted and reflected images.

The phenomenon was known to *Euclid*, *c.* 300 B.C., and to *Ptolemy*, 100 A.D., and the problem was studied by *Bacon*, 1214, and *Kepler*, 1571, but the laws of the phenomenon were not established until *Snell*, 1621, formulated them.

1. Laws of Refraction

Let AO (Fig. 48) represent a ray of light incident at O on the surface of separation of the media, M and M′, and let OB represent the refracted ray. Then, if NON′ be the normal to the surface at O, the angle, AON, is the **angle of incidence**, and the angle, BON′, is the corresponding **angle of refraction.**

The laws of refraction, as established by experiment, refer to the relative position and magnitude of these angles, and may be stated:—

(i) *The angles of incidence and refraction lie in the same plane—that is, the incident and refracted rays, and the normal at the point of incidence, all lie in the same plane.*

(ii) *For the same two media, the ratio of the sine of the angle of incidence to the sine of the angle of refraction is always a constant quantity.*

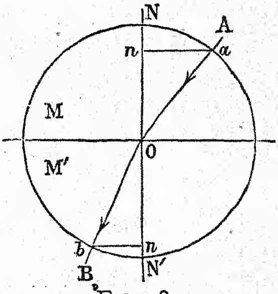


FIG. 48.

The second law is generally known as the law of sines, and is often referred to as Snell's law. Without employing the term *sine*, the law may be explained by a geometrical construction:—

With centre, O (Fig. 48) and any radius, Oa, describe the circle, aNbN′, cutting OA and OB in a and b. From a and b draw perpendiculars, an and bn', on the normal, NN′. Then the law may be expressed by stating that, for the same two media, the ratio $\frac{an}{bn'}$, is constant. This is evident because, $\sin \text{AON} = \frac{an}{\text{O}a}$, and $\sin \text{BON}' = \frac{bn'}{\text{O}b}$;

$$\therefore \quad \frac{\sin \text{AON}}{\sin \text{BON}'} = \frac{an}{\text{O}a}\bigg/\frac{bn'}{\text{O}b} = \frac{an}{bn'} = \text{constant.}$$

2. Experimental Verification of the Laws of Refraction

The laws stated above may be verified roughly by means of the apparatus shown (Fig. 49). A cylindrical glass vessel, VV, is fixed in a suitable stand, with its axis horizontal and its circular section vertical. A circular scale, graduated in degrees, is fitted or engraved round its circumference. The position of the mirror, m, is adjusted until a pencil of parallel rays of light is reflected, ACA′, to cut the scale at points, A and A′, equally distant from the zero. It is then evident that the pencil passes through the centre of the circular scale, C. Next, the vessel is half-filled with water, holding a small quantity of freshly precipitated silver chloride in suspension, and the surface of the water is adjusted accurately on a level with the centre, C, of the scale. Thus, the pencil of light reflected from the mirror, m, is incident in the plane of the scale on the surface of the water at C. The path of the refracted pencil is rendered visible by the light scattered by the particles of silver chloride, and is seen to be deviated from the direction of the incident pencil immediately on entering the water.

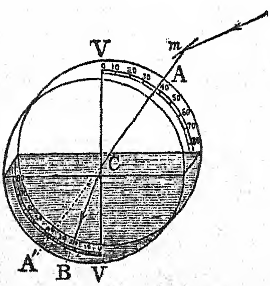


FIG. 49.

If the scale is accurately vertical, its plane will contain the normal to the surface of the water at the point of incidence, and the refracted pencil will be seen to lie in this plane, thus verifying the first law of refraction.

Also, if the magnitudes of the angles of incidence and refraction be observed on the circular scale, for several different values of each, it will be found that, in accordance with the law of sines, the ratio of the sine of the angle of incidence to the sine of the corresponding angle of refraction is constant.

A simpler verification of the laws may be performed with a rectangular block of glass and a number of pins:—

Experiment. To verify the laws of refraction.—Fix the block of glass, AB (Fig. 50), on a sheet of cartridge paper, and mark the outline in pencil. Fix a pin, P, into the paper, and close to the glass. Arrange other pins, P_1, P_2, P_3, P_4, at convenient distances from one another and from P. With the eye on a level with the block and looking through it towards PP_1, place two pins, Q_1, R_1, Q_1 being in contact with the glass and R_1 some distance away, so that R_1, Q_1, P, P_1, appear in a straight line. Do the same with R_2, Q_2, P, P_2, and R_3, Q_3, P, P_3, etc.

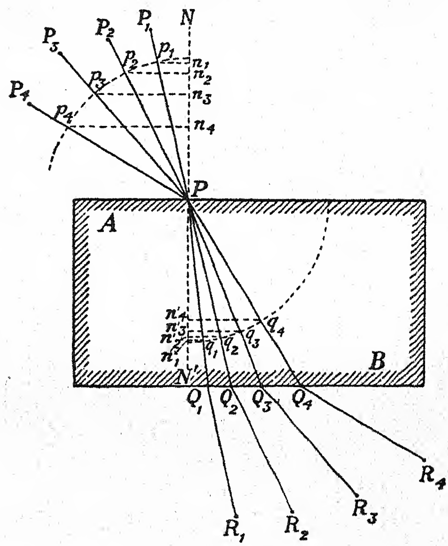


FIG. 50.

Remove AB, and join up the pinpricks by straight lines. Draw the normal, NN′, at P, and measure the angles, P_1PN, P_2PN, P_3PN, . . . Q_1PN', Q_2PN', Q_3PN', . . . with a protractor. Look up the values of the sines of these angles in a book of tables, and show that—

$$\frac{\sin P_1PN}{\sin Q_1PN'} = \frac{\sin P_2PN}{\sin Q_2PN'} = \frac{\sin P_3PN}{\sin Q_3PN'} = \ldots = \text{a constant, } \mu \text{ say.}$$

If a protractor is not available, describe a circle with P as radius cutting the rays in $p_1, p_2, p_3, \ldots q_1, q_2, q_3 \ldots$ Draw perpendiculars, p_1n_1, p_2n_2, p_3n_3, . . . q_1n_1', q_2n_2', q_3n_3' . . . to NN′. Measure these with a pair of dividers and a diagonal scale, and show that—

$$\frac{p_1n_1}{q_1n_1'} = \frac{p_2n_2}{q_2n_2'} = \frac{p_3n_3}{q_3n_3'} = \ldots = \text{the same constant, } \mu.$$

Observe also that P_1P, P_2P, P_3P, . . . are parallel to Q_1R_1, Q_2R_2, Q_3R_3, . . . respectively, showing that the directions of the rays have not been changed, but that the rays have been shifted laterally by a distance which increases with the angle of incidence.

3. Refractive Index

It has been shown that, when a ray of light is refracted from one medium, *a*, into another medium, *b*, the *ratio* of the sine of the angle of incidence to the sine of the angle of refraction is constant. This ratio is the **relative index of refraction** from the medium, *a*, into the medium, *b*. Of, if ${}_a\mu_b$ represent this index, and if ϕ and ϕ' denote respectively the angles of incidence and refraction, then—

$$\text{Index of refraction from } a \text{ into } b = \frac{\text{sin of angle of incidence in } a}{\text{sin of angle of refraction in } b}$$

$$\text{or, } {}_a\mu_b = \frac{\sin\phi}{\sin\phi'} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

It has been established by experiment that the path of a ray of light is reversible. Thus, if BO (Fig. 48) be taken to represent the incident ray, then OA will be the path of the refracted ray. This fact is evidently expressed by writing—

$${}_b\mu_a = \frac{\sin\phi'}{\sin\phi} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(ii)}$$

$$\text{From (i) and (ii) above, } {}_a\mu_b \cdot {}_b\mu_a = \frac{\sin\phi}{\sin\phi'} \cdot \frac{\sin\phi'}{\sin\phi} = 1.$$

$$\text{Hence, } {}_a\mu_b = \frac{1}{{}_b\mu_a}, \text{ and } {}_b\mu_a = \frac{1}{{}_a\mu_b} \quad \ldots\ldots\ldots \text{(1)}$$

This result may be stated in words by saying that, *if ${}_a\mu_b$ denotes the index of refraction from a to b, then $\frac{1}{{}_a\mu_b}$ denotes the index of refraction from b to a.* For example, if the index of refraction from air to water be $\frac{4}{3}$, then the index of refraction from water to air is $\frac{3}{4}$.

When a ray of light is refracted *from vacuum into any medium*, the index of refraction from *vacuum into that medium* is called the **absolute refractive index**, or the **refractive index** of the medium.

If a ray of light passes from a given medium, through a layer of another medium bounded by parallel planes, into the medium in which it was travelling originally, it is known from experiment that the *initial and final directions of the rays are parallel.* This either may be taken as an experimental fact, or may be deduced from results already obtained from experimental data (see Art. 2).

Thus, let AA and BB (Fig. 51) represent the parallel surfaces of separation of a layer of the medium, *b*, from the medium, *a*, and let RN, R′N′, represent the path of a ray travelling from *a* through *b* into *a* again. Then, it is evident that the angles, R′N′*n*′ and RN*n*, are equal, for they have respectively the same relation to the equal angles, O′N′N and ONN′. Hence, R′N′ is parallel to RN, but is not in the same straight line with it.

It follows from this that, when a ray of light passes from one medium through any number of layers of different media, having parallel surfaces of separation, back into the same medium, then the initial and final directions are parallel. The lateral shifting of the rays explains the displacement of a body seen through a thick glass plate.

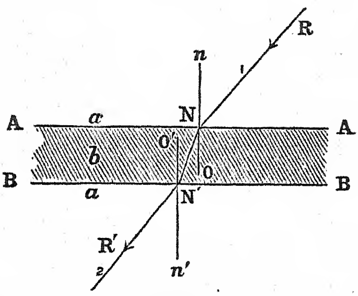


FIG. 51.

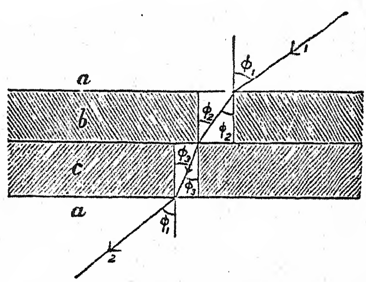


FIG. 52.

Consider the case for the three media, *a*, *b*, *c* (Fig. 52). Here:—

$$_a\mu_b = \frac{\sin\phi_1}{\sin\phi_2},\ _b\mu_c = \frac{\sin\phi_2}{\sin\phi_3},\ _c\mu_a = \frac{\sin\phi_3}{\sin\phi_1};$$

$$\therefore\ _a\mu_b \cdot {_b\mu_c} \cdot {_c\mu_a} = 1;$$

$$\text{i.e. } _a\mu_b \cdot {_b\mu_c} = \frac{1}{_c\mu_a} = {_a\mu_c} \text{ from (1) above.}$$

$$\text{or } _a\mu_c = {_a\mu_b} \cdot {_b\mu_c} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

This relation is easily remembered by noticing that *a* and *c* are the initial and final suffixes on *each* side. Compare the suffixes *w* and *g* below. In words, the index of refraction from a to c is equal to the index of refraction from a to b multiplied by the index of refraction from b to c.

This is an important relation, and enables the relative index of refraction from *a* to *c* to be determined, given the indices of refraction from *a* to *b* and from *b* to *c*. For example, if the index of refraction

from air to glass, ${}_a\mu_g$, is $\frac{3}{2}$, and that from air to water, ${}_a\mu_w$, is $\frac{4}{3}$, then the index of refraction from water to glass, ${}_w\mu_g$, is given by—

$$ {}_w\mu_g = {}_w\mu_a \cdot {}_a\mu_g = \frac{1}{{}_a\mu_w} \cdot {}_a\mu_g = \frac{{}_a\mu_g}{{}_a\mu_w}, $$

$$ \text{or, } {}_w\mu_g = \tfrac{3}{2} \times \tfrac{3}{4} = \tfrac{9}{8}. $$

This relation also enables another relation to be established between the relative refractive index for any two media and the *absolute* refractive indices of those media. Thus, if ${}_v\mu_a$ denote the absolute refractive index of the medium, a, and ${}_v\mu_b$ that of b, then

$$ {}_a\mu_b = {}_a\mu_v \cdot {}_v\mu_b; $$

$$ \therefore {}_a\mu_b = \frac{{}_v\mu_b}{{}_v\mu_a} \quad \ldots\ldots\ldots\ldots\ldots\ldots (3) $$

That is, *the relative refractive index from a to b is the ratio of the absolute refractive index of b to the absolute refractive index of a.*

Example.—*The absolute refractive indices of diamond and glass are respectively $\frac{5}{2}$ and $\frac{3}{2}$. Find the relative indices of refraction from glass to diamond, and from diamond to glass.*

Here, if ${}_g\mu_d$ denote the relative refractive index from glass to diamond, then by (3) above,

$$ {}_g\mu_d = \frac{{}_v\mu_d}{{}_v\mu_g} = \tfrac{5}{2}/\tfrac{3}{2} = \tfrac{5}{2} \times \tfrac{2}{3} = \tfrac{5}{3}, $$

and by (1) above, ${}_d\mu_g = \frac{3}{5}$.

It should be noticed here that, as a general rule, a ray of light in passing from one medium into a denser medium is bent towards the normal, while in passing into a rarer medium it is bent away from the normal. This is equivalent to stating that if the medium, b, is denser than a, then

$$ {}_a\mu_b > 1, \text{ and } {}_b\mu_a = \frac{1}{{}_a\mu_b} < 1, $$

which expresses the case for refraction from b into the rarer medium, a. Since all media are denser than vacuum, it follows that all absolute refractive indices should be greater than unity. This holds for all familiar transparent media, but not for all media without exception. It has been shown (see page 353) that certain metals have refractive indices considerably less than unity. See Table I., page 353.

So far the refractive index has been considered merely as a geometrical relation, established by experiment, between the

directions of the incident and refracted rays. When considered in connexion with the wave theory of light, however, a definite physical meaning can be attached to this constant. It can be shown that the refractive index from any medium, a, into another medium, b, is the ratio of the velocity of light in a to its velocity in b. That is—

$$ {}_a\mu_b = \frac{V_a}{V_b}, $$

where V_a denotes the velocity of light in a, and V_b the velocity of light in b. (See Chapter XIV., Art. 5.)

This ratio differs for waves of different wave-length, being *greater* the *shorter* the wave-length; and, as difference of wave-length in light corresponds to difference of *colour*, it follows that the value of the refractive index depends on the colour of the light which suffers refraction. Light of the *greatest wave-length* and *lowest refractive index* is of a deep *red* colour, and that of the *shortest wave-length* and *highest refractive index* is coloured *violet*. Between these two extremes, the refractive index increases as the wave-length decreases, and the colour of the light shades off from red through orange, yellow, green, and blue to violet.

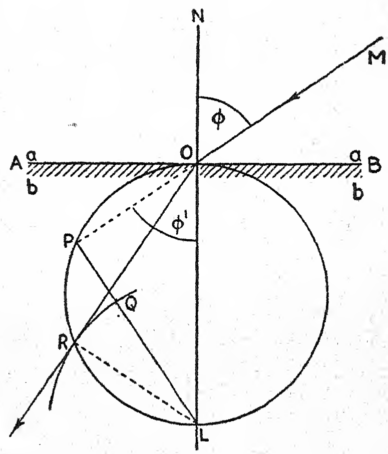


FIG. 53.

4. Geometrical Construction for Refraction

A simple geometrical construction for the directions of the incident and refracted rays is afforded by the fact that angles in a semicircle are right angles.

Let AB (Fig. 53) be the surface of separation between two media, a and b, and let MO be a ray in the medium, a, incident on the surface at O.

Draw NOL, the normal at O, and on any convenient length, OL, as diameter, describe the circle, OPRL. Produce MO to cut the circle at P. Divide LP at Q so that ${}_a\mu_b$.LQ = LP. With L

as centre describe the arc, QR, cutting the circle in R. Draw OR; it is the refracted ray.

$$\text{For, } \sin \phi = \sin \text{LOP} = \frac{\text{LP}}{\text{LO}} = {}_a\mu_b \cdot \frac{\text{LQ}}{\text{LO}} = {}_a\mu_b \frac{\text{LR}}{\text{LO}}$$

$$i.e. \sin \phi = {}_a\mu_b \sin \text{LOR} = {}_a\mu_b \sin \phi' ;$$

$$\therefore \frac{\sin \phi}{\sin \phi'} = {}_a\mu_b.$$

Another construction is as follows:—Let AB (Fig. 54) be any incident ray on a surface, *mm*, separating the media, *a* and *b*. Draw any normal, NAA′, cutting the incident ray in A, and choose A′ on it so that BA′ = ${}_a\mu_b$.BA. Draw A′B and produce it to C; BC is the refracted ray.

$$\text{For, } \frac{\sin \phi}{\sin \phi'} = \frac{\sin \text{BAN}}{\sin \text{BA'N}} = \frac{\text{BN/BA}}{\text{BN/BA'}} = \frac{\text{BA'}}{\text{BA}} = {}_a\mu_b, \text{ by construction.}$$

5. Critical Angle

All possible values of an angle of incidence or refraction must evidently lie between 0° and 90°. Now, when a ray of light passes from a rarer into a denser medium, it is refracted towards the normal—that is, the angle of refraction, ϕ', is less than the angle of incidence, ϕ. Therefore, whatever be the value of ϕ, between 0° and 90°, that of ϕ' must also lie between 0° and 90°, and consequently refraction is always possible.

However, when a ray of light passes from a denser into a rarer medium, it is refracted away from the normal—that is, the angle of refraction, ϕ', is greater than the angle of incidence, ϕ. Hence, when ϕ passes a certain limit at which ϕ' becomes equal to 90°, refraction is no longer possible, and the incident ray is totally reflected at the surface of separation of the media, back into the denser medium.

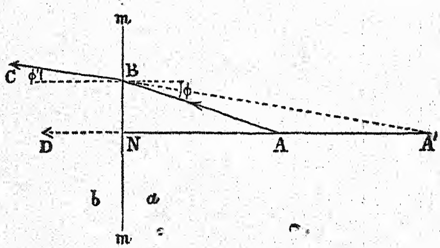


FIG. 54.

Let *mm* (Fig. 55) represent the surface of separation of any two media, *a* and *b*, of which *b* is the rarer, and let IO represent a ray of light incident at O, at a small angle, ION, and refracted along OR. As the angle of incidence increases and the incident

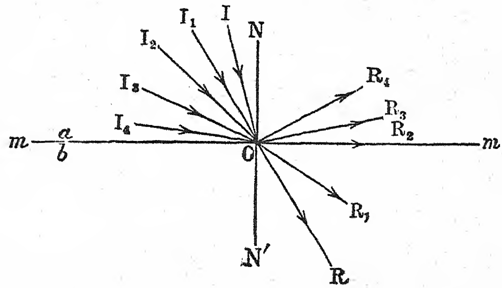


FIG. 55.

ray takes the positions, I_1O, I_2O, the angle of refraction also increases, and the refracted ray takes successively the corresponding positions, OR_1, OR_2. The angle of refraction being greater than the angle of incidence, however, a position is reached at I_2O where the angle of refraction, R_2ON', becomes equal to 90°, and the refracted ray, OR_2, travels along the surface of separation of the media.

The angle of incidence, I_2ON, at which this takes place is the **critical angle** for the media, *a* and *b*. As the angle of incidence becomes greater than I_2ON, the ray is no longer refracted into the medium, *b*, but is **totally reflected** from the surface, *mm*, in accordance with the ordinary laws of reflection. That is, as the incident ray passes into the positions, I_3O, I_4O, it is reflected from *mm* along the corresponding paths, OR_3, OR_4.

Hence, *when refraction takes place from a denser into a rarer medium, the angle of incidence, which corresponds to an angle of refraction of* 90°, *is called the critical angle for the given media.* At this angle refraction ceases and total reflection from the surface of separation of the media—reflection back into the denser medium—begins.

It should be noticed that for angles of incidence between 0° and the critical angle, only a *portion* of the light incident on the surface of the rarer medium is reflected at that surface, the remainder being refracted and scattered (see page 15). For angles of incidence greater than the critical angle, the incident light is almost totally reflected, no portion of it being refracted.

The value of the critical angle is determined readily for any media when the relative refractive index for those media is known. Thus, let ${}_a\mu_b$ denote the refractive index from *a* to *b*. Then, if θ denote the critical angle (Art. 4),

$$ {}_a\mu_b = \frac{\sin\phi}{\sin\phi'} = \frac{\sin\theta}{\sin 90^\circ} = \frac{\sin\theta}{1} = \sin\theta. $$

That is, *the critical angle for refraction from a medium, a, into a rarer medium, b, is the angle whose sine is the relative refractive index from a to b,* or

$$ {}_a\theta_b = \sin^{-1}{}_a\mu_b \quad \dots\dots\dots\dots\dots\dots\dots (4)$$

This value of θ for air and water is about 48° 30′, and for air and glass it ranges from 38° to 41° according to the nature of the glass.

Example.—*Find the critical angle for water and glass, given that the refractive index from air to glass is $\frac{3}{2}$, and that from air to water $\frac{4}{3}$.*

Of the two media, glass is the denser, and, using relations (1) and (2) above,

$$ {}_g\mu_w = {}_g\mu_a \cdot {}_a\mu_w = \tfrac{2}{3} \cdot \tfrac{4}{3} = \tfrac{8}{9}.$$

Now, if ${}_g\theta_w$ denote the required critical angle, then

$$ {}_g\theta_w = \sin^{-1}{}_g\mu_w = \sin^{-1}\tfrac{8}{9};$$

$$\therefore\ {}_g\theta_w = 66°44'.$$

6. Total Internal Reflection

As seen above, total reflection occurs when a ray of light, travelling in the denser of two media, is incident on the surface of separation at an angle greater than the critical angle for the media.

This phenomenon is exhibited readily by means of the apparatus shown in Fig. 49. The position of the mirror, *m*, is changed and adjusted so as to reflect a beam of light upwards through the water into the air, along the path, BCA. As the angle of incidence is increased slowly, the refracted beam gradually approaches the surface of the water, and finally, when the critical angle is passed, suffers total reflection at the surface of separation of the water and air, and is seen in the water as if reflected by a mirror coincident with this surface.

Simple illustrations of the phenomenon are often seen. Thus, if a glass vessel containing water be held above the level of the eye, and the surface of separation of the water from the air be observed from below, it appears as a brilliant reflecting surface. If a spoon be placed in the water, then, on looking up at the water surface, the part of the spoon above the water cannot be seen at all, but the part immersed is reflected brilliantly as if by a silvered mirror [Fig. 56 (*a*)].

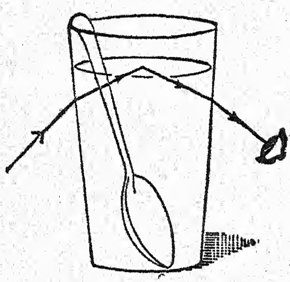


FIG. 56 (*a*).

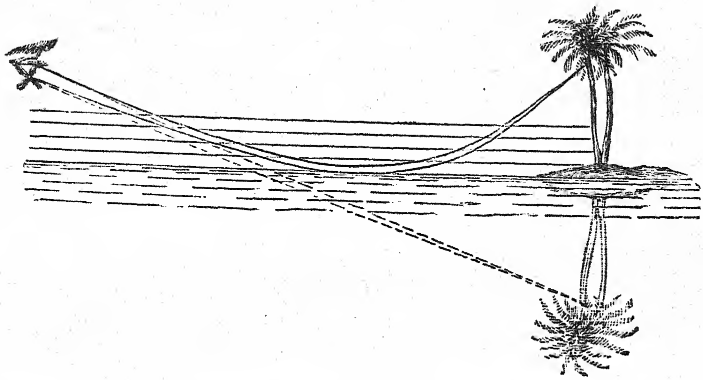


FIG. 56 (*b*).

Similarly, the edge of a crack in a pane of glass, seen obliquely, exhibits the same effect; as does also the surface of a glass tube, held obliquely in a beaker of water, when looked at through the sides of the beaker.

The brilliancy of many precious stones is due to their large refractive indices and, therefore, small critical angles. When light enters a cut diamond at any face, it finds it very difficult to get out at most of the other faces, and hence bright beams issue at the one or two faces available for emergence.

The **mirage** is a phenomenon, on a large scale, due to total reflection from layers of air. In hot sandy deserts inverted images of distant objects and of the sky are often seen reflected as from a lake. By contact with the hot sand the lower layers of air become so heated that, up to the height of a few feet, the density increases upwards. Rays of light from a distant object, entering these layers of rarefied air obliquely downwards, become deviated upwards more and more by refraction the lower they penetrate, and finally strike a layer at an angle greater than the critical angle (Fig. 56*b*). Here total reflection takes place, and, the rays becoming deviated more and more in traversing the denser layers above, at last reach the observer's eye as if they came from a point as far below the reflecting layer as the object is above it, while at the same time the object is seen directly by rays which do not pass down into the reflecting layer. Thus, in the figure, the appearance is that of a palm-tree standing by a pool of water.

In the Arctic regions, inverted images of ships and other objects are sometimes seen in the air, even though the objects themselves

may be below the horizon. This is due to the very low temperature of the ice and sea cooling the lower atmospheric layers so much that the density increases rapidly downwards. Then rays of light, passing obliquely upwards from the objects into the rarer layers,

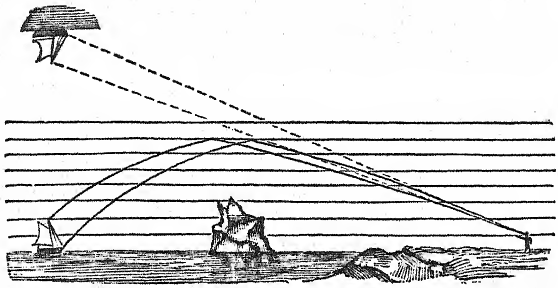


FIG. 57.

become more and more deviated downwards until they suffer total reflection, as in the previous case and as shown in Fig. 57.

7. Geometrical Construction for Critical Angle

Let O be a point in the plane of separation, AB (Fig. 58), of two media, a and b. It is required to find the direction of a ray of light which, passing through the medium, b, and incident on AB at the point, O, is just totally reflected.

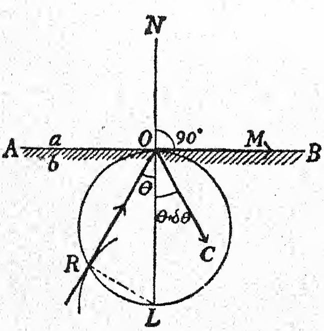


FIG. 58.

Draw NOL, normal at O, and on any convenient length, OL, as diameter describe a circle, LRO. With L as centre and radius equal to $\frac{\text{OL}}{{}_a\mu_b}$, strike an arc cutting the circle in R. Then RO is the direction of the critical incident ray.

For, since ORL is a right angle, $\sin \text{ROL} = \frac{\text{RL}}{\text{OL}} = \frac{1}{{}_a\mu_b} = {}_b\mu_a$. Hence, the angle, ROL, is equal to θ, the critical angle.

The refracted ray, OM, just skims the surface of separation, AB. If the angle, ROL, exceeds θ by a very small amount, $\delta\theta$, all the light is reflected, taking the direction OC.

It should be noted that in Art. 5, it was shown that $\sin \theta = {}_a\mu_b$, where θ is the critical angle and a the *denser* medium. In the present case, b is the denser medium, and $\sin ROL = \sin \theta = {}_b\mu_a$.

8. Refractive Index of Glass by Total Reflection

The glass block used in Art. 2 above may also be used to determine the refractive index from air to glass, by an experiment in which the light is totally reflected from one face of the block.

Experiment. Place the block of glass upon a sheet of paper as before, and mark the outline in pencil. With the eye in the neighbourhood of P_4 (Fig. 59), sight a pin placed at P_1 by observing its reflection in the face, BC.

Next insert pins at P_2 and P_3 so that P_1, P_2, P_3, P_4 appear in the same straight line. Remove the block, and produce P_1P_2 and P_4P_3 to meet AB and DC in M and N. Between M and N, the ray has been reflected at the face, BC.

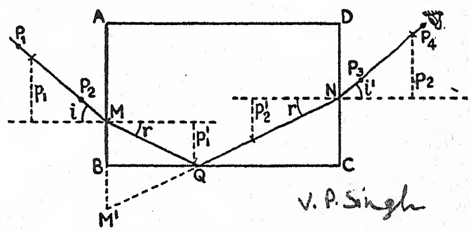


FIG. 59.

To find its path, produce AB to M', making BM' equal to BM. Join NM', cutting BC in Q. Then, $P_1P_2MQNP_3P_4$ is the complete path of the ray from P_1 to the eye.

Draw normals at M and N, and from the values of the angles, i, r, i', r', or from the lengths, p_1, p_1', p_2, p_2', calculate two values of μ.

9. Refractive Index of a Liquid by Total Reflection

Let RA (Fig. 60) be a ray of light passing through a liquid, l, and incident at A on a parallel-sided plate of glass, g. If i is the angle of incidence, the angle of refraction, r, is given by—

$$\frac{\sin i}{\sin r} = {}_l\mu_g = {}_l\mu_a \times {}_a\mu_g = \frac{{}_a\mu_g}{{}_a\mu_l}.$$

The ray passes through the glass and meets the second surface at B. If air be the medium beyond the plate, the angle of refraction, i', is given by—

$$\frac{\sin i'}{\sin r} = {}_a\mu_g.$$

The angle, i', is always greater than i, and hence if i be gradually increased, eventually the light will be totally reflected at B. If θ and ϕ be the values of i and r when this occurs, then—

$$\frac{\sin\theta}{\sin\phi} = \frac{{}_a\mu_g}{{}_a\mu_l}, \text{ and } \sin\theta = \frac{{}_a\mu_g}{{}_a\mu_l}\sin\phi.$$

$$\text{Also, } \frac{\sin 90^\circ}{\sin\phi} = {}_a\mu_g, \text{ or } \sin\phi = \frac{1}{{}_a\mu_g}.$$

$$\text{Hence, } \sin\theta = \frac{1}{{}_a\mu_l} = {}_l\mu_a,$$

so that θ is the critical angle from the liquid to air (Art. 5).

The problem resolves itself, therefore, into an accurate determination of this angle, θ. When i is less than the critical angle, the ray, RA, will travel through the plate, but when i is equal to or greater than the critical angle, the ray is totally reflected at the glass-air surface and no light passes through. This method is due to Wollaston, and the manner of using it will now be described.

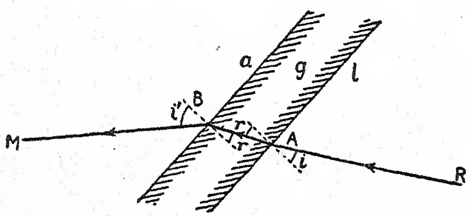


FIG. 60.

Experiment. To determine the refractive index of a liquid by total reflection.—Two fairly thin glass plates, about 4 cm. long and 3 cm. wide, are separated by pieces of microscope cover slips, placed at the corners, and then cemented together at the edges, thus forming a glass cell, A (Fig. 61), containing a thin film of air. A is fixed in a metal clip, B, which is supported by a vertical spindle, C, working in the centre of the top of a flat wood box. By means of a head, D, the cell can be rotated, the amount of rotation being indicated by the motion of a pointer, E, over a graduated circle, F. The wood in the middle of two opposite sides of the box is cut away and replaced by two glass plates on which sheets of tinfoil have been pasted. Two narrow rectangular strips of tinfoil are removed from the sheets and these serve as slits, M and R.

The liquid whose refractive index is to be determined is placed in a glass vessel, G, whose sides must be plane and parallel, and the vessel placed as shown in the diagram. A source of monochromatic

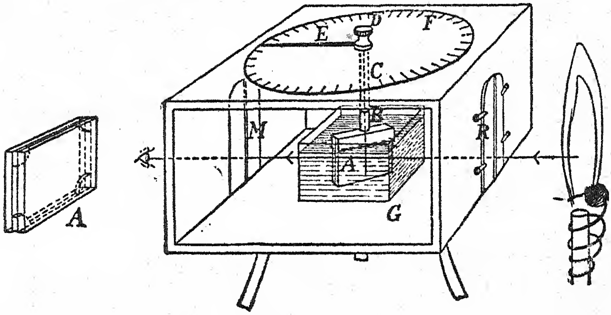


FIG. 61.

light is then placed in front of R. For this purpose a Bunsen burner and a piece of salt may be used.

Placing the eye beyond M and looking along MR, turn D so that A is nearly broadside on to the light. R is then easily seen. Now turn D gradually to the right or left until R just disappears. The ray, RA, is now making an angle, θ, with the normal. Take the reading of E. Now rotate the cell back to its original position, and then beyond it until R just disappears once more. The two positions of the cell are represented by A_1A_1, A_2A_2 (Fig. 62), and it will be seen that the angle through which the cell has been rotated is equal to twice the critical angle, for sodium yellow light, of the liquid contained in G.

When F is graduated to read quarter degrees, values of the critical angle can be determined quickly and accurately, yielding values of the refractive index correct to one-half per cent.

10. Deviation Produced by Refraction

Let AO (Fig. 63) represent the incident ray, and OB the refracted ray. Then, the deviation, D, produced by the refraction at O is expressed by—

$$D = \angle A'OB = \angle A'ON' - \angle BON';$$

$$\therefore D = \phi - \phi' \quad \ldots\ldots(5)$$

where ϕ denotes the angle of incidence and ϕ' the angle of refraction.

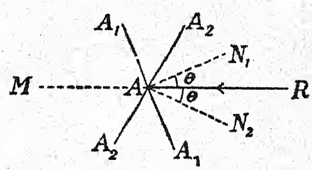


FIG. 62.

Since $\sin \phi = \mu \sin \phi'$, where μ denotes the corresponding refractive index, it is evident that if ϕ gradually

increases, ϕ' also increases, but at a slower rate. Hence, the deviation increases as the angle of incidence increases.

When the angle of incidence is zero, then the angle of refraction is zero also, and therefore no deviation is produced. Thus, when a ray is incident along the normal to the surface of separation of two media, it does not suffer deviation, but continues its path in the same straight line.

11. Image by Refraction at a Single Plane Surface

So far the refraction of a single ray only has been dealt with. The refraction of small pencils *directly* incident on the surface of separation of the media will now be considered. A pencil of light is said to be directly incident on a surface when the axis of the pencil is perpendicular to that surface.

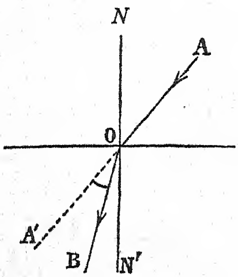


FIG. 63.

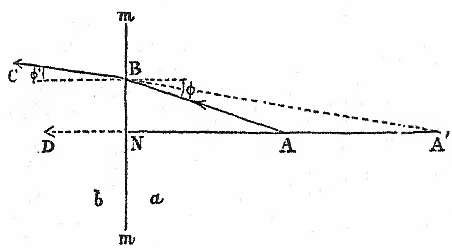


FIG. 64.

Let *mm* (Fig. 64) represent the surface of separation of two media, *a* and *b*, of which *b* is the denser, and let AB represent one of the extreme rays of a diverging pencil of light directly incident on *mm* along AN. AB is refracted at B along BC, while AN, being normal to *mm*, passes along ND without suffering deviation. The focus of the refracted pencil will now be found at the point, A′, from which BC and ND apparently diverge. It thus appears that A and A′ are conjugate foci, and that A′ may be considered as the image of A, formed by refraction at the surface, *mm*. It now remains to determine the relation between the distances of A and A′ from that surface.

Let ϕ and ϕ' denote the angles of incidence and refraction, and μ the refractive index from *a* to *b*. In this case and in what follows, μ always denotes the refractive index for refraction *in the direction*

in which the light is travelling. Then, since ϕ and ϕ' are equal to the angles, BAN and BA'N respectively,

$$\mu = \frac{\sin \phi}{\sin \phi'} = \frac{\sin \mathrm{BAN}}{\sin \mathrm{BA'N}} = \frac{\mathrm{BN}.\mathrm{BA'}}{\mathrm{BA}.\mathrm{BN}} = \frac{\mathrm{BA'}}{\mathrm{BA}}.$$

But, if BN is small—that is, if the incident pencil is small—then BA and BA' are approximately equal to NA and NA', and

$$\mu = \frac{\mathrm{NA'}}{\mathrm{NA}}.$$

Adopting the notation used in the case of spherical mirrors (page 39), but taking numerical values regardless of sign, let NA be denoted by u and NA' by v. Then $\mu = v/u$, or :—

$$v = \mu . u \quad \ldots\ldots\ldots\ldots (6)$$

Expressing this in words, *the distance of the image from the plane refracting surface is μ times that of the object from the surface.*

This explains why, on looking vertically downwards, the depth of a pool of water appears to be only about three-fourths of the real depth.

Let O (Fig. 65) represent an object at the bottom of the pool. Then, after refraction at the surface of the water, the small direct pencil incident along ON appears to diverge from I—that is, the object, O, is seen at I. From relation (6) above,

$$\mathrm{IN} = \mu . \mathrm{ON}.$$

Now, μ from water to air is $\frac{3}{4}$, and, therefore, IN $= \frac{3}{4}$ON.

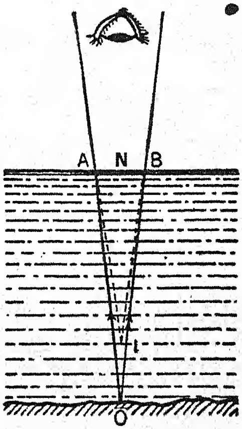


FIG. 65.

In exactly the same way, the apparent thickness of a plate of glass, or other transparent medium, as seen by an eye looking along a normal to the surface of the plate, is less than its actual thickness. For, if O (Fig. 66) represent an object close to the face of the plate remote from the eye, then its apparent position is at I—that is, IN is the apparent thickness of the plate of actual thickness, ON. Hence, if t denotes the thickness of the plate, its apparent thickness is given by μt, where μ is the refractive index *from the medium into air.* The result is true only in the case of small direct pencils.

Example.—*A small air bubble in a piece of glass with a plane surface is* 3 *in. below that surface. Find its apparent distance from an eye looking at it,*

along a normal to the surface, from a point 8 *in. from the surface. The refractive index from air to glass is* $\frac{3}{2}$.

Here, applying the relation, $v = \mu u$, and remembering that the light is supposed to be travelling from glass to air, and that therefore $\mu = \frac{2}{3}$,

$$v = \frac{2}{3} \times 3 = 2 \text{ in.}$$

Hence, the apparent distance of the bubble from the eye $= 8 + 2 = 10$ in.

The method of real and apparent thickness may be applied to the experimental determination of refractive index.

Experiments. To determine the refractive index of a solid (or liquid) by noting the apparent thickness.—(i) Cut a thin slip of gummed paper and stick it in a vertical position at O to the edge of the glass block, AB (Fig. 66), used in previous experiments. Mark an outline of the block on paper, and draw in the normal, ONP. With the eye on this normal, look at O through the glass. It appears to be at I. To locate I, place a pin on the normal and, with the eye in several positions not far distant from the normal, adjust it until its image by reflection coincides with I. Note its final position, P. Remove the block, and make IN = NP. Then, I is the position of the image of O, and

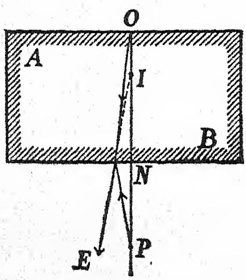


FIG. 66.

$$\mu = \frac{\text{ON}}{\text{IN}}.$$

(ii) A more accurate method is to use a low-power microscope which has a vertical adjustment and a fine scale by which the vertical motion can be measured. Cut a small star of paper, place it on the table and focus the microscope on it. Read the scale, *a*. Place the glass block on the paper star, and elevate the microscope until it is again in focus. Again read the scale, *b*. Place another small paper star on the top of the block, and again elevate the microscope until this is in focus. Again read the scale, *c*. The difference between *b* and *c* is equal to IN, and that between *a* and *c* is ON. Then, $\mu = \frac{\text{ON}}{\text{IN}}$, as before.

If the refractive index of a liquid is required, the liquid must be placed in a glass cell, whose walls are thin, plane, and parallel. The operations are then the same as for the solid block, the refraction of the glass walls being neglected.

12. Caustic by Refraction

Strictly, I (Fig. 65) is the conjugate focus of O only for a very narrow pencil of light. For a pencil of wide angle we get the phenomenon that we have already met with a spherical mirror (Chapter IV., Art. 10), namely that the rays of light from a point on the object do not intersect at a single point after refraction, but along a curve (Fig. 67). From the laws of refraction it is evident that, if we have a pencil of angle 2θ, where θ is the critical angle, the rays near the boundary of the pencil will appear after refraction to come, not from I, but from points near N. Rays more oblique than this will be totally reflected. Conversely, an eye placed underneath the water and looking upwards will see images of objects above the water, most of them greatly distorted, all contained within a cone of semi-vertical angle θ (the critical angle). The water surface outside this cone acts as a mirror and forms images of objects below it.

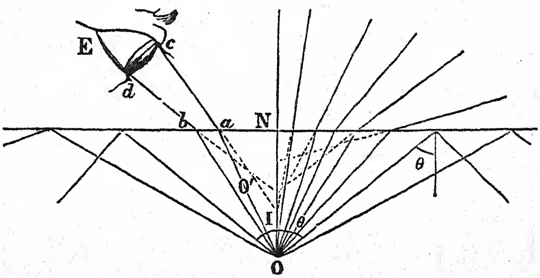


FIG. 67.

If now an oblique conical pencil, O*abcd*, emanating from O and entering an eye placed at E, is considered, it does not diverge from a definite point after refraction. Two special cases may be considered. If the pencil be much narrower horizontally than vertically, it may be regarded as a number of rays all lying in, or close to, the vertical plane in which the figure is drawn. This will diverge almost exactly from O′ after refraction. If the pencil be wider horizontally than in the plane of the figure, it can be regarded as an aggregate of rays all equally inclined to ON, and these diverge from I after refraction. For other forms of pencil, neither of these results is true, but the emergent pencil consists of rays, every one of which very nearly cuts two "*focal lines,*" one at I in the plane of the figure, and one at O′ perpendicular to it.

If the point, O, is being observed with two eyes situated on a level and at the same distance from ON, these receive two narrow pencils which appear to come from the same point, I, *on the normal,* ON. If it is being observed with two eyes in the same vertical

plane through ON—that is, the plane of the figure—the rays received appear to diverge from a point, O′, *not on the normal,* ON, *but on the observer's side of it.* This case is well represented by Fig. 67, if *c* and *d* are taken to be the positions of the eyes of the observer. If the eyes are held obliquely, the image seen is confused and cannot be located at a definite point.

Thus the apparent thickness of the medium becomes less and less as it is viewed more and more obliquely, and finally becomes zero when the direction of vision is parallel to its surface. This explains why the flat bottom of a vessel full of water appears to be slightly concave; the points vertically below the eye are seen by direct pencils, but the surrounding points by slightly oblique pencils, so that the water appears to be shallower as the range of vision travels outwards from the point vertically below the eye. If the eye be moved along parallel to the surface of the water, this appearance of concavity moves along with it, and thus an apparent wave motion is given to the bottom of the vessel. For the same reason, the depth of a pool of water appears to increase as it is approached, and vice versa.

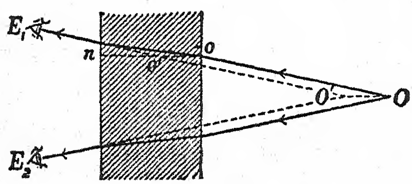


FIG. 68.

On the right-hand side of Fig. 67 it will be noticed that the backward prolongations of the refracted rays are tangential to a curve which is known as a *virtual caustic by refraction* (see also page 56). This is a double curve with a cusp at I, and may be drawn, in the case of a glass block, by using pins to locate the directions of the refracted rays.

13. Image by Direct Refraction through a Plate

When a plate of glass, or any transparent substance, is interposed between the eye and a near object, the distance of the object from the eye is apparently diminished. This is evidently due to the apparent diminution in the thickness of the plate by refraction. If t denotes the actual thickness, then the apparent thickness, t', is given by the relation, $t' = \mu t$, where μ denotes the refractive index *from the plate to air*, and the position of the object is apparently nearer the eye by a distance, $(t - \mu t)$ or $t\,(1 - \mu)$.

Thus, an object at O (Fig. 68) is seen at O′, the virtual focus of the refracted pencils which enter the eyes at E_1, E_2. OO′ represents

the apparent change of position, and, being equal to OO′, is apparently equal to *on* — *o′n*. Thus, if OO′ be denoted by *d*—

$$d = t\,(1 - \mu) \quad \text{.....................} (7)$$

If the refractive index *from air to the plate* be used, the relation becomes—

$$d = \frac{t\,(\mu - 1)}{\mu}.$$

14. Multiple Images by a Plate with Parallel Faces

Let O (Fig. 69) represent an object placed in front of the plate. Rays of light reach the nearest face of the plate in all directions from O. Consider the ray, O*a*. It is partially reflected from the first face at *a*, and an image due to this reflection is seen at I. A portion of the light incident at *a*, however, is refracted into the plate along *ab*, and, on incidence on the second face of the plate at *b*, a portion is reflected along *bc*, the remainder being refracted out into the air. The portion travelling along *bc* again suffers partial reflection and refraction at *c*, and the emergent ray, *cf*, gives rise to another image at I′, fainter than the first at I because of the loss of light at *b* and *c*.

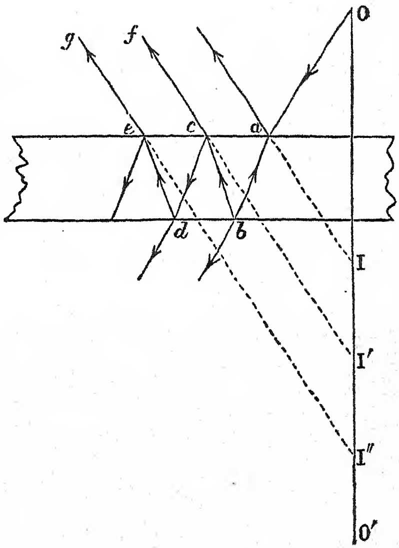


FIG. 69.

Similarly, after reflection at *d*, and refraction at *e*, the light emergent along *eg* gives rise to another image at I″, fainter than that at I′, and so on.

In this way, by continued reflection and refraction, a series of images is formed on the line, IO′, each member of the series becoming fainter and fainter as the number of reflections by which it is produced is increased.

When standing in front of a thick plate-glass mirror, there is no apparent confusion in the reflection in it, because the images formed by the two surfaces are almost exactly superposed, and still more because the second image, due to the silvered *back* surface,

quite overpowers by its brilliancy the feeble first image formed by the front surface of glass. If, however, a finger, or a candle flame, is held near the glass, and its reflection viewed obliquely, both these images, partly overlapping each other, are seen at once, the second much brighter than the first. On looking more obliquely, the images separate more widely, the first becoming brighter and the second less bright, and a third image, fainter than either, appears. On looking still more obliquely, a fourth, and perhaps a fifth, image will be seen, and the first image is now the brightest, the others showing a gradual diminution of brightness.

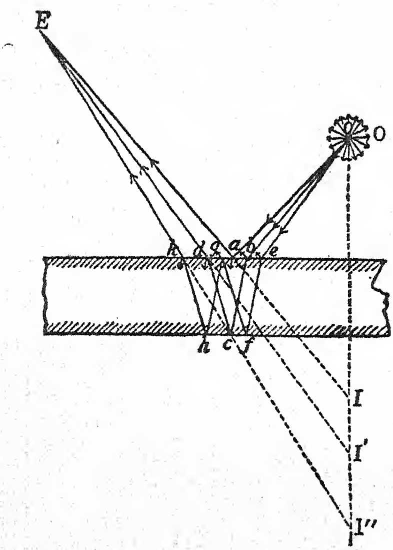


FIG. 70.

To explain this, a modification of Fig. 69 must be employed. In this figure, the parallel rays from *a*, *c*, and *e* are too widely separated to enter the eye at the same time, and, even if they did, they would all blend together to give a single image at infinity.

Let O and E (Fig. 70) represent object and eye. Then, all the rays, or more strictly the axes of the narrow diverging pencils, by which O is seen by E, must, on emergence from the plate, converge to the eye. O emits light in all directions. Of the rays emitted, one, such as O*a*, is partly reflected at the front face of the plate, and the reflected part passes to the eye along *a*E. The other part penetrates the glass, but, since none of this on emergence will enter E, it may be neglected.

O*b*, suffers reflection internally at *c*, and enters the eye by the path O*bcd*E: parts are reflected at *b*, *d*, but may be neglected.

A third ray, O*e*, reaches the eye by the path O*efghk*E. Parts of it reflected at *e* and *k*, and emergent at *g*, need not be considered.

Other rays can be treated in a similar manner, and, if each of these rays is considered as the axial ray of a small conical pencil, it will be obvious that these pencils will appear, to an eye at E, to come from images I, I′, I″, which are on the normal to the plate through O, but which are not quite equidistant from each other.

The number of images seen depends upon the polish of the reflecting surfaces, for, after a certain number of reflections and refractions, the quantity of light reaching the eye becomes too small to excite the sensation of vision, and the loss of light by reflection at any surface depends upon the degree of polish of that surface.

In observing this phenomenon experimentally, it will be noticed that the first image increases in brightness as the angle at which it is seen is increased, and, at very oblique incidence, it becomes much the brightest. This shows that the quantity of light reflected from a glass surface increases as the angle of incidence increases.

In ordinary looking-glasses these multiple images are scarcely ever noticed, and are of no importance. In optical instruments, however, they would be most inconvenient, and their formation is prevented by silvering the glass on the *front* surface and polishing the silver deposit as highly as possible. When great brilliancy is not required, a very fair single image reflector may be obtained by coating the back surface of a piece of plate glass with lamp-black, which absorbs all the light except that which forms the first image.

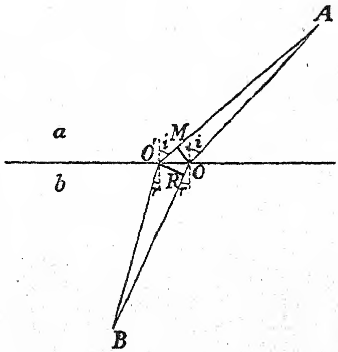


FIG. 71.

If the luminous object, O (Fig. 70), is at a great distance away, so that the rays, Oa, Ob, Oc, etc., may be considered parallel to each other, only one beam, and that of parallel rays, will enter the eye. Hence, only one image is seen in the plate or mirror. If, however, the plate is not exactly uniform, or its faces not plane and parallel, more than one image of a distant object will be seen, and thus this affords a severe test of the *goodness* of a plate.

15. Principle of Least Time

It has been proved above (see page 20) that the path taken by light when reflected is that which makes the time of passage least. Similarly, the path taken by light when refracted is such that the time taken is least—that is, the time taken by light to travel from A to B by the path, AOB (Fig. 71), is less than that taken along

any other path. The time is given by $\frac{AO}{v_a} + \frac{OB}{v_b}$, where v_a is the velocity of light in medium *a*, and v_b the velocity in *b*.

Consider a ray, AO′B, very near AOB, so that the angles of incidence and refraction are approximately the same for both rays. Let OM be drawn perpendicular to AO′, and O′R perpendicular to OB. Now—

$$\sin i = \sin O'OM = \frac{O'M}{OO'}, \text{ and } \sin r = \sin OO'R = \frac{OR}{OO'};$$

$$\therefore {}_a\mu_b = \frac{\sin i}{\sin r} = \frac{O'M}{OO'} \cdot \frac{OO'}{OR} = \frac{O'M}{OR};$$

$$\text{but, } {}_a\mu_b = \frac{v_a}{v_b}; \quad \therefore \frac{v_a}{v_b} = \frac{O'M}{OR}, \text{ or } \frac{O'M}{v_a} = \frac{OR}{v_b}.$$

Thus, the time taken by light in travelling from M to O′ in the medium, *a*, is the same as that taken in travelling from O to R in the medium, *b*. Also, AO is very nearly equal to AM, and BO′ to BR; hence, the light takes the same time to travel from A to B along the two paths, AOB, AO′B.

Now, it can be proved mathematically that, when a function is gradually varying, its variation is zero when near a maximum or a minimum. In this case, therefore, the time is either a maximum or a minimum. It is certainly not a maximum, however, and hence it must be a minimum (cf. page 89).

16. Opacity of Mixtures of Transparent Substances

If some paraffin oil and water, or any two other liquids, which are not miscible and which have no chemical action on each other, be shaken in a test-tube, the mixture becomes opaque like milk. The result of the agitation is to break up each liquid into a multitude of minute drops, each one of which retains all the transparency that the oil and water in bulk possessed. However, when a ray of light falls on the mixture, and encounters first a drop, say, of water, a certain proportion of the light will be reflected from the first surface of the drop, the remainder passing through the drop until it encounters a neighbouring drop of oil. Here, another reflection takes place, and the weakened ray passes through the drop of oil until it encounters a drop of water, when further reflection and further weakening take place. Since there must be many such reflecting surfaces in a thin layer of the mixture, it will be apparent that the light will be unable to penetrate directly to any considerable

depth, and the opacity is explained; and the milky whiteness also is explained, for the mixture scatters the light freely instead of allowing it to pass freely through it away from the eye.

Foam is white and opaque for similar reasons, since it is a mixture of minute particles of air and water, both of which are separately transparent. *Milk* also owes its whiteness to the same cause, for it consists of a multitude of minute globules of transparent fat floating in a transparent watery liquid. *Snow* and *crushed glass* are white and opaque for similar reasons.

If two transparent liquids of precisely the same absolute refractive index are shaken together, no such results follow, for there are no reflections at the bounding surfaces. Again, if a colourless transparent solid is immersed in a colourless transparent liquid of the same refractive index, the solid would be invisible. Indeed, Rayleigh has shown that in a field of uniform illumination any transparent body would be invisible, even if the body and the surrounding medium were of different refractive index. An approach to the condition may be made by immersing a glass rod in glycerine, when it will be found that the presence of the rod might be easily overlooked on a casual glance.

A fibre of cotton when seen under the microscope is nearly transparent, but paper, which is a feltwork of such fibres, is opaque because the interstices between the fibres are occupied by air, which has a very different refractive index from the cotton. In the manufacture of tracing-paper, the air is replaced by greasy substances, whose refractive index approaches much more nearly to that of the cotton fibres than in the case of air, and consequently there is much less internal reflection and more transparency than in the untreated paper. The increased transparency of a linen lantern screen on wetting is similarly explicable.

17. Atmospheric Refraction

If the earth had no atmosphere, the rays of light proceeding from a celestial body would travel in straight lines right up to the observer's eye or telescope, and the body would be seen in its actual direction. But, when a ray, S*a* (Fig. 72), meets the uppermost layer, AA′, of the earth's atmosphere, it is refracted, and its direction changed to *ab*. On passing into a denser stratum, BB′, of air, it is further bent into the direction, *bc*, and so on. Thus, on reaching the observer, the ray is travelling in a direction, OT, different from its original direction, but in the same vertical plane. The body is seen, therefore, in the direction, OS′, although its real

direction is aS or OS. Also, since the successive horizontal layers of air, AA′, BB′, CC′, . . . are of increasing density, the effect of refraction is to bend the ray towards the normal to the surfaces of separation—that is, towards the vertical. Hence, *the apparent altitude of a star is increased by refraction.*

In reality, the density of the atmosphere increases *gradually* as the earth is approached, instead of changing abruptly at the planes, AA′, BB′, CC′, . . . Consequently, the ray describes a curved path, instead of the polygonal path, SabcO, but the general effect is the same.

The elevation due to refraction increases as the horizon is approached from the zenith. At the horizon the elevation is about 33′, and consequently, a celestial body appears to rise or set when it is 33′ below the horizon. Thus, the effect of refraction is to accelerate the time of rising, and to retard by an equal amount the time of setting of a celestial body. In particular, the sun and moon, whose angular diameters are 32′ and 31′ respectively, appear to be just above the horizon when they are really just below.

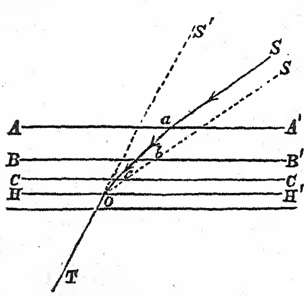


FIG. 72.

When the sun or moon is near the horizon, it appears to be distorted into a somewhat oval shape. This is due to refraction. The whole disc is raised by refraction, but the refraction increases as the altitude diminishes, so that the lower limb is raised more than the upper limb, and the vertical diameter appears contracted. The horizontal diameter is unaffected by refraction, since its two extremities are simply raised. Hence, the disc appears somewhat flattened or elliptical, instead of truly circular.

The apparent position of terrestrial objects also suffers from the same influence, distant bodies appearing elevated above their true position. The duration of *twilight* (see page 33) is also increased by refraction.

The air near the ground is more or less disturbed by convection currents, and thus the refractive index will vary from point to point, even at the same level. The rays of light from a star, besides being bent as explained above, will deviate occasionally from their position in an atmosphere at rest, and so the light from the star will sometimes be concentrated at a point and sometimes

decreased in intensity. To a fixed observer, therefore, the star begins to *twinkle* or scintillate.

That planets do not twinkle as much as stars is due to the angular size of the former. The directions of the rays from different parts of the planetary discs may vary very much, but the sum of the number of rays received by any given area, even if small, is very nearly constant and uniform illumination results. This explanation is borne out by the fact that a star seen through a large telescope does not twinkle, the average amount of light falling on such a large area as that of the object glass of the telescope being approximately constant.

Taneja

CHAPTER VI

REFRACTION THROUGH PRISMS

THE passage of light through a prism is a special case of refraction at the *plane* surfaces of separation of transparent media. In this chapter, however, what is called *dispersion*—that is, the splitting up of a compound beam of light according to the wave-lengths of its different constituents—which is also produced by the passage of the light through a prism, will not be considered. What follows, therefore, must be understood to deal with the refraction of rays and pencils of light of *definite wave-length*, and thus of definite refractive index and *colour*. Such light is sometimes referred to as *monochromatic* or *homogeneous* light, and is obtained conveniently, of a yellow colour, from a Bunsen flame coloured by the presence of a salt of sodium.

1. Prisms

From an optical point of view, a **prism** is any portion of a transparent medium lying between two plane faces inclined to each other at any angle. The line of intersection of these faces is known as the **refracting edge**, or **edge** of the prism, and a section of the prism at any point in its length, perpendicular to this edge, is called a **principal section**. The **refracting angle**, or angle of the prism is the angle between its faces, as measured by the corresponding plane angle of the principal section.

The prisms generally used for optical experiments are triangular, in the geometrical sense of the term. The principal sections of such prisms are equilateral, isosceles, or scalene triangles, according to the purpose for which the prism is intended. When the principal section is equilateral, the angle at each edge is equal to 60°, and thus there is no advantage in having three edges. With an isosceles section, there are two different angles available, and with a scalene section the angle at each edge is different, and thus the prism is equivalent to three prisms, considered in the optical sense.

2. Refraction through a Prism

In dealing with refraction through a prism, only the case where the plane of incidence and refraction is coincident with a principal section of the prism will be considered.

Let ABC (Fig. 73) represent a principal section of a prism, and BAC the refracting angle considered. Then, if the material of the prism be of greater absolute refractive index than the external medium, a ray, RN, incident on the face, AC, at N is deviated towards the normal on entering the prism, and, taking the path, NN′, is incident on the face, AB, at N′, where it is deviated away from the normal, and leaves the prism by the path, N′R′.

Thus, the ray, RN, after refraction through the prism, is *deviated* from its original direction, RN, and finally travels along the path, N′R′. The deviation (see page 73) is evidently measured by the angle, roN′. Its magnitude is found to depend on the path of the ray through the prism, and its direction is always *away from the refracting edge of the prism.* There is one position for which this deviation is a minimum—*when the prism is so placed that the incident and emergent rays make equal angles with the normals at the respective faces, then the deviation is a minimum,* and the prism is said to be in the **position of minimum deviation.** This can be proved theoretically (Art. 5). Many experiments illustrating minimum deviation will be dealt with later.

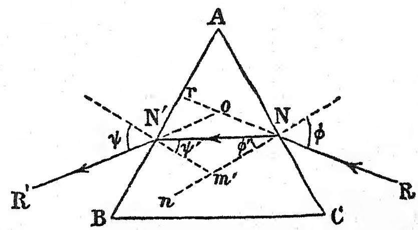


FIG. 73.

The minimum deviation produced by any prism depends on the refracting angle of the prism and the refractive index of its material relative to the external medium. An important relation between these quantities will now be established.

Let ϕ and ϕ' (Fig. 73) denote the angles of incidence and refraction at N, and ψ' and ψ the corresponding angles at N′. At N′ the angle, NN′m, is the angle of incidence, but, for the sake of symmetry with ϕ', it is here denoted by ψ' and not by ψ. Then, if D denote the deviation produced,

$$D = ro\mathrm{N}' = o\mathrm{NN}' + o\mathrm{N}'\mathrm{N},$$

$$\textit{i.e.}\ D = (\phi - \phi') + (\psi - \psi') = (\phi + \psi) - (\phi' + \psi') \ldots . (1)$$

But, since the angle between any two lines is equal to that between lines perpendicular to them,

$$nm\mathrm{N}' = \mathrm{BAC} = \mathrm{A}, \text{ the angle of the prism.}$$

$$\text{Now, } nm\mathrm{N}' = (\phi' + \psi'),$$

$$\text{hence, } \mathrm{A} = (\phi' + \psi') \ldots\ldots\ldots\ldots\ldots . (2)$$

Substituting this value in (1),

$$D = (\phi + \psi) - A \dots\dots\dots\dots\dots\dots (3)$$

Now, when the prism is in the position of minimum deviation, the ray passes *symmetrically* through the prism, and consequently, $\phi = \psi$, and $\phi' = \psi'$. Therefore, from (3),

$$D = 2\phi - A, \text{ and } \phi = \frac{D + A}{2} \dots\dots\dots\dots (4)$$

$$\text{Also, from (2), } 2\phi' = A, \text{ and } \phi' = \frac{A}{2} \dots\dots (5)$$

But, if μ denote the refractive index of the material of the prism relative to the external medium, then

$$\mu = \frac{\sin \phi}{\sin \phi'}.$$

Therefore, substituting from (4) and (5),

$$\mu = \frac{\sin \dfrac{D + A}{2}}{\sin \dfrac{A}{2}} \dots\dots\dots\dots\dots\dots (a)$$

This relation, in connexion with refraction through a prism in the position of minimum deviation, is of great practical importance.

Example.—*The refracting angle of a prism is 60°, and the minimum deviation produced in a pencil of monochromatic light is 40°. Find the refractive index of the material of the prism, given that sin 50° = 0·766.*

$$\text{Applying the relation, } \mu = \frac{\sin \frac{1}{2}(A + D)}{\sin \frac{1}{2}A},$$

$$\mu = \frac{\sin \frac{1}{2}(60° + 40°)}{\sin \frac{1}{2}(60°)} = \frac{\sin 50°}{\sin 30°}.$$

$$\text{Thus, } \mu = \frac{0·766}{0·5} = 1·53.$$

When the angle of the prism is small, a very convenient expression for D may be obtained from the relation just established. If the angle of the prism is small, the angle of incidence for minimum deviation must be small in order that the ray may pass symmetrically through the prism. Then, the angles, ϕ and ϕ' (Fig. 73), may be considered as equal approximately to the sines of these angles, and $\mu = \phi/\phi'$. Therefore, substituting from (4) and (5),

$$\mu = \frac{D + A}{A}, \text{ or } D = (\mu - 1)A \dots\dots\dots\dots (b)$$

3. Conjugate Foci by Refraction through a Prism

It is a general rule that, when any quantity is passing through its maximum or minimum value, a small change in the variable concerned produces very little effect on the magnitude of the quantity itself. For example, the magnitude of the deviation produced by refraction through a prism depends upon the path of the rays; but, when the prism is in the position of minimum deviation, any small change in the path produces little change in the magnitude of the deviation. Hence, for rays passing through a prism by paths near to that of minimum deviation, the deviation which each undergoes is very approximately the same and very nearly equal to the minimum value.

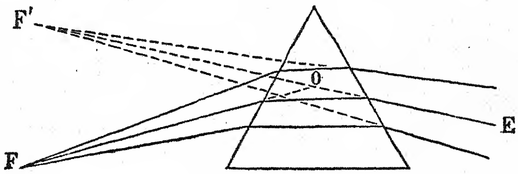


FIG. 74.

Hence, if a *small* pencil of rays proceeding from F (Fig. 74) is incident on the face of a prism at such an angle that the axis passes along the path of minimum deviation, then all the rays will be deviated to an approximately equal extent, and therefore, on emergence, will be inclined to one another at nearly the same angle as before incidence. Hence, if produced backwards, the rays of the pencil appear to come from a point, F′, such that F′O = FO. Similarly, if the path of the pencil is reversed, a convergent pencil having its focus at F′ would converge to the point, F, after refraction through the prism.

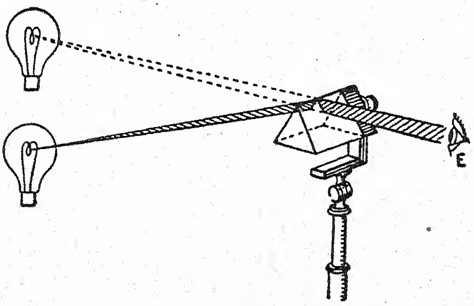


FIG. 75.

F and F′ are thus conjugate foci. Further, if the pencil be incident near the refracting edge of a prism of small refracting angle, placed in the position of minimum deviation for the axis of the prism, the thickness of the prism may be neglected, and, it may be stated that conjugate foci are on the same side of the prism and equidistant from its edge.

It follows, from what has been said above, that if an object be placed at F, its image is seen at F′ by an eye placed at E. This image is evidently *virtual* and displaced from the position of the object towards the edge of the prism (Fig. 75).

4. Practical Illustration of Minimum Deviation

The following experiment will illustrate that there is a position of minimum deviation for a prism, and explain how it is used in the determination of refractive index.

Experiment. To illustrate minimum deviation and to determine the refractive index of the material of a prism.—Take a prism, P (Fig. 76), and cut out a circular piece of cardboard, C, so that, when the prism is mounted on it, its edges stand vertically over the circumference of the disc. Attach C to P by means of soft wax.

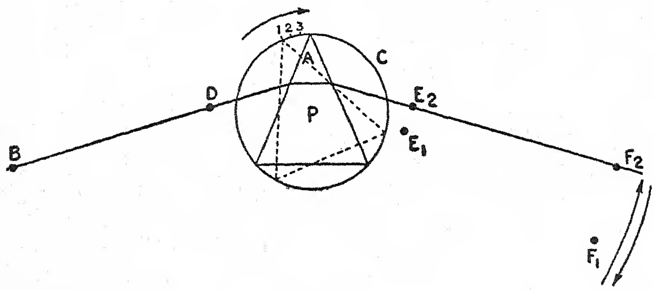


FIG. 76.

Fasten a sheet of drawing-paper to a board and on it describe a circle of the same size as C. Draw a straight line, BD, across the paper and place two pins at B and D. Place P and C in position, one face of P being nearly normal to BD.

Look through the prism from the other face and place two pins, E_1 and F_1, in the paper on the same side as the eye, so that E_1, F_1, and the images of B and D appear in a straight line. The images of B and D will appear a trifle indistinct and coloured, and the alignment must be effected by means of their centres. Test the observation by placing the eye beyond B and looking along BDE_1F_1. Mark the position of A on the paper with a pencil point, 1, remove the pins and rule in the line, E_1F_1.

Still keeping C exactly over the pencilled circle, turn it around so that A moves a little to the right. Repeat the observations. Denote the new positions of A and the pins by 2, E_2, and F_2

respectively. Repeat for several positions of A, until the angle of incidence is large and the images of the pins become very indistinct.

It will be found that as P is turned around, the line of pins on the eye side of the prism moves up to a position, E_2F_2, and then retreats. Remove the prism and cardboard. Mark in the position of the prism which gave E_2F_2, produce BD and F_2E_2 to meet the faces of the prism, and show that the angle of incidence is equal to the angle of emergence. Now measure the angle between the prolongations of BD and F_2E_2. This is the angle of minimum deviation, D (Art. 2).

The angle of minimum deviation may also be found by drawing a graph, plotting the angle of incidence against the angle between the prolongations of BD and FE, measured for the several positions of the prism and the pins. A curve will be obtained which clearly shows the angle required.

Measure the angle, A, of the prism by means of a protractor, and find the refractive index of the material of the prism by means of the relation,

$$\mu = \frac{\sin \frac{1}{2}(D + A)}{\sin \frac{1}{2}A}.$$

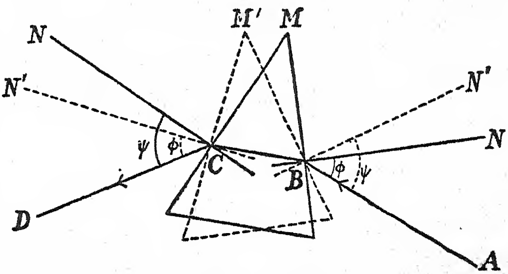


FIG. 77.

A more exact method of measuring the angle of minimum deviation and the angle of the prism will be considered below (see page 141).

5. Deviation is Minimum when Light Passes through the Prism Symmetrically

Assuming that experiment shows that there is only one position of the prism with reference to the incident ray which makes the deviation a minimum, it is easy to show theoretically that this position occurs when the incident and emergent rays occupy symmetrical positions with respect to the prism.

For, if the incident ray, AB (Fig. 77), is fixed in direction, and the position of the prism, M, is such as to make the deviation of the ray, ABCD, a minimum when the angles of incidence and emergence are ϕ and ψ respectively, ϕ and ψ being unequal to

each other, then also will the deviation be a minimum when the angles of incidence and emergence are respectively ψ and ϕ. So that there are two positions of the prism, M and M′ (Fig. 77), which render the deviation a minimum. The experiment above shows this to be false, and it will be noted that the two cases merge into one only when ϕ and ψ are equal to each other.

An approximate theoretical proof is as follows:—It can be proved simply that an angle increases more rapidly than its *sine*, a glance at a table of sines showing this. From this it follows that as an angle of incidence increases, the angle of refraction also increases, but at a lesser rate, and, therefore, that the angle of deviation increases (see page 73).

Consider now a ray passing symmetrically through a prism as in Fig. 73. For this ray, $\phi = \psi$, and $\phi' = \psi'$. Now suppose another ray passes for which ϕ' is slightly increased. Since $\phi' + \psi' = A$, ψ' will decrease by an equal amount. ϕ will increase and ψ will decrease, but $\phi - \phi'$ will be greater than $\psi - \psi'$; that is, the increment of ϕ will be greater than the decrement of ψ. Hence, the total deviation of this new ray, which equals $\phi + \psi - A$, will be greater than the deviation of the symmetrical ray. In other words, the deviation is least when $\phi = \psi$.

This result is of interest in connexion with the theory of spherical aberration of lenses. We have seen that a prism forms the best image of a point-source when used in the position of minimum deviation. In the next chapter we shall see that the action of lenses is not unlike that of prisms, and this leads us to expect that there would be an analogous condition for the optimum performance of a lens. It is, in fact, found that spherical aberration is a minimum when the rays of light are deviated approximately equal amounts at the two surfaces of a lens. (See Chapter VII., Art. 19.)

CHAPTER VII

REFRACTION AT SPHERICAL SURFACES. LENSES

IN the last two chapters, the problems of refraction at the surface of separation of two transparent media have been considered for the case when the surface of separation is plane. In order to understand the behaviour of a lens, however, it is necessary to consider the case of refraction when the surface of separation is not plane but spherical, since, generally speaking, a lens is defined as any medium bounded by two curved surfaces, or by one curved surface and the other plane.

The behaviour of a lens may also be considered by means of the behaviour of a prism dealt with in the last chapter.

In what follows below, the two methods of consideration will be fully dealt with.

V. P. Singh

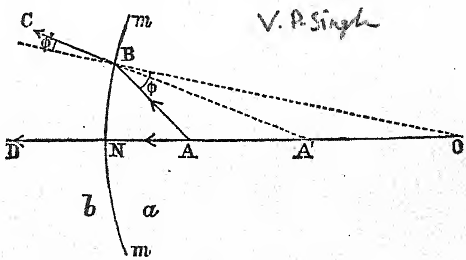


FIG. 78.

1. Refraction at a Single Spherical Surface

Let *mm* (Fig. 78) represent the concave spherical surface of separation of two media, *a* and *b*, of which *b* is the denser, and let AB represent an extreme ray of a small pencil of light, diverging from A, and incident *directly* on *mm* along AN. AB is refracted at B along BC, while AN, being normal to *mm*, passes along ND without undergoing deviation.

The virtual focus of the refracted pencil will now be at the point, A′, from which BC and ND apparently diverge. Or A′ is the conjugate focus of A, and may be considered as the image of A, formed by refraction at the spherical surface, *mm*.

Let O represent the centre of curvature of *mm*. Then OB is the normal at B, and the angles of incidence and refraction, ϕ and

ϕ', are respectively equal to, or supplementary to ABO and A′BO. Hence, if μ denote the refractive index from a to b,

$$\mu = \frac{\sin \phi}{\sin \phi'} = \frac{\sin \phi}{\sin \text{BOA}} \cdot \frac{\sin \text{BOA}}{\sin \phi'};$$

$$\therefore \mu = \frac{\sin \phi}{\sin \text{BOA}} \cdot \frac{\sin \text{BOA}'}{\sin \phi'}.$$

But, from the triangles, BOA and BOA′,

$$\frac{\sin \phi}{\sin \phi'} = \frac{\text{AO}}{\text{BA}} \cdot \frac{\text{BA}'}{\text{A}'\text{O}} = \mu.$$

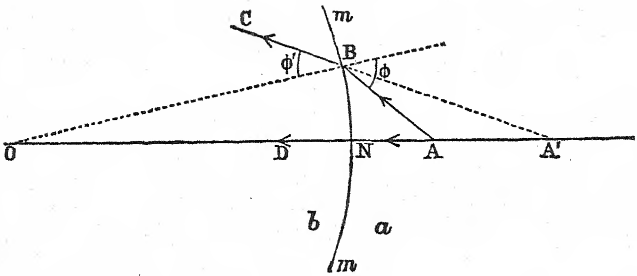


FIG. 79.

If BN be sufficiently small, then BA and BA′ are approximately equal to NA and NA′ respectively. Hence:—

$$\mu = \frac{\text{AO}}{\text{NA}} \cdot \frac{\text{NA}'}{\text{A}'\text{O}}.$$

As before, let NA be represented by u, NA′ by v, and NO by r. †Then, with the first sign convention (see page 40),

$$\mu = \frac{\text{AO}}{\text{NA}} \cdot \frac{\text{NA}'}{\text{A}'\text{O}} = \frac{(r - u)\, v}{u\,(r - v)}.$$

Similarly, for a convex spherical surface of separation of the media (Fig. 79), it may be shown that

$$\mu = \frac{\text{AO}}{\text{NA}} \cdot \frac{\text{NA}'}{\text{A}'\text{O}}.$$

† A reader using the Real is Positive sign convention should omit the passages between asterisks and read instead the alternative passages at the end of the chapter.

In this case, however, AO = AN + NO = $u + (-r) = u - r$, and A′O = A′N + NO = $v + (-r) = v - r$. Therefore, whether *mm* be concave or convex,

$$\mu = \frac{(r-u)\,v}{u\,(r-v)}.$$

From this relation:—$\mu ru - \mu uv = rv - uv$,

and $\mu ru - rv = uv\,(\mu - 1)$.

Dividing through by *uvr*, we get

$$\frac{\mu}{v} - \frac{1}{u} = \frac{\mu - 1}{r} \quad \dots\dots\dots\dots\dots\dots (1)$$

This is a general relation, applicable to all cases of refraction of small direct pencils at a *single* plane or spherical surface of separation of two media, whose relative refractive index *for the direction in which the light is travelling* is denoted by μ. When *mm* is a plane surface (see page 75), then r is infinite and $\frac{\mu-1}{r} = \frac{\mu-1}{\infty} = 0$, and the relation reduces to $\frac{\mu}{v} - \frac{1}{u} = 0$, or $v = \mu u$, a result obtained previously.

2. The Principal Foci of a Spherical Refracting Surface

In the above relation, when v is infinite, the value of u is called the *first principal focal distance*. Denoting it by f_1:—

$$f_1 = -\frac{r}{\mu - 1}.$$

The point on the principal axis at a distance of f_1 from the pole is called the *first principal focus*, and rays proceeding from it (f_1 positive), or to it (f_1 negative), are refracted so as to proceed parallel to the axis.

Similarly, by making u infinite, the value of v is called the *second principal focal distance*. Denoting it by f_2:—

$$f_2 = \frac{\mu r}{\mu - 1}.$$

The point on the principal axis at a distance of f_2 from the pole is called the *second principal focus*, and rays originally proceeding parallel to the axis are refracted so as to diverge from it (f_2 positive) or converge to it (f_2 negative).*

3. Construction for Refraction at a Spherical Surface

Let MJ (Fig. 80) be any ray incident on the surface of radius, OJ. With O as centre, describe two circles of radii, $\mu . OJ$ and $\frac{OJ}{\mu}$ respectively, where μ is the refractive index. Produce MJ to cut the outer circle in R. Join OR, cutting the inner circle in S. Then JS is the refracted ray.

For since $OS/OJ = OJ/OR$, the triangles, OSJ, OJR, are similar. Hence, the angle OJS = the angle ORJ, and therefore

$$\frac{\sin OJR}{\sin OJS} = \frac{\sin OJR}{\sin ORJ} = \frac{OR}{OJ} = \mu,$$

from which it is seen that the angle, OJS, is the angle of refraction.

It is to be noted that all rays directed towards R are refracted so as to pass through S. Similarly, all rays coming from S are refracted so as to appear to come from R. R and S are conjugate foci for *all* rays, and the surface is said to be *aplanatic* (see Art. 4).

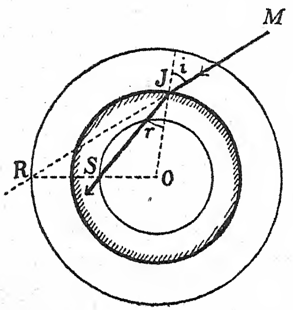


FIG. 80.

*Example.—*A gold-fish globe of radius 6 in. is filled with water. Determine the apparent position of inside the globe, 4 in. from its surface, when seen by an eye looking along the radius of the globe.*

The surface at which refraction takes place is spherical and, neglecting the action of the glass of the globe, the relation,

$$\frac{\mu}{v} - \frac{1}{u} = \frac{\mu - 1}{r},$$

may be used, in which $u = 4$ in., $r = 6$ in., and $\mu = \frac{3}{4}$ (water to air). Thus:—

$$\frac{3}{4v} - \frac{1}{4} = -\frac{1}{24} \text{ and } \frac{3}{4v} = \frac{5}{24}.$$

$$\text{Hence, } 20v = 72 \text{ and } v = 3{\cdot}6 \text{ in.}$$

That is, the apparent position of the point is inside the globe on the radius passing through its real position, and 3·6 in. from the surface. *

4. Lenses

A lens may be defined generally as a portion of a refracting medium enclosed between two surfaces of definite geometrical form and having a common normal. Usually these surfaces are portions

of the surfaces of spheres or plane surfaces, and the medium most generally employed is glass. Lenses of such a form may be considered as solids of revolution. For example, if any one of the sections shown in Figs. 81 and 82 be supposed to revolve about a central horizontal axis in the plane of the diagram, the solid described by such revolution determines the form of the lens corresponding to that section. It is usual to divide lenses into two classes:—

(1) *Convex or Convergent Lenses*

Of these there are three chief forms, as shown in Fig. 81:—

(*a*) Double convex,

(*b*) Plano-convex,

(*c*) Concavo-convex, or convergent meniscus.

The distinguishing characteristic of these lenses is that they are *thicker at the centre than at the edges.*

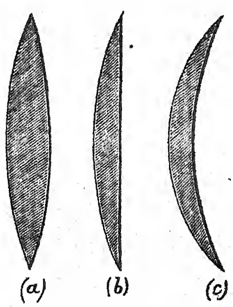


FIG. 81.

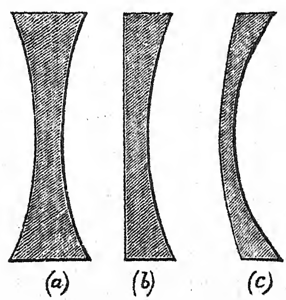


FIG. 82.

(2) *Concave or Divergent Lenses*

Of these also there are three chief forms, as shown in Fig. 82:—

(*a*) Double concave,

(*b*) Plano-concave,

(*c*) Convexo-concave, or divergent meniscus.

The distinguishing characteristic of these lenses is that they are *thinner at the centre than at the edges.*

The action of any of these forms of lens on a pencil of rays of light passing through them depends on the refractive index of the medium of which they are made, *relative* to the surrounding medium. Usually the medium is glass surrounded by air—that is, the medium of the lens is of higher refractive index than the surrounding

medium. In this case, *convex lenses cause the rays of a pencil to become more convergent* after passing through them, and for this reason are called **convergent** lenses. Similarly, *concave lenses cause the rays of a pencil to become more divergent* after passing through them, and for this reason are called **divergent** lenses.

When, however, the refractive index of the medium of the lens is *less* than that of the surrounding medium, then a *convex* lens acts as a *divergent* lens, and a *concave* lens as a *convergent* lens.

This action of convex and concave lenses may be explained in the following way:—The section of a double convex lens may be considered as similar to that of two prisms placed base to base (Fig. 83). Consider the rays, PA and PB, incident on the prisms at A and B. As explained above (page 87), these rays are deviated

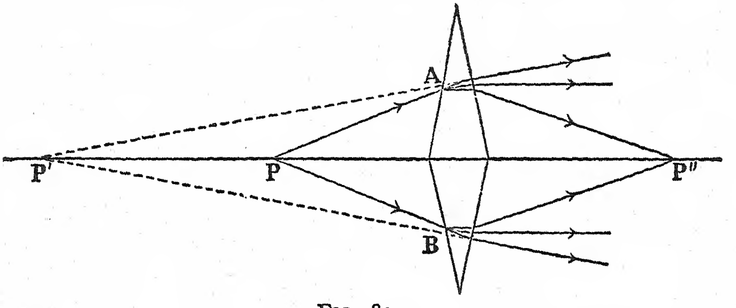


FIG. 83.

away from the edges of the prisms on which they are incident, and are thus *less* divergent after refraction. The paths of the rays, PA and PB, after passing through the lens, depend on the magnitude of the deviation produced. They may either diverge from P′, run parallel, or, if the deviation be sufficiently great, converge to a point, P″.

Similarly, the section of a double concave lens may be considered as similar to that of two prisms placed apex to apex (Fig. 84). In this case the rays, PA and PB, are refracted away from the edges of the prisms—that is, from the centre of the lens—and, after refraction, appear to diverge from the point, P′. The rays are thus *more* divergent after passing through the lens than before.

In the cases of the prisms shown in Figs. 83 and 84, the positions of P′ and P″ will depend on the positions of A and B, but in the case of a lens, owing to the curvature of the surface, all rays coming from P would, after refraction, pass through the same point. When

this is accurately the case, the curvatures of the surfaces of the lens are specially adapted to the existing conditions, and the lens is said to be aplanatic. For ordinary lenses, with spherical or plane surfaces, this is only approximately the case, and the defect resulting from this lack of accuracy is known as *spherical aberration* (see Art. 19). However, when the surfaces of the lens are only very small portions of spherical surfaces, spherical aberration is almost negligible, and, for all practical purposes, the lens is aplanatic.

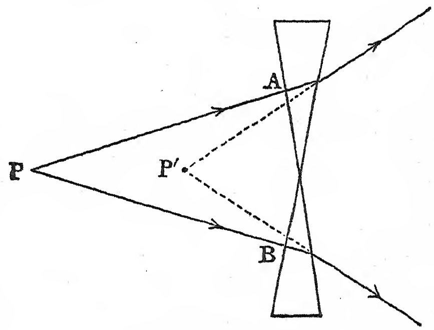


FIG. 84.

A lens, being a solid of revolution, is symmetrical about its centre, and hence all sections passing through the axis of revolution are similar. Thus it follows that what has been explained above, for one section, is true for all similar sections, and, consequently, if a *pencil* of light, diverging from P, be refracted through a lens, all the rays are deviated symmetrically, and, after refraction, pass through the same point.

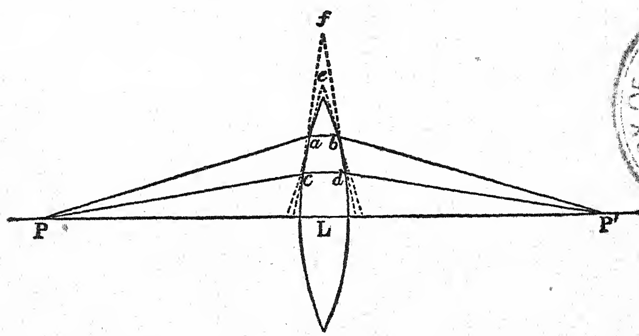


FIG. 85.

Consider the refraction of the rays, P*ab*P′ and P*cd*P′, through the lens, L (Fig. 85). It is evident from the figure that, in order that the rays may pass through P′, the deviation of P*ab*P′ must be greater than that of *PcdP′*. At *a* and *b* draw tangent planes to the surfaces of the lens meeting at *e*, and at *c* and *d* draw tangent planes meeting in *f*.

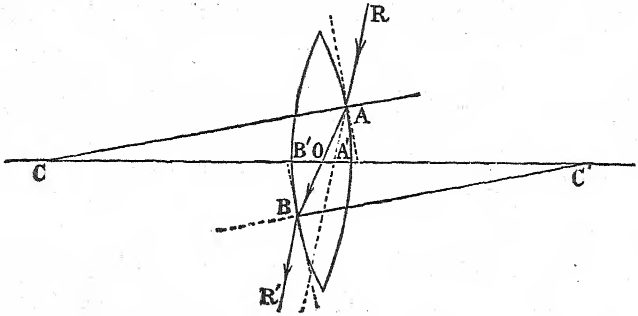


FIG. 86.

Now the deviation in the case of the ray, P*ab*P′, is that due to the prism of refracting angle, *aeb*, and the deviation for the ray, P*cd*P′, is that due to the prism of angle, *cfd*. But, when the angle of the prism is small, the deviation produced is approximately proportional to the angle of the prism (page 88). Therefore, in this case, the deviation for the ray, P*ab*P′, is greater than that for P*cd*P′, and thus it is possible for both rays to pass through P′.

5. Definitions

The **principal axis** of a lens coincides with its axis of revolution. When the surfaces of the lens are spherical, this axis passes through the centres of curvature of these surfaces; when one surface is plane, and the other spherical, it passes through the centre of curvature of the spherical surface and is normal to the plane surface.

The **optical centre** of a lens is *that point on the principal axis through which pass all rays, or, in certain cases, the prolongations of the portions of such rays which are within the lens, having their paths parallel before and after refraction through the lens.*

Let C and C′ (Fig. 86) be the centres of curvature of the two spherical surfaces of a lens. Draw any radius, CA, and through C′ draw the radius, C′B, parallel to CA. Join AB, cutting the principal axis, CC′, in O. Then O is the optical centre of the lens.

For, if AB represent the path, *through the lens*, of the ray, RABR′, then, by construction, AB makes equal angles at A and B with the normals, CA and C′B, and, consequently, the incident and emergent rays, RA and BR′, also make equal angles with these normals, and are therefore parallel. The action of the lens is, under the conditions considered, exactly similar to that of a plate enclosed by the parallel tangent planes at A and B (see page 63).

This is true for *any* two parallel radii, CA and C′B, and hence O is the optical centre of the lens, as defined above. From the triangles, AOC and BOC′,

$$\frac{CO}{C'O} = \frac{CA}{C'B} = \frac{CA'}{C'B'}.$$

$$\text{Hence, } \frac{CA'}{C'B'} = \frac{CO}{C'O} = \frac{CA' - CO}{C'B' - C'O} = \frac{OA'}{OB'}.$$

Thus, *the point,* O, *divides the thickness of the lens into segments which are proportional to the radii of curvature of the adjacent surfaces.*

In the cases of double convex and double concave lenses, the optical centre lies in the interior of the lens; in the cases of plano-convex and plano-concave lenses, it is situated on the spherical surfaces; and, in the cases of convergent or divergent meniscus lenses, it lies outside the lens on the same side as the surface of lesser radius of curvature.

Although the incident and emergent rays, RA and BR′, are parallel, they are not in the same straight line. If, however, the thickness of the lens be small, the displacement produced is negligible, and it may be stated that *all rays passing through the optical centre of the lens suffer no deviation, but continue their course in the same straight line.*

Any line, other than the principal axis, passing through the optical centre is called a *secondary axis.*

6. Principal Focus and Focal Length

When a parallel pencil of light is incident on a lens in a direction parallel to the principal axis of the lens, the rays, after refraction through the lens, converge to or diverge from a point on the principal axis. This point is called the **principal focus** of the lens, and its distance from the optical centre of the lens, measured along the principal axis, is called the **focal length** of the lens.

In the case of a convex lens of any form, the parallel pencil of rays is made to *converge* to a point, F (Fig. 87), on the *other side of the lens.* A concave lens of any form causes the rays to *diverge* from a point, F (Fig. 88), on the *same side of the lens* as the

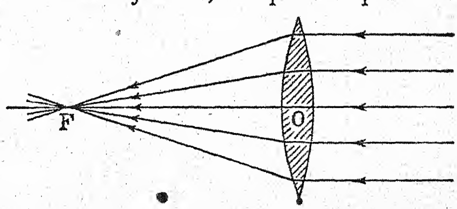


FIG. 87.

incident pencil. These two facts should be carefully noted. In the examples given, the incident light is shown coming from the right of the lens; in Fig. 87, F is behind the lens—that is, on the left of the lens, and in Fig. 88, F is in front of the lens—that is, on the right of the lens. †Applying the usual *sign* convention (see page 40), distances being measured from O, the focal length of a *convex* lens is *negative*, and that of a *concave* lens is *positive*. If the pencil of parallel light is incident in a direction parallel to a secondary axis, AF′ (Fig. 89), inclined at a *small* angle to the principal axis, the focus of the refracted pencil is on the secondary axis at a point, F′, such that OF′ is approximately equal to the focal length of the lens.

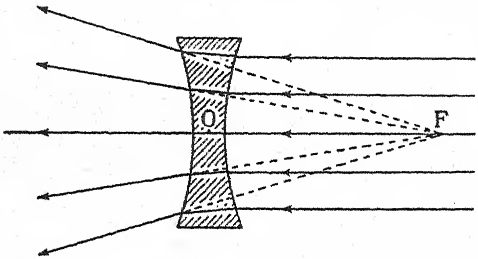


FIG. 88.

7. Path of a Ray through a Lens

Let C and C′ (Fig. 90) denote the centres of curvature of the surfaces of the lens, L, and let the ray, PA, meet the surface of the lens at A. Join CA and produce it to N; then, CAN is the normal at A, and the ray, PA, is refracted into the lens along AB, making an angle, BAC, with the normal such that

$$\frac{\sin PAN}{\sin BAC} = \mu,$$

where μ denotes the refractive index of the material of the lens relative to the surrounding medium. Similarly at B, the ray meets the second surface of the lens, and is refracted along BP′ in such a direction that

$$\frac{\sin P'BN}{\sin C'BA} = \mu.$$

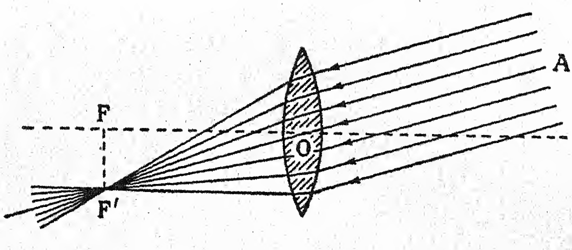


FIG. 89.

† Using the Real is Positive convention, the focal length of a convex lens is positive because the focus is real, while that of a diverging lens is negative.

To determine the path of any given ray by this construction would be a very troublesome process. It is important, therefore, to note two particular cases in which the path is determined readily.

(1) *Any ray passing through the optical centre of a lens continues its course in the same straight line* (Art. 5). For ordinary purposes, the optical centre of a *thin* lens may be taken at any point on the principal axis in its thickness.

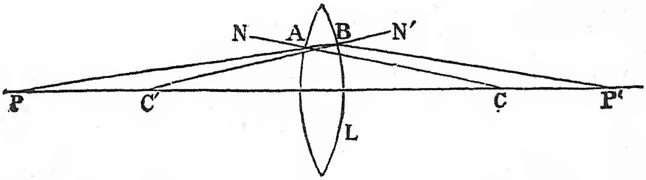


FIG. 90.

(2) *When the incident ray is parallel to the principal axis, the refracted ray passes through the principal focus* (Art. 6).

These two rules are used extensively in drawing diagrams showing the formation of images of objects produced by lenses. They may be compared with the two rules used in the case of reflection by spherical mirrors (see page 48).

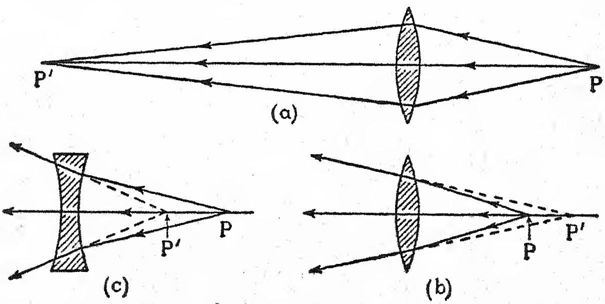


FIG. 91.

8. Conjugate Foci

When a pencil of rays of light, diverging from a point, P (Fig. 91), on the principal axis of a lens, is refracted through the lens, the focus of the refracted pencil is another point, P′, also on the principal axis. These points, P and P′, are called **conjugate foci.**

It should be noted that, with the convex lens in Fig. 91 (*a*), P and P′ are on opposite sides of the lens, whilst in Fig. 91 (*b*), P and P′ are on the same side of the lens. With the concave lens in Fig. 91 (*c*), P and P′ are on the same side of the lens. These facts will be dealt with later (Art. 10). It may be stated here, however, that if the conjugate foci are both *real*, the image of an object placed at either focus is formed at the other; but, if one of the foci is *virtual*, then the image of an object placed at that focus is not formed at the other, but rays converging to the *virtual* focus are refracted through the conjugate focus. In other words, the optical relation between conjugate foci assumes the reversibility of the direction of the light.

When the point, P, is on any secondary axis *inclined at a small angle to the principal axis*, the point, P′, is also on that secondary axis. It is important to notice, however, that secondary axes have not the same relation to lenses as they have to spherical mirrors. In the case of spherical mirrors, the secondary axes have exactly the same geometrical relation to the spherical reflecting surface as the principal axis, but in the case of lenses, this is not so and refraction along secondary axes involves several complications which cannot be considered now. When, however, the angle between a secondary axis and the principal axis is *small*, the laws applicable to refraction along the principal axis may be applied with approximately correct results.

9.* General Relation for Lenses

The relation between the distances of conjugate foci from the optical centre of a lens and the focal length of the lens will be deduced by a simple geometrical method later (Art. 13). Here, however, a relation between these distances will be established by the application of the relation applying to refraction at a *single* spherical surface deduced above (Art. 1). In this relation,

$$\frac{\mu}{v} - \frac{1}{u} = \frac{\mu - 1}{r},$$

μ represents the relative refractive index from one medium to the other in the direction taken by the rays of light.

Let *abcd* (Fig. 92) represent a lens, and let r denote the radius of curvature of the surface, *ad*, and s the radius of curvature of the surface, *cb*. Then, first considering refraction at the surface, *ad*, the pencil of rays diverging from p_1, appears, after refraction, to

diverge from p. If Op_1 be denoted by u, and Op by v', and the refractive index from air to the lens by μ, then

$$\frac{\mu}{v'} - \frac{1}{u} = \frac{\mu - 1}{r} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

In this investigation, the thickness of the lens is assumed to be negligibly small compared with u and v, so that O may be taken as a point on either of the spherical surfaces, *ad* and *cb*.

After this first refraction at the surface, *ad*, the pencil diverging from p may be supposed to be incident on the surface, *cb*, and to undergo refraction at that surface from the lens into air, the focus of the emergent rays being at p'. Now, if μ denote the refractive index from air to the lens, then $\frac{1}{\mu}$ denotes the refractive index from

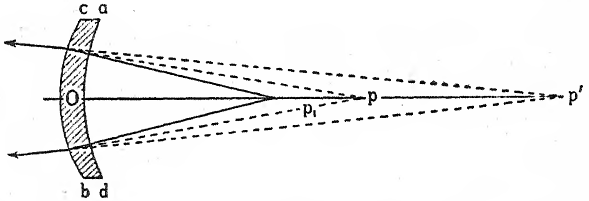


FIG. 92.

the lens to air (see page 62). Hence, if Op' be denoted by v,

$$\frac{\frac{1}{\mu}}{v} - \frac{1}{v'} = \frac{\frac{1}{\mu} - 1}{s},$$

$$\text{or } \frac{1}{v} - \frac{\mu}{v'} = \frac{1 - \mu}{s} = -\frac{\mu - 1}{s} \quad \ldots\ldots\ldots\ldots (2)$$

Adding the equations (1) and (2),

$$\frac{1}{v} - \frac{1}{u} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right) \quad \ldots\ldots\ldots\ldots\ldots (3)$$

This establishes a relation between u and v, the distances of the conjugate foci from the lens, and the radii of curvature of the spherical surfaces.

If u be infinite—that is, if the incident rays are parallel, then the emergent pencil passes through the *principal focus*, and v

becomes equal to the *focal length* of the lens (Art. 6). Therefore, if the focal length be denoted by f,

$$\frac{1}{f} - \frac{1}{\infty} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right);$$

$$\therefore \frac{1}{f} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right).$$

Substituting in equation (3), then,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \qquad \text{.......................(4)}$$

which is the most important relation applying to lenses.

For the sake of clearness, the details of Fig. 92 have been so chosen that all the distances are positive. Wherever reference is made to *sign*, it must be understood that the convention explained above (page 40) has been adopted. If due regard be paid to sign the different relations established here may be obtained in the same form for all cases of refraction through a lens. The general relation,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f},$$

is therefore applicable to all cases, and, if p' be considered as the image of p_1, establishes a relation between the distances of the object and image from the optical centre of the lens and the focal length of the lens.

Example.—*Find the focal length of a double concave lens, the radii of curvature of its surfaces being* 25 *cm. and* 50 *cm. respectively, and the refractive index of the material being* 1·5.

Using the relation, $\frac{1}{f} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right)$, and supposing the light to be incident on the more concave surface,

$$\mu = 1{\cdot}5,\ r = 25 \text{ cm., and } s = -50 \text{ cm.}$$

$$\text{Then, } \frac{1}{f} = (1{\cdot}5 - 1)\left(\tfrac{1}{25} + \tfrac{1}{50}\right) = \tfrac{1}{2} \times \tfrac{3}{50} = \tfrac{3}{100}.$$

$$\text{Thus, } f = 33\tfrac{1}{3} \text{ cm.}$$

Supposing the light to be incident on the other surface,

$$\mu = 1{\cdot}5,\ r = 50 \text{ cm., and } s = -25 \text{ cm.}$$

$$\text{Then, } \frac{1}{f} = (1{\cdot}5 - 1)\left(\tfrac{1}{50} + \tfrac{1}{25}\right) = \tfrac{3}{100};$$

$$\therefore f = 33\tfrac{1}{3} \text{ cm. as before.}$$

10.* Relative Positions of Point Object, P, and Image, P′

FIRST METHOD.—By application of the above relation, the position of P′, corresponding to any given position of P, can be determined. Thus, taking the case of a convex lens, and assuming P to be at infinity,

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}, \text{ and } \frac{1}{v}-\frac{1}{\infty}=\frac{1}{f}; \quad \therefore v=f.$$

This means that the image, P′, is at the principal focus, F—that is, at a distance, f, on the other side of the lens. Note that f is negative for a convex lens.

If P is at a distance in front of the lens *numerically* equal to twice the focal length, f, then,

$$\frac{1}{v}+\frac{1}{2f}=\frac{1}{f}, \text{ and, } \frac{1}{v}=\frac{1}{2f}, \text{ or, } v=2f.$$

This means that the image, P′, is at a distance numerically equal to $2f$ on the other side of the lens. Note that v is again negative.

If P is at a distance in front of the lens *numerically* equal to the focal length, f, then—

$$\frac{1}{v}+\frac{1}{f}=\frac{1}{f}, \text{ and, } \frac{1}{v}=0, \text{ or, } v=\infty.$$

If P is at a distance in front of the lens *numerically* equal to $\frac{f}{3}$, then,

$$\frac{1}{v}+\frac{3}{f}=\frac{1}{f}, \text{ and, } v=-\frac{f}{2}.$$

This means that the image, P′, is at a distance numerically equal to $f/2$ *in front of* the lens—that is, on the same side of the lens as the object, P. Note that in this case v is positive, and the image is *virtual*.

Again, taking the case of a concave lens, and assuming P to be at infinity:—

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}, \text{ and, } \frac{1}{v}-\frac{1}{\infty}=\frac{1}{f}, \text{ or } v=f.$$

This means that the image, P′, is at the principal focus, F—that is, at a distance, f, on the same side of the lens as P. It should be remembered that f is positive for a concave lens. Note that v is positive and the image is *virtual*.

By proceeding as indicated above, it can be shown that, in the case of a convex lens, as P moves from infinity in front of the lens up to a distance equal to f from the lens, P′ moves from F on the other side of the lens to infinity. So far the image has been *real*. As P continues to move from a distance, f, in front of the lens up to the lens, P′ disappears at infinity on the other side of the lens, reappears at infinity in front of the lens, and moves from infinity up to the lens. During this stage the image has been *virtual*.

Similarly, it can be shown that, in the case of a concave lens, as P moves from infinity in front of the lens up to the lens, P′ moves from F in front of the lens up to the lens. The image is always *virtual* except for a *convergent* pencil.

SECOND METHOD.—A second method of tracing the path of P′ as P moves from infinity on one side of the lens up to the lens, and then to infinity on the other side of the lens, is similar to the second method for spherical mirrors (see page 44). Thus, taking the general relation, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we have:—

$$uf - vf = uv.$$

$$\text{That is, } f^2 + uf - vf - uv = f^2,$$

$$\text{and } (f + u)(v - f) = f^2.$$

Let x and x' denote the distances of object, P, and image, P′, measured from F′ and F respectively, F being the principal focus and F′ a point distant f from the lens but on the other side of it (see Fig. 99), the ordinary convention being used. Then,

$$x = u + f, \text{ and } x' = -(f - v),$$

$$\text{and } xx' = -f^2.$$

Note that, f^2 being positive, x and x' are always of different sign—that is, the object is on the opposite side of F′ that the image is of F. This leads to the following results:—

I. CONVEX LENSES

(1) When P is at infinity in front of the lens (Fig. 87)—that is, when the incident rays are parallel to the principal axis, P′ is at the principal focus, F, behind the lens. Or, when $x = +\infty$, $x' = 0$.

(2) When P is in front of the lens at a distance *numerically* equal to $2f$, then P′ is at an equal distance behind the lens. Or when $x = -f$, $x' = +f$.

(3) When P is at F′ in front of the lens (Fig. 93)—that is, when the incident rays diverge from a point in front of the lens at a distance from its optical centre equal to the focal length, then P′ is at infinity on the other side of the lens, and the refracted rays are parallel to the principal axis. Or, when $x = 0$, $x' = \infty$.

(4) When P is at O, P′ is also at O. Or, when

$$x = +f, x' = -f.$$

These are the *most important* results for *convex* lenses. To summarise, as P travels from infinity on the left up to F′ on the left (Fig. 99), P′ travels in the same direction from F on the right to infinity on the right. As P travels from F′ on the left to O, P′, after disappearing at infinity on the right, reappears at infinity on the left and travels in the same direction as P to O. For the sake of completeness, the cases of the *virtual object* must be added:—

(5) When P is at F—that is, when the incident rays *converge* to the principal focus behind the lens, then P′ is midway between O and F. Or, when

$$x = 2f, x' = -\frac{f}{2}.$$

(6) When P is at infinity behind the lens, the incident rays are parallel, and P′ is at F, as in (1). Or, when

$$x = -\infty, x' = 0.$$

V.P. Singh

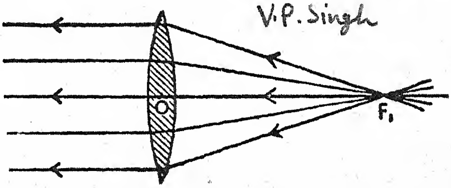


FIG. 93.

In cases such as (5) and (6), when the incident rays converge towards a point, P, and do not actually pass through it, the point can be described as a virtual object. Similar definitions apply to mirrors. Thus a plane mirror produces a real image of a virtual object (see page 21).

II. CONCAVE LENSES

(1) When P is at infinity in front of the lens (Fig. 88)—that is, when the incident rays are parallel to the principal axis, P′ is at the principal focus, F, in front of the lens. Or, when $x = +\infty$, $x' = 0$.

(2) When P is at F in front of the lens (Fig. 94), then P′ is midway between F and O—that is, at the point, h. Or, when $x = 2f$, $x' = -\frac{f}{2}$.

(3) When P is at O, P′ is also at O. Or, when $x = +f, x' = -f$.

These are the *most important* results for *concave* lenses. To summarise, as P travels from infinity in front of the lens, on the right, up to O, P′ travels in the same direction from F in front of the lens up to O. For the sake of completeness, the following cases must be added:—

(4) When P is at F′—that is, when the incident rays *converge* to a point behind the lens at a distance from the optical centre equal to the focal length of the lens, then P′ is at infinity. Or, when $x = 0, x' = -\infty$.

This is indicated (Fig. 94) by the two dotted lines from the right converging towards F′, and turned by the lens into the parallel dotted lines to the left.

(5) When P is at a distance behind the lens *numerically* equal to $2f$, P′ is the same distance in front of the lens. Or, when $x = -f$, $x' = +f$.

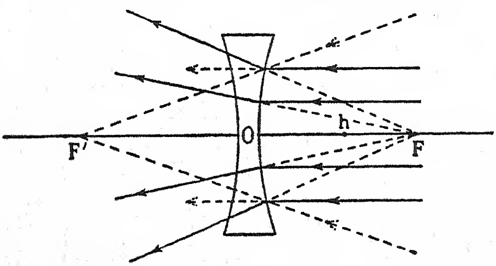


FIG. 94.

(6) When P is at infinity behind the lens, the incident rays are parallel, and, as in (1), P′ is at F. Or, when $x = -\infty, x' = 0$.

Of the above results, I., (1), (3), (6), and II., (1), (4), (6), are summed up in the statement that, when a parallel pencil of light is incident on a lens, the focus of the refracted pencil is at the principal focus of the lens. I., (4), and II., (3) are evident and easily remembered. I., (2), (5), and II., (2), (5) may be deduced from the general relation, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$.

General Rule. When an image is formed by refraction through a lens, any motion of the object along the principal axis of the lens produces a corresponding motion of the image in the same direction along the principal axis.

The application of this rule is simple. For example, from I., (1) and (3), it is seen that, as P moves from infinity to F′, P′ moves in the same direction from F to infinity behind the lens. Similarly,

from II., (2) and (3), as P moves from F to O, P′ moves in the same direction from h to O (Fig. 94). *

11. Crown Glass Lenses

When drawing to scale diagrams of lenses of known focal lengths, it is not sufficient, as in the case of spherical mirrors, to know the radii of curvature of its surfaces, since the focal length depends upon the refractive index of the material of the lens as well as upon these radii. However, in the case of lenses of *crown glass* there is a simple rule which expresses with a near approach to accuracy the relation between focal length and radii of curvature.

The refractive index of crown glass is approximately 1·5, and on substitution in the general relation†—

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right); \qquad \therefore\ \frac{1}{f} \approx \frac{1}{2}\left(\frac{1}{r} - \frac{1}{s}\right).$$

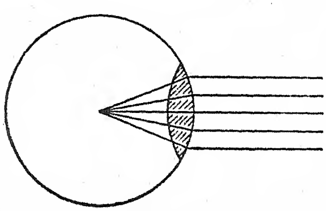


FIG. 95.

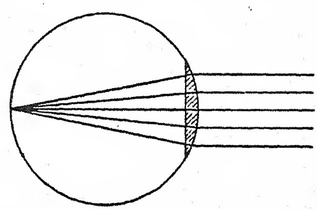


FIG. 96.

In the case of a double convex lens whose surfaces are of the same radius of curvature, $s = -r$, and therefore—

$$\frac{1}{f} \approx \frac{1}{2}\cdot\frac{2}{r} \approx \frac{1}{r}, \text{ or } f \approx r.$$

In other words, the principal focus of the lens is approximately at the centre of a sphere of which the front surface of the lens forms a part (Fig. 95). In the case of a plano-convex lens:—

$$\frac{1}{f} \approx \frac{1}{2}\cdot\frac{1}{r}, \text{ or, } f \sim 2r.$$

That is, if the spherical surface is in front, the principal focus of the lens is on the circumference of the sphere of which the front surface forms a part (Fig. 96). Similar relations exist for equi-concave and for plano-concave lenses.

† This relation holds also if the Real is Positive sign convention is used, f and r being positive for a bi-convex lens, while s is negative.

12. General Construction for Images formed by Lenses

Let AB (Figs. 97 and 98) represent an object placed on the principal axis of a lens. To determine the position of the image of the point, A, it will be sufficient to determine the point of intersection, after refraction through the lens, of any two rays originally diverging from A. It has been seen (Art. 7) that the path of a ray

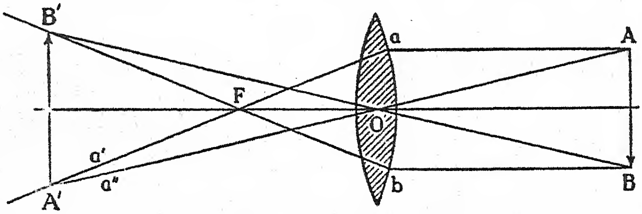


FIG. 97.

is readily determined when it is incident parallel to the principal axis, or passes through the optical centre of the lens. Consider, then, rays coming from A along two such paths.

The ray, A*a*, incident parallel to the principal axis, is refracted along *aa'* in a direction passing through F, the principal focus of the lens. The ray, AO, passing through O, the optical centre of

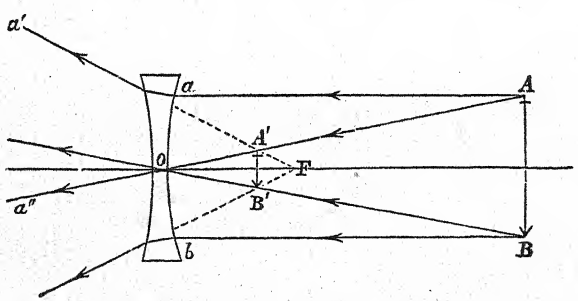


FIG. 98.

the lens, is not deviated, but continues its course along the straight line, AO*a''*. The two refracted rays, *aa'* and O*a''*, actually intersect (Fig. 97), or appear to intersect (Fig. 98), at A', which is, therefore, the image of A. Similarly, the image of B is formed at B', and the images of all points between A and B being assumed to lie between A' and B', the complete image, A'B', is determined.

When the rays intersect (Fig. 97), the image formed is said to be real, but when they only apparently intersect (Fig. 98), the image is said to be virtual. *A real image is always formed on the side of the lens opposite to that on which the object is situated*, and *may be received on a screen*, or seen by an eye, so placed as to receive the rays involved in its formation and at a distance from the image not less than the *distance of distinct vision* (see page 187). A virtual image, having no real existence, cannot be said to be formed anywhere, but *it is always seen on the side from which the light travels by an eye placed on the opposite side of the lens; a virtual image cannot be received on a screen.*

The above should be studied carefully before proceeding further. Note again that F, the principal focus of the convex lens (Fig. 97), is behind the lens, on the opposite side of the lens to the object, AB, whilst F, the principal focus of the concave lens (Fig. 98), is in front of the lens, on the same side of the lens as the object, AB. Note also again the point, F′, on the opposite side of the lens to F and the same distance from the optical centre, O, sometimes called the *second principal focus*. Do not confuse F and F′.

13.* Relative Position of Finite Object and Image

The relation, $1/v - 1/u = 1/f$, deduced for conjugate foci (Art. 9), evidently establishes a relation between the distances of a finite object and its image from the optical centre of the lens, since an image is an assemblage of foci, conjugate to corresponding points on the object. This relation may be proved geometrically:—

Let AB and A′B′ (Fig. 99) represent a finite object and its image respectively, formed by the lens, L. The construction for the image is identical with that explained above (Art. 12), with the addition of a ray from A through F′ which, after refraction by the lens, travels parallel to the principal axis. From the triangles, A′B′F and aOF,

$$\frac{A'B'}{aO} = \frac{B'F}{OF} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Similarly, from the triangles, A′B′O and ABO, we have $A'B'/AB = OB'/OB$. But, $AB = aO$;

$$\therefore \frac{A'B'}{aO} = \frac{OB'}{OB} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Thus, from (1) and (2):—

$$\frac{B'F}{OF} = \frac{OB'}{OB}.$$

If now, OB be denoted by u, OB′ by v, OF by f, and the usual sign convention (page 40) be observed, then—

$$\frac{-v-(-f)}{-f}=\frac{-v}{u}, \text{ for the convex lens,}$$

$$\text{and, } \frac{f-v}{f}=\frac{v}{u}, \text{ for the concave lens.}$$

$$\text{In both cases:— } uf-uv=vf,$$

$$\text{i.e. } uf-vf=uv.$$

Dividing by uvf, then—

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}.$$

*

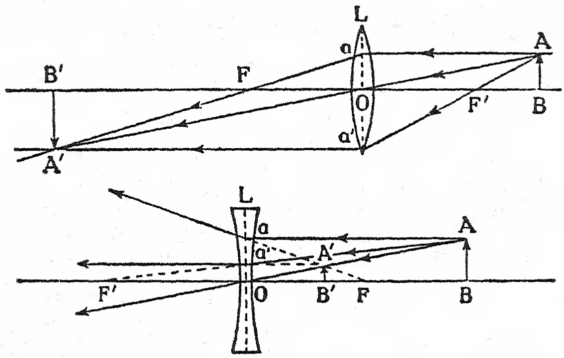


FIG. 99.

14. Relative Positions of Finite Object, AB, and Image, A′B′

The variation of the position of the image with that of the object may be traced by the same method as that adopted for conjugate foci (Art. 10). The following cases should be noted:—

I. CONVEX LENSES

(1) A real object at a distance from the lens greater than the focal length has a *real* image, on the other side of the lens, also at a distance greater than the focal length, and this image is *inverted*. Fig. 97 represents this case, AB being the object and A′B′ the image. If the distance of this object from the lens is *less* than twice the focal length, the image is *greater* than the object—that is, *magnified*. If the distance of this object from the lens is *greater*

than twice the focal length, the image is *less* than the object—that is, *diminished*. If the distance of this object from the lens is *equal to* twice the focal length, the image is the *same size* as the object, and its distance from the lens, on the opposite side, is also equal to twice the focal length. In all these cases the image, being real, can be obtained on a screen.

With the object, AB (Fig. 97), at a very great distance in front of the lens—at infinity, say—the real, inverted, and diminished image is at F, on the other side of the lens. As the object moves towards the lens, the image on the other side of the lens moves in the same direction, away from the lens. When the object has moved up to a distance, $2f$, in front of the lens, the image has moved to a distance, $2f$, on the other side of the lens, and is now the same size as the object. As the object moves from a distance, $2f$, up to a distance, f, in front of the lens, the image—now magnified—moves from a distance, $2f$, to infinity on the other side of the lens.

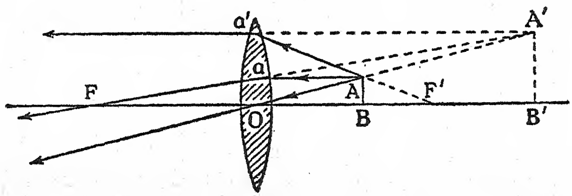


FIG. 100.

In these cases, the numerical value of f is taken whichever sign convention is in use.

(2) A real object, at a distance from the lens *less* than the focal length, has a *virtual* image on the same side of the lens as the object, and this image is *erect*. Fig. 100 represents this case. The image is *larger* than the object—that is, *magnified*.

As the object moves from a distance, f, in front of the lens up to the lens, the image, having disappeared at infinity on the other side of the lens reappears at infinity in front of the lens, and moves from an infinite distance up to the lens. It is magnified, finally becoming equal to the object at the lens.

For the sake of completeness, the case of the virtual object is given:—

(3) If the object be virtual, the image lies between the lens and the object, and its distance from the lens is less than the focal length. It is real, erect, and diminished.

II. CONCAVE LENSES

(1) A real object always has a *virtual, erect* image, nearer the lens than the object, and *diminished*. It is always nearer the lens than the principal focus. Fig. 98 represents this case.

With the object at infinity, the virtual image is at the principal focus. As the object moves from infinity up to the lens, the virtual image moves from the principal focus up to the lens.

For the sake of completeness, the cases of the virtual object are given:—

(2) A virtual object, nearer the lens than the principal focus, has a real, erect, magnified image, more distant than the object. Fig. 98 represents this case, if the direction of the rays be reversed.

(3) A virtual object, further from the lens than the principal focus, has a virtual inverted image further from the lens than the second principal focus on the other side of the lens.

It should be noted that, when the object is real, both lenses and spherical mirrors form images which are erect when virtual, and inverted when real.

15.* Relative Size of Finite Object and Image

The magnification produced by a lens is expressed by the ratio, $\frac{\text{Image}}{\text{Object}}$. When the image is erect, the ratio is considered to be positive; when inverted, the ratio is taken as negative.

Let AB (Fig. 99) represent the object, and A′B′ the image.

(1) From the triangles, AOB and A′OB′:—

$$\frac{A'B'}{AB} = \frac{OB'}{OB} = \frac{-v}{u};$$

$$\therefore \frac{\text{Image}}{\text{Object}} = \frac{v}{u},$$

the magnification being of negative sign in the upper figure.

(2) From the triangles, OF′a' and BF′A:—

$$\frac{a'O}{AB} = \frac{OF'}{BF'} = \frac{OF'}{OB - OF'}.$$

$$\text{But, } a'O = A'B',$$

$$\text{and } \frac{A'B'}{AB} = \frac{OF'}{OB - OF'} = \frac{-f}{u - (-f)};$$

$$\therefore \frac{\text{Image}}{\text{Object}} = \frac{f}{u+f},$$

account being taken of the sign.

(3) From the triangles, FB′A′ and FOa:—

$$\frac{A'B'}{Oa} = \frac{FB'}{FO} = \frac{FO - B'O}{FO}$$

$$\text{But, } Oa = AB,$$

$$\text{and, } \frac{A'B'}{AB} = \frac{FO - B'O}{FO} = \frac{f-v}{f};$$

$$\therefore \frac{\text{Image}}{\text{Object}} = -\frac{v-f}{f}.$$

Thus, if the magnification is denoted by m,

$$m = \frac{\text{Image}}{\text{Object}} = \frac{v}{u} = \frac{f}{u+f} = -\frac{v-f}{f}.$$

These three relations may be connected very simply by the general relation, $1/v - 1/u = 1/f$, by eliminating either u or v. Thus, for example, multiplying both sides of the relation by u,

$$\frac{u}{v} - 1 = \frac{u}{f},$$

$$\text{or } \frac{u}{v} = 1 + \frac{u}{f} = \frac{u+f}{f}.$$

$$\text{Then, } \frac{v}{u} = \frac{f}{u+f}.$$

From this relation the relative sizes of object and image may be determined without finding the position of the image.

Considering the relation:—

$$\frac{\text{Image}}{\text{Object}} = \frac{v}{u},$$

the following results are readily deduced:—

(1) If $v > u$, the image is greater than the object;

(2) If $v = u$, the image is equal in size to the object;

(3) If $v < u$, the image is less than the object.

Applying these results to the general case, I., (1) (Art. 13), three particular cases are obtained, according as the image is *magnified*,

equal to the object, or *diminished.* In the second case, when the image is equal to the object, for a *convex* lens, $v = -u$, and therefore in the relation,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}, \quad -\frac{2}{u} = \frac{1}{f}, \text{ or, } u = -2f.$$

Thus, when the image formed by a convex lens is equal to the object, the distance of both image and object from the lens is equal to twice the focal length of the lens, and hence *the distance between the object and the image is numerically equal to four times the focal length of the lens.* It may also be proved readily that this distance is the minimum distance between object and image in the case under consideration.

It may also be noted that, in order to get a real magnified image, the object must be placed at a distance from the lens numerically greater than the focal length but less than twice the focal length of the lens.

Examples.—(1) *An object is placed* 12 *in. from a convex lens of* 8 *in. focal length. Find the position and nature of the image.*

Here, in the relation, $1/v - 1/u = 1/f$, we have $u = 12$ in., and $f = -8$ in. for a *convex* lens. Hence:—

$$\frac{1}{v} - \frac{1}{12} = \frac{1}{-8},$$

$$\text{and } \frac{1}{v} = -\frac{1}{8} + \frac{1}{12} = -\frac{1}{24};$$

$$\therefore v = -24 \text{ in.},$$

or, the image is 24 in. on the other side of the lens.

$$\text{Again, } \frac{\text{Image}}{\text{Object}} = \frac{v}{u} = \frac{-24}{12} = -2,$$

or, the image is *twice* the size of the object, and is *real* and *inverted.*

(2) *An object,* 3 *cm. long, is placed* 10 *cm. from a concave lens of* 20 *cm. focal length. Find the size and nature of the image.*

$$\frac{\text{Image}}{\text{Object}} = \frac{f}{u+f} = \frac{20}{10+20} = \frac{2}{3};$$

$$\therefore \frac{\text{Length of image}}{3} = \frac{2}{3},$$

i.e. length of image = 2 cm., and the image is *virtual* and *erect.*

A more usual, but less direct, method of solving this problem is to determine v first, and then to determine the size and nature of the image as in Example (1).

(3) *A concave lens of* 12 *in. focal length is placed on the axis of a concave mirror of* 12 *in. radius at a distance of* 6 *in. from the mirror. An object is so placed that light from it passes through the lens, is reflected from the mirror, again passes through the lens, and forms an inverted image coincident with the object. Where must the object be placed?*

In problems such as this, where by reflection and refraction the image of the object is made to coincide with the object itself, the solution is simple, if it is remembered that rays diverging from a point in the object, *on the principal axis*, return to the same point, and therefore travel to and fro by the same paths. But, if a ray, after reflection by a spherical mirror, return along its incident path, it follows that it must be travelling *along a normal to the mirror.*

In this case, after the first refraction through the lens, the rays of the refracted pencil—originally diverging from a point in the object on the principal axis—are normal to the mirror, and therefore diverge from its centre of curvature. To find the position of the object, therefore, it is necessary only to find a point on the principal axis such that rays diverging from this point appear, after refraction through the lens, to diverge from the centre of curvature of the mirror.

Thus, in the relation, $1/v - 1/u = 1/f$, we have $v = 6$ in., $f = 12$ in., and

$$\frac{1}{6} - \frac{1}{u} = \frac{1}{12}; \quad \therefore \ u = 12 \text{ in.}$$

Or, the object must be placed 12 in. from the lens on the side remote from the spherical mirror.

(4) *When a luminous point is placed on the principal axis of a convex lens,* A, *and at a distance, a, from it, an image is formed* 10 *in. from the lens on the other side. If a second lens,* B, *is placed close to* A, *the image is* 15 *in. away. Determine the focal length of the lens,* B, *and state whether it is convex or concave.*

The action of the lens, B, is evidently to cause a pencil of rays, originally converging to a point, P, 10 in. behind the lenses, to become less convergent and to converge to a point, P′, 15 in. behind the lenses. Thus, P and P′ are conjugate foci with respect to the lens, B, P′ being the image of P. Hence, in the relation, $1/v - 1/u = 1/f$, we have $u = -10$ in. and $v = -15$ in.;

$$\therefore \ \frac{1}{-15} - \frac{1}{-10} = \frac{1}{f}.$$

$$\text{Thus } \frac{1}{f} = -\frac{1}{15} + \frac{1}{10} = \frac{1}{30}, \text{ or } f = 30 \text{ in.}$$

That is, the lens is *concave*, and its focal length is 30 in.

16. Rays by which an Image is Seen by the Eye

If the object be near the principal axis of the lens, the eye should also be near the axis. It will then receive rays which have passed through the lens. The case of a convex lens producing a

real image is illustrated (Fig. 101). AB is the object, A′B′ the image found as described above (Art. 12).

To find the course of the rays by which the point, A, is seen, join A′ to the extremities of the pupil of the eye; then, produce the lines so obtained backwards to cut the lens in the points, *aa*, and join *aa* to A. The rays by which A is seen are all included in the incident pencil, A*aa*. Similarly for the point, B.

The case of the virtual image formed by a concave lens is also illustrated (Fig. 102), the construction being exactly similar.

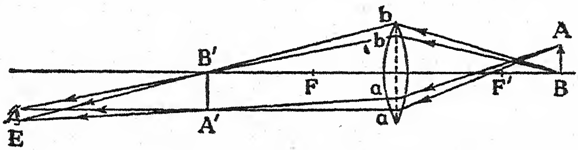


FIG. 101.

17.* Combination of Lenses in Contact

Let two thin lenses of focal lengths, f_1 and f_2, be placed in contact. The problem is to determine the focal length of a single lens which is optically equivalent to this combination.

Suppose that light from a point, P (Fig. 103), at a distance, u, from O, the optical centre of the combination, is incident first on the lens of focal length, f_1. The thickness of the lenses is assumed to be so small, compared with other distances involved, that the optical centre may be taken at any point in their combined thickness.

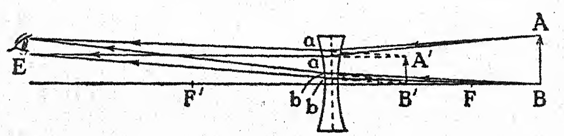


FIG. 102.

Then, *considering the action of this lens only*, the focus of the refracted pencil will be at a point, P′, at a distance, v', from the lens, such that

$$\frac{1}{v'} - \frac{1}{u} = \frac{1}{f_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

But this refracted pencil passes through the second lens and, after doing so, is refracted through another point, P″, at a distance, v, from the lens, such that

$$\frac{1}{v} - \frac{1}{v'} = \frac{1}{f_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

The combined action of the lenses is thus to cause a pencil of rays diverging from P, at a distance, u, from O, to be refracted through P″, at a distance, v, from O. Therefore, if F be the focal length of the combination,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{F} \quad \text{......................(3)}$$

But, by adding (1) and (2),

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2} \quad \text{.....................(4)}$$

Therefore, from (3) and (4),

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}, \text{ or } F = \frac{f_1 f_2}{f_1 + f_2}.$$

Thus, a single lens of focal length, $\frac{f_1 f_2}{f_1 + f_2}$, is optically equivalent to two thin lenses in contact, of focal lengths, f_1 and f_2. When the positions of equivalent lens and combination are the same, the image produced by the equivalent lens is in the same place and is the same size as that produced by the combination.

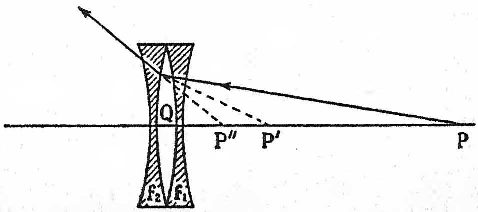


FIG. 103.

By extending the problem to a number of thin lenses in contact, we have:—

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} + \ldots, \text{ etc.}, = \Sigma\frac{1}{f}. \quad *$$

18. Focal Power of a Lens

The reciprocal of the focal length in *metres* of a lens is called the **focal power** of the lens. The unit of power is that possessed by a lens of one metre focal length and is called the **dioptre**. The power of a converging lens is considered *positive*, and that of a diverging lens *negative*, so that the sign of the power is the reverse of that of the focal length.†

Thus a convex lens of 50 cm. focal length has a power of 2 dioptres, and is known as a +2D lens, while a concave lens of

† The sign of power of a lens is the same as the sign of its focal length when the Real is Positive convention is used.

25 cm. focal length has a power of 4 dioptres, and is known as a − 4D lens.

This system of units is used widely by industrial workers in optics, such as opticians. The Real is Positive convention gives a positive focal length to a converging lens, in contradistinction to Art. 6, and the student must be on the look out for this possible confusion; the context will usually reveal which type of lens is meant.

Using the definition of power, the theorem proved above (Art. 17) may be expressed:—**The power of a combination of lenses in contact is equal to the sum of the powers of the constituent lenses.** Thus since:—

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$D = D_1 + D_2.$$

19. Spherical Aberration by Refraction

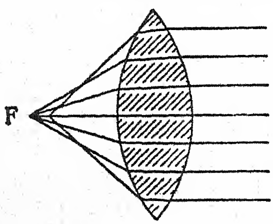


FIG. 104.

In discussing conjugate foci and the formation of images above, cases in which the curvatures of the surfaces of the lens are small have been considered only. If the surfaces be greatly curved—that is, if the radii of curvature are small—rays diverging from any point in the object are not all brought together at one conjugate focus, but those rays which pass furthest from the optical centre of the lens have a focal distance shorter than that found by the rules indicated.

This is illustrated (Fig. 104) in the case of a convex lens, where the marginal rays are seen to intersect before the central rays which intersect at the principal focus, F. This wandering of the intersections of the refracted rays from the principal focus is known as *spherical aberration* (see also Art. 4).

Experiment. To illustrate the different focal lengths of the central and marginal parts of a convex lens.—Take a convex lens of large aperture and of short focal length, and, covering the centre of the lens with a large disc of black paper, use it to produce an image of a candle flame on a screen. Then, without altering the relative positions of candle, lens, and screen, replace the paper disc by a black paper ring which covers all the lens except a small central circle. The image of the flame will now be indistinct, and the

screen will have to be moved further from the lens to make the image sharp.

Since spherical surfaces are usually employed, all lenses are in some degree subject to this defect, but if the curvature is small, the defect is very slight. As with spherical mirrors, this defect may be reduced to any desired degree by the use of diaphragms which cover up either the central or marginal portions of the lens. Either of these courses reduces brilliancy, and, in many applications, it is most undesirable to reduce the available aperture of the lens (see Chapter XV., Art. 8).

Spherical aberration may also be reduced by using a plano-convex lens, instead of a double convex lens, with the spherical surface facing the rays which are the more nearly parallel to the axis. Each ray is then very nearly equally deviated at the two surfaces, and it can be shown mathematically that under these conditions the aberration is a minimum. This fact is utilised in the construction of optical instruments. The two deviations may be made exactly equal by a suitable choice of the radii of curvature of the surfaces of the lens. Thus, for example, in the case of crown glass, for which the refractive index is 1·5 approximately, the aberration produced if the incident light is parallel by a double convex lens of given focal length is a minimum when the radius of curvature of the second surface is six times that of the first surface. Such a lens is called a *crossed* lens. In practice such a lens is little better than a plano-convex one. For flint glass (refractive index 1·6) the crossed lens is plano-convex.

20. Caustic by Refraction

A close inspection of Fig. 104 will show that the intersection of the refracted rays from different parts of the lens must give rise, when the curvature of the surface is great, to a luminous curved surface. Such a surface is known as a *real caustic by refraction*, just as similar effects produced by concave spherical mirrors are known as real caustics by reflection (see page 56).

The presence of a caustic surface affords a simple method of illustrating spherical aberration to an audience. If the region on the left-hand side of the lens used above (Fig. 104) is filled with smoke, the caustic surface is rendered very apparent.

APPENDIX TO CHAPTER VII

ALTERNATIVE TREATMENT OF CERTAIN SECTIONS IN TERMS OF THE REAL IS POSITIVE SIGN CONVENTION

1. Refraction at a Single Spherical Surface

Then, with the Real is Positive sign convention (see p. 41) we have u positive, v negative because the image is virtual, and r negative because the image would be virtual for parallel light, so that the corresponding focal distance is negative. r is of the same sign as the focal distance if $\mu > 1$ (see Art. 2).

Then $\mu = \dfrac{\text{AO}}{\text{NA}} \cdot \dfrac{\text{NA}'}{\text{A}'\text{O}} = \dfrac{(-r-u)\,(-v)}{u\,(r+v)} = \dfrac{v\,(r+u)}{u\,(r+v)}.$

Similarly, for a convex spherical surface of separation of the media

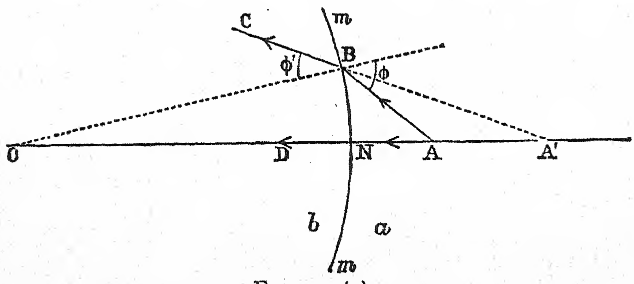


FIG. 104 (*a*).

[Fig. 104 (*a*)], it may be shown that

$$\mu = \frac{\text{AO}}{\text{NA}} \cdot \frac{\text{NA}'}{\text{A}'\text{O}}$$

r is positive in this case, because parallel light would now give a *real* image. As Fig. 104 (*a*) is drawn, the image is virtual, so v is negative, but for AN large the image would be real. Thus $\text{AO} = \text{AN} + \text{NO} = u + r$, $\text{A}'\text{O} = \text{A}'\text{N} + \text{NO} = -v + r$, so that

$$\mu = \frac{(r+u)\,v}{u\,(r+v)}$$

which is identical with the relation obtained for the concave surface.

From this relation: $\mu ru + \mu rv = rv + ru.$

Dividing through by *uvr* we get

$$\frac{\mu}{v}+\frac{1}{u}=\frac{\mu-1}{r}.$$

This is a general relation, applicable to all cases of refraction of small direct pencils at a *single* plane or spherical surface of separation, whose relative refractive index, *for the direction in which the light is travelling*, is denoted by μ. When $m.m$ is a plane surface (see p. 75), r is infinite and $\frac{\mu-1}{r}=0$ and the relation reduces to $-\frac{\mu}{v}=\frac{1}{u}$, a result obtained previously.

2. The Principal Foci of a Spherical Refracting Surface

In the above relation, when v is infinite, the value of u is called the *first principal focal distance*. Denoting it by f_1

$$f_1=\frac{r}{\mu-1}.$$

The point on the principal axis at a distance of f_1 from the pole is called the *first principal focus*, rays proceeding from it (f_1 positive) or to it (f_1 negative) are refracted so as to proceed parallel to the axis, a distance corresponding to a *real* focus being *positive*.

Similarly, by making u infinite, the value of v is called the *second principal focal distance*. Denoting it by f_2:—

$$f_2=\frac{\mu r}{\mu-1}.$$

The point on the principal axis at a distance of f_2 from the pole is called the *second principal focus*, and rays originally proceeding parallel to the axis are refracted so as to converge to it (f_2 positive) or to proceed from it (f_2 negative).

Thus, both f_1 and f_2 are of the same sign as r if $\mu > 1$, *i.e.* if the light proceeds from one medium to an optically denser one. If the surface of the denser medium is convex, the surface has a converging effect, so parallel light forms a real image and f_1 and f_2 are positive.

3. Construction for Refraction at a Spherical Surface

Example.—*A gold-fish globe of radius 6 in. is filled with water. Determine the apparent position of a point inside the globe, 4 in. from its surface, when seen by an eye looking along the radius of the globe.*

The surface at which refraction takes place is spherical and, neglecting the action of the glass of the globe, we have

$$\frac{\mu}{v} + \frac{1}{u} = \frac{\mu - 1}{r}.$$

Here $u = 4$ in., $\mu = \frac{3}{4}$ water to air and $r = 6$ in. The focal distances are positive because the surface of the water is convex, and r is to be reckoned negative because $\mu < 1$.

Thus:— $$\frac{3}{40} + \frac{1}{4} = -\frac{\frac{1}{4}}{-6} = +\frac{1}{24} \text{ and } \frac{3}{40} = -\frac{5}{24}.$$

Hence, $-20v = 72$ or $v = -3{\cdot}6$ in.

The image is thus virtual, and the point appears to be 3·6 in. from the surface of the globe on the radius through its real position.

9. General Relation for Lenses

The relation between the distances of conjugate foci from the optical centre of the lens will be deduced by a simple geometrical method later (Art. 13). Here, however, a relation between these distances will be established by the application of the relation

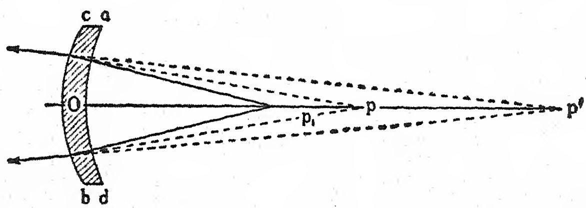


FIG. 104 (*b*).

applying to refraction at a *single* spherical surface deduced above (Art. 1). In this relation

$$\frac{\mu}{v} + \frac{1}{u} = \frac{\mu - 1}{r}$$

μ represents the relative refractive index from one medium to the other, in the direction taken by the rays of light.

Let *abcd* [Fig. 104 (*b*)] represent a lens, and let r denote the radius of curvature of the surface, *ad*, and s the radius of curvature of the surface, *cb*. Then, first considering refraction at the surface, *ad*, the pencil of rays diverging from p_1, appears. After refraction, to diverge from p. If Op_1 is denoted by u and Op by v' and the refractive index from air to the lens by μ, then

$$\frac{\mu}{v} + \frac{1}{u} = \frac{\mu - 1}{r} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

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As Fig. 104 (*b*) is drawn, u is positive (real object), v' is negative because the image is virtual, and r is negative because μ is greater than unity and the surface of the optically denser medium is concave so that parallel light would appear to diverge from a virtual focus.

We are assuming the thickness of the lens negligibly small compared with u and v, so that O may be taken as a point on either of the spherical surfaces *ad* and *cb*.

After refraction at the surface *ad* the pencil apparently diverging from p may be supposed incident on the surface, *cb*, and to undergo refraction at that surface from the lens into the air, the focus of the emergent rays being at p'. Now, if μ is the refractive index from air to the lens, $\frac{1}{\mu}$ is the refractive index from the lens to air (see p. 62). If Op is denoted by u', and Op' by $-v$, we have

$$\frac{\frac{1}{\mu}}{v}+\frac{1}{u'}=\frac{\frac{1}{\mu}-1}{s} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

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In Fig. 104 (*b*) v is negative because p' is a virtual image, but u' is positive because, as far as the second surface is concerned, the image would be formed in exactly the same way as it would be if the rays of light came from a real object at p. Thus Op must be reckoned as negative for the first surface and positive for the second surface. Thus, the question of signs has to be considered afresh for each surface in turn. It remains to fix the sign of s. As Fig. 104 (*b*) is drawn the surface *cb* has a converging effect, so its focal distances are both positive, but s has to be reckoned negative because the refractive index is less than unity for the direction of travel of the light. To find the relation between u and v for the whole lens, we use equations (1) and (2), remembering that $v'=-u'$ for the reason just given. From equation (2) we have, by multiplying both sides by μ,

$$\frac{1}{v}+\frac{\mu}{u'}=\frac{1-\mu}{s}.$$

Adding this to equation (1) we obtain

$$\frac{1}{u}+\frac{1}{v}=(\mu-1)\left(\frac{1}{r}-\frac{1}{s}\right) \quad \ldots\ldots\ldots\ldots\ldots \quad (3)$$

where the signs of r and s have to be determined in the manner explained above, so that real focal distances are positive. As this criterion is a little complicated to apply in practice, we give here a simpler one which gives the same results. *Surfaces which are convex to the direction of travel of the incident light are to be*

reckoned as of positive radius, surfaces concave to this direction have negative radii. Thus, if the lens is bi-convex the radius of the face first reached by the light is to be reckoned positive, while that of the second face is negative.

Equation (3) gives the relation between the distances of object and image from the lens. If u is infinite—that is if the incident rays are parallel, the emergent pencil passes through the *principal focus*, and v becomes equal to the *focal length* of the lens (Art. 6). Therefore, if the focal length be denoted by f,

$$\frac{1}{f} + \frac{1}{\infty} = \frac{1}{f} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right),$$

so that equation (3) becomes

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f},$$

which is exactly the same as the formula for mirrors obtained previously (Chapter IV., Art. 4). Since a converging lens brings parallel light to a *real* focus, it is clear that its focal length is *positive*, while a diverging lens has a negative focal length. The student may, by trying various cases, satisfy himself that the arguments which led to equations (1) to (3) hold, whether μ is greater or less than 1, whatever may be the signs of u, v, u', v', r and s, and whichever surface the light is incident on first.

Example.—*Find the focal length of a double concave lens, the radius of curvature of its surfaces being* 25 *cm. and* 50 *cm. respectively, and the refractive index of the material being* 1·5.

Using the relation $\frac{1}{f} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right)$, and supposing the light to be incident on the more concave surface,

$$\mu = 1{\cdot}5,\ r = -25 \text{ cm.},\ s = +50 \text{ cm.}$$

Then, $$\frac{1}{f} = (1{\cdot}5 - 1)\left(-\frac{1}{25} - \frac{1}{50}\right) = -\frac{1}{2} \times \frac{3}{50} = -\frac{3}{100}.$$

Thus, $f = -33\frac{1}{3}$ cm.

Supposing the light to be incident on the other surface

$$\mu = 1{\cdot}5,\ r = -50 \text{ cm.},\ s = +25 \text{ cm.}$$

Then, $$\frac{1}{f} = (1{\cdot}5 - 1)\left(-\frac{1}{50} - \frac{1}{25}\right) = -\frac{3}{100};$$

$\therefore f = -33\frac{1}{3}$ cm., as before.

10. Relative Positions of Point Object P, and Image P′

FIRST METHOD.—By application of the above relation, the position of P′ corresponding to any given position of P, can be

determined. Thus, taking the case of a convex lens, and assuming P to be at infinity,

$$\frac{1}{v}+\frac{1}{\infty}=\frac{1}{f}, \text{ and } v=f.$$

This means that the image P′ is at the principal focus, F, that is, at a distance, f, on the other side of the lens and is real. Note that f is positive for a convex lens.

If P is at a distance in front of the lens equal to twice the focal length, f, then—

$$\frac{1}{v}+\frac{1}{2f}=\frac{1}{f}, \text{ or } v=2f.$$

This means that the image P′ is real, and is therefore at a distance $2f$ on the other side of the lens.

If P is at a distance in front of the lens equal to the focal length f, then—

$$\frac{1}{v}+\frac{1}{f}=\frac{1}{f}, \text{ or } v=\infty,$$

that is, the emergent beam of light is parallel and the image is at infinity. It may be regarded either as a real image at $+\infty$, or as a virtual image at $-\infty$.

If P is at a distance in front of the lens equal to $\frac{f}{3}$, then

$$\frac{1}{v}+\frac{3}{f}=\frac{1}{f}, \text{ and } v=-\frac{f}{2}.$$

This means that the image, P′, is at a distance equal to $f/2$ *in front of* the lens, the image being *virtual*, and therefore on the same side of the lens as the object P.

Again, taking the case of a concave lens, and, assuming P to be at infinity:—

$$\frac{1}{v}+\frac{1}{\infty}=\frac{1}{f}, \text{ or } v=f.$$

This means that the image P′ is at the principal focus, F, that is at a distance, f, on the same side of the lens as P. Since f is negative for a concave lens, v is negative, and the image is *virtual*.

By proceeding as indicated above, it can be shown that in the case of a convex lens, as P moves from infinity in front of the lens up to a distance equal to f from the lens, P′ moves from F on the other side of the lens to infinity. So far the image has been *real*. As P continues to move from a distance, f, in front of the lens up

to the lens, P′ disappears at infinity on the other side of the lens, reappears at infinity in front of the lens, and moves from infinity up to the lens. During this stage the image has been *virtual*.

Similarly, it can be shown that, in the case of a concave lens, as P moves from infinity in front of the lens up to the lens, P′ moves from F in front of the lens up to the lens. The image is always *virtual* except for a *convergent* pencil.

SECOND METHOD.—A second method of tracing the path of P′ as P moves from infinity on one side of the lens up to the lens, and then to infinity on the other side of the lens, is similar to the second method for spherical mirrors (see p. 44). Thus, taking the general relation, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$, we have,

$$uf + vf = uv.$$

That is, $$f^2 - uf - vf + uv = f^2$$

and $$(u - f)(v - f) = f^2.$$

Let x and x' denote the distances of object, P, and image, P′ measured from F′ and F respectively, F being the principal focus and F′ a point distant f from the lens, but on the other side of it [see Fig. 104 (*c*)]. We chose the signs of x and x' so that $x = u - f$, and $x' = v - f$, the signs of u, v, and f being determined by the conventions already explained. This gives us $xx' = f^2$, just as for mirrors, but the interpretation of the formula is slightly different, because x and x' are now measured from two distinct points F′ and F respectively. Since f^2 is positive whatever the nature of the lens, x and x' are always of the same sign. It may be verified by trying different cases that the object is always on the opposite side of F′ that the image is of F. This leads to the following results:—

I. CONVEX LENSES

(1) When P is at infinity in front of the lens (Fig. 87)—that is when the incident rays are parallel to the principal axis, P′ is at the principal focus f behind the lens. Or when $x = \infty$, $x' = 0$.

(2) When P is in front of the lens at a distance equal to $2f$, then P′ is at an equal distance behind the lens. Or when $n = f$, $x' = f$.

(3) When P is at F′ in front of the lens (Fig. 93)—that is, when the incident rays diverge from a point in front of the lens at a

distance from its optical centre equal to the focal length, then P′ is at infinity on the other side of the lens, and the refracted rays are parallel to the principal axis. Or, when $x = 0$, $x' = \infty$.

(4) When P is at O, P′ is also at O. Or when $x = -f, x' = -f$. These are the *most important* results for *convex* lenses. To summarise, as P travels from infinity on the left up to F′ on the left (Fig. 99), P′ travels in the same direction from F on the right to infinity on the right. As P travels from F′ on the left to O, P′, after disappearing at infinity on the right, reappears at infinity on the left and travels in the same direction as P to O. For the sake of completeness, the cases of the *virtual object* must be added:—

(5) When P is at F, that is, when the incident rays *converge* to the principal focus behind the lens, then P′ is midway between O and F. Or when

$$x = 2f, x' = \frac{f}{2}.$$

(6) When P is at infinity behind the lens, the incident rays are parallel, and P′ is at F, as in (1). Or, when

$$x = -\infty, x' = 0.$$

In cases such as (5) and (6), when the incident rays converge towards a point, P, and do not actually pass through it, the point can be described as a virtual object. Similar definitions apply to mirrors. Thus a plane mirror produces a real image of a virtual object (see page 21).

II. CONCAVE LENSES

(1) When P is at infinity in front of the lens (Fig. 88)—that is, when the incident rays are parallel to the principal axis, P′ is at the principal focus, F, in front of the lens. Or, when $x = +\infty$, $x' = 0$.

(2) When P is at F in front of the lens (Fig. 94), then P′ is midway between F and O—that is, at the point h. Or, when $x = -2f$, $x' = -f/2$. Note that f is negative for a concave lens.

(3) When P is at O, P′ is also at O. Or, when $x = -f, x' = -f$. These are the *most important results* for *concave* lenses. To summarise, as P travels from infinity in front of the lens on the right, up to O, P′ travels in the same direction from F in front of the lens up to O. For the sake of completeness, the following cases must be added:—

(4) When P is at F′—that is, when the incident rays *converge* to a point behind the lens at a distance from the optical centre equal to the focal length of the lens, then P′ is at infinity. Or, when $x = 0$, $x' = -\infty$.

(5) When P is at a distance behind the lens *numerically* equal to $2f$, P′ is the same distance in front of the lens. Or, when $x = f$, $x' = f$.

(6) When P is at infinity behind the lens, the incident rays are parallel, and, as in (1), P′ is at F. Or, when $x = -\infty$, $x' = 0$.

Of the above results, I. (1), (3), (6), and II. (1), (4), (6) are summed up in the statement that, when a parallel pencil of light is incident on a lens, the focus of the refracted pencil is at the principal focus of the lens. I. (4) and II. (3) are evident and easily remembered. I. (2), (5), and II. (2), (5), may be deduced from the general relation $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$.

General Rule.—When an image is formed by refraction through a lens, any motion of the object along the principal axis of the lens produces a corresponding motion of the image in the same direction along the principal axis. This holds whatever may be the signs of u, v and f.

The application of the rule is simple. For example, from I. (1) and (3), it is seen that, as P moves from infinity to F′, P moves in the same direction from F to infinity behind the lens. Similarly, from II. (2) and (3), as P moves from F to O, P′ moves in the same direction from h to O (Fig. 94).

13. Relative Position of Finite Object and Image

The relation $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ deduced for conjugate foci (Art. 9), evidently establishes a relation between the distances of a finite object and its image from the optical centre of the lens, since an image is an assemblage of foci, conjugate to corresponding points on the object. This relation may be proved geometrically:—

Let AB and A′B′ [Fig. 104 (*c*)] represent a finite object and its image respectively, formed by the lens, L. The construction for the image is identical with that explained above (Art. 12), with the addition of a ray from A through F′ which, after refraction by

the lens, travels parallel to the principal axis. From the triangles, A′B′F and aOF,

$$\frac{A'B'}{aO} = \frac{B'F}{OF} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Similarly from the triangles, A′B′O and ABO, we have A′B′/AB = OB′/OB. But, AB = aO;

$$\therefore \quad \frac{A'B'}{aO} = \frac{OB'}{OB} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Thus, from (1) and (2):—

$$\frac{B'F}{OF} = \frac{OB'}{OB}.$$

Then, if u, v and f have their usual meanings and the sign convention is observed, this relation becomes

$$\frac{v-f}{f} = \frac{v}{u} \text{ for the convex lens,}$$

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and
$$\frac{-f+v}{-f} = \frac{-v}{u} \text{ for the concave lens.}$$

In both cases:— $uf + vf = uv.$

Dividing by uvf, then $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$.

15. Relative Size of Finite Object and Image

The magnification produced by a lens is expressed by the ratio $\frac{\text{Image}}{\text{Object}}$. When the image is erect, the ratio is considered to be positive; when inverted, the ratio is taken as negative.

Let AB [Fig. 104 (c)] represent the object and A′B′ the image.

(1) From the triangles, AOB and A′OB′:—

$$\frac{AB'}{AB} = \frac{OB'}{OB} = \frac{v}{u}.$$

$$\therefore \frac{\text{Image}}{\text{Object}} = \frac{-v}{u},$$

the magnification being of negative sign in the upper figure.

(2) From the triangles, OF′a' and BF′A

$$\frac{a'\mathrm{O}}{\mathrm{AB}} = \frac{\mathrm{OF'}}{\mathrm{BF'}} = \frac{\mathrm{OF'}}{\mathrm{OB} - \mathrm{OF'}}.$$

$$\text{But,} \quad a'\mathrm{O} = \mathrm{A'B'},$$

and
$$\frac{\mathrm{A'B'}}{\mathrm{AB}} = \frac{\mathrm{OF'}}{\mathrm{OB} - \mathrm{OF'}} = \frac{-f}{-u+f};$$

$$\therefore \frac{\text{Image}}{\text{Object}} = \frac{f}{f-u},$$

account being taken of the sign.

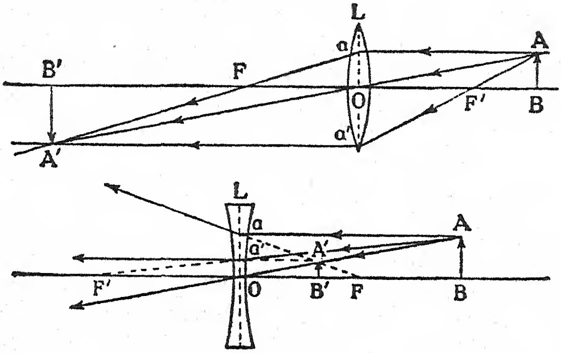


FIG. 104 (c).

(3) From the triangles, FB′A′ and FOa

$$\frac{\mathrm{A'B'}}{\mathrm{O}a} = \frac{\mathrm{FB'}}{\mathrm{FO}} = \frac{\mathrm{B'O} - \mathrm{FO}}{\mathrm{FO}}.$$

$$\text{But,} \quad \mathrm{O}a = \mathrm{AB}$$

and,
$$\frac{\mathrm{A'B'}}{\mathrm{AB}} = \frac{\mathrm{B'O} - \mathrm{FO}}{\mathrm{FO}} = \frac{v-f}{f},$$

$$\therefore \frac{\text{Image}}{\text{Object}} = -\frac{v-f}{f}.$$

Thus, if the magnification is denoted by m,

$$m = \frac{\text{Image}}{\text{Object}} = -\frac{v}{u} = \frac{f}{f-u} = \frac{f-v}{f}.$$

These three relations may be connected very simply by the general

relation $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$, by eliminating either u or v. Thus, for example, multiplying both sides of the relation by u,

$$\frac{u}{v}+1=\frac{u}{f},$$

or

$$\frac{u}{v}=\frac{u}{f}-1=\frac{u-f}{f}.$$

Then,

$$\frac{-v}{u}=\frac{f}{f-u}.$$

From this relation the relative sizes of object and image may be determined without finding the position of the image.

Considering the relation:—

$$\frac{\text{Image}}{\text{Object}}=-\frac{v}{u},$$

the following results are readily deduced:—

(1) If v is numerically greater than u, the image is greater than the object.

(2) If v is numerically equal to u, the image is equal in size to the object.

(3) If v is numerically less than u, the image is less than the object.

Applying these results to the general case, I. (1) (Art. 13), three particular cases are obtained, according as the image is *magnified, equal to the object, or diminished.* In the second case, when the image is equal to the object, for a *convex* lens, $v = u$, and therefore in the relation

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f},\quad \frac{2}{u}=\frac{1}{f} \text{ or } u=2f.$$

Thus, when the image formed by a convex lens is equal to the object, the distance of both image and object from the lens is equal to twice the focal length of the lens, and *hence the distance between the object and image is equal to four times the focal length of the lens.* It may also be proved readily that this distance is the minimum distance between object and image in the case under consideration.

It may also be noted that, in order to get a real magnified image, the object must be placed at a distance from the lens

numerically greater than the focal length, but less than twice the focal length of the lens.

Examples.—(1) *An object is placed* 12 *in. from a convex lens of* 8 *in. focal length. Find the position and nature of the image.*

Here, in the relation, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$, we have $u = 12$ in. $f = 8$ in. for a convex lens. Hence,

$$\frac{1}{v} + \frac{1}{12} = \frac{1}{8},$$

$$\text{and } \frac{1}{v} = \frac{1}{8} - \frac{1}{12} = \frac{1}{24};$$

$$\therefore\ v = +\ 24 \text{ in.}$$

Or the image is *real* and is therefore 24 in. on the far side of the lens.

$$\text{Again, } \frac{\text{Image}}{\text{Object}} = -\frac{v}{u} = -\frac{24}{12} = -2.$$

i.e. the image is twice the size of the object and *inverted.*

(2) *An object,* 3 *cm. long, is placed* 10 *cm. from a concave lens of* 20 *cm. focal length. Find the size and nature of the image.*

$$\frac{\text{Image}}{\text{Object}} = \frac{f}{f-u} = \frac{-20}{-20-10} = \frac{2}{3};$$

$$\therefore\ \frac{\text{Length of Image}}{3} = \frac{2}{3}.$$

i.e. length of image = 2 cm. and the image is *erect* and therefore *virtual.*

A more usual, but less direct, method of solving this problem is to determine v first, and then to determine the size and nature of the image, as in Example (1).

(3) *A concave lens of* 12 *in. focal length is placed on the axis of a concave mirror of* 12 *in. radius at a distance of* 6 *in. from the mirror. An object is so placed that light from it passes through the lens, is reflected from the mirror, again passes through the lens, and forms an inverted image coincident with the object. Where must the object be placed?*

In problems such as this, where by reflection and refraction the image of the object is made to coincide with the object itself, the solution is simple, if it is remembered that rays diverging from a point in the object, *on the principal axis,* return to the same point, and therefore travel to and fro by the same paths. But, if a ray, after reflection by a spherical mirror, return along its incident path, it follows that it must be travelling *along a normal to the mirror.*

In this case, after the first refraction through the lens, the rays of the refracted pencil—originally diverging from a point in the object on the principal axis—are normal to the mirror, and therefore diverge from its

centre of curvature. To find the position of the object, therefore, it is necessary only to find a point on the principal axis such that rays diverging from this point appear, after refraction through the lens, to diverge from the centre of curvature of the mirror.

Thus, in the relation, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$, we have $v = -6$ in. (negative because the image is virtual), $f = -12$ in.

$$-\frac{1}{6}+\frac{1}{u}=-\frac{1}{12}; \qquad \therefore\ u = 12 \text{ in.}$$

Or, the object must be placed 12 in. from the lens on the side remote from the mirror.

(4) *When a luminous point is placed on the principal axis of a convex lens,* A, *and at a distance, a, from it, an image is formed* 10 *in. from the lens on the other side. If a second lens,* B, *is placed close to* A, *the image is* 15 *in. away. Determine the focal length of the lens,* B, *and state whether it is convex or concave.*

The action of the lens, B, is evidently to cause a pencil of rays converging to a point P, 10 in. behind the lenses, to become less convergent and to converge to a point, P′, 15 in. behind the lenses. Thus, P and P′ are conjugate foci with respect to lens, B, P′ being the image of P. Hence, in the relation, $1/v + 1/u = 1/f$, we have $u = -10$ in. (virtual object), and $v = +15$ in. (real image);

$$\therefore\ \frac{1}{15}-\frac{1}{10}=\frac{1}{f}.$$

$$\text{Thus } \frac{1}{f}=-\frac{1}{30}, \quad \text{or} \quad f=-30 \text{ in.}$$

That is, the lens is *concave*, and its focal length is 30 in.

17. Combination of Lenses in Contact

Let two thin lenses of focal lengths, f_1 and f_2, be placed in contact. The problem is to determine the focal length of a single lens which is optically equivalent to this combination.

Suppose the light from a point, P (Fig. 103), at a distance, u, from O, the optical centre of the combination is incident first on the lens of focal length f_1. The thickness of the lenses is assumed to be so small, compared with the other distances involved, that the optical centre may be taken at any point in their combined thickness. Then, considering the action of this lens only, the focus of the refracted pencil will be at a point P′, at a distance v' from the lens, such that:—

$$\frac{1}{v'}+\frac{1}{u}=\frac{1}{f_1} \quad \text{.......................} \quad (1)$$

But this refracted pencil passes through the second lens, and after doing so, its focus will be another point, P″, at a distance v from the lens, such that

$$\frac{1}{v}+\frac{1}{u'}=\frac{1}{f_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

where u' is the distance of P′ from the second lens.

We have now to determine the relation between v' and u', the distances of P′ from the first and second lenses respectively, sign being taken into account. Since the lenses are thin, and in contact, these distances are *numerically* equal, but we still have to determine their signs. Two cases are possible:—

(*a*) The image P′ may be real. In this case v' is positive (real image), but the pencil crosses the second lens *before* reaching P′, so that P′ is a virtual object and u' is negative.

(*b*) The image P′ may be virtual. v' is now negative, but the image is now behind the lens, and, so far as the second lens is concerned, the situation is exactly the same as if the light were coming from a real object at P′. Thus u' is to be reckoned positive.

Thus, we have in all cases $v' = -u'$. Adding equations (1) and (2) we get:—

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f_1}+\frac{1}{f_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

Thus the combined action of the two lenses is to refract through P″, at a distance v a pencil originally diverging from P at a distance u. Thus, if F be the focal length of the combination

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}, \text{ or } F=\frac{f_1 f_2}{f_1+f_2}.$$

The signs of v and u are determined by the usual conventions, just as for a single lens, and, from equation (3), it is clear that the relation between u and v is exactly the same as for a single thin lens. By extending the work to any number of lenses in contact, we find

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}\ldots\ldots,$$

where care must be taken to give the proper signs to f_1, f_2, etc.

In this investigation we have again met the difficulty which occurred in the derivation of the relation between the focal length

of a lens and its radii of curvature (Art. 9), namely that the question of whether an intermediate image is to be regarded as real or virtual cannot be settled once for all, but has to be considered afresh for each refraction in turn. The derivation of the lens formula takes account of this circumstance for the two surfaces of each separate lens, but it is still possible for the same image to be "real" from the point of view of one lens and "virtual" from the point of view of another lens. It is, however, not *universally* true that quantities like u' and v' in the cases considered above are of *opposite* sign. For example, if we have two lenses *not* in contact, it is possible to form a real image *between* the lenses. In such a case, *both* u' and v' would have to be reckoned positive because such an image is both real itself, and acts as a real object for the next lens.

CHAPTER VIII

EXPERIMENTS ON THE OPTICAL CONSTANTS OF MIRRORS, LENSES, AND PRISMS

IT cannot be too strongly emphasised that the study of the subject of Optics is greatly helped by the performance of experimental work, in which the various phenomena can be reproduced and observed, and in which the many relationships, deduced theoretically by simple mathematical means, covering these phenomena can be tested by the student. Actually, the apparatus required for such experimental work is neither complicated nor expensive. Very simple apparatus is, in fact, all that is necessary.

In this chapter will be described forms of apparatus suitable for such experimental work, together with a series of experiments which will cover almost all the points dealt with theoretically in the preceding chapters.

1. The Optical Bench

This apparatus is of such frequent use in optical measurements that it is advisable, at this stage, to consider briefly its construction and method of use.

In one of its simplest forms, the optical bench consists of a heavy base board, BB (Fig. 105) of hard wood, about two metres long, and having a deep, wide groove running along the middle of its upper face. A scale, showing centimetres and millimetres, is cut parallel to the groove in such a way that the edge of the groove is also the measuring edge of the scale. The inside edges of the groove are not vertical, but are cut obliquely as shown at S. A set of uprights, constructed to hold suitably mounted mirrors, lenses, screens, etc., are fitted into small base boards, which are so made that they can be pushed

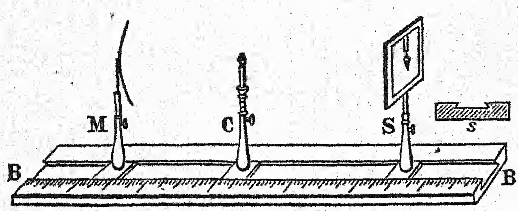


FIG. 105.

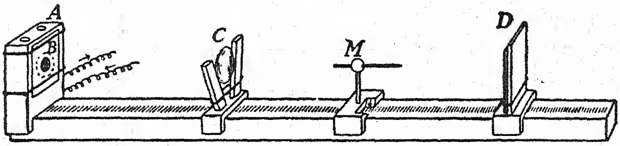


FIG. 106.

into the groove in BB at one end, and moved along to any position on the bench. This position is indicated, with reference to the scale of the bench, by means of a fine index line cut on the base of the upright in the plane of its vertical axis. The mounting of the mirrors, lenses, etc., should be effected in such a way that the point, to or from which measurements are to be made, is on the axis of the upright.

The optical bench is used chiefly for the experimental determination of the optical constants of spherical mirrors and lenses. Fig. 105 shows a concave mirror mounted on a stand, M, a candle in another upright, C, and a screen of thin, white, unglazed paper mounted on a frame in a third stand, S. As shown, an image of the candle is focused on the screen. The distances between M and C, and M and S, can be read off on the scale.

For more accurate work, however, a candle flame is not a very suitable source of light, and a better source can be made as follows:—An incandescent electric lamp is enclosed in a box, A (Fig. 106), a portion of the front of which has been cut away. A white cardboard screen, B, containing a large hole over which a piece of fine wire gauze is fixed, is fastened to the front of the box by india-rubber bands. The strongly illuminated gauze serves as a luminous object, and, being in a vertical plane, measurements can be accurately made, using a metal measuring rod, M, of known length, provided with pointed ends. When measuring distances, its ends are brought into contact with the different surfaces, and readings taken from the index line cut on its base.

C (Fig. 106) shows a suitable carrier for small mirrors or lenses. It consists of a sliding base supporting two adjustable arms grooved on the inside; the edges of the mirror or lens are placed within the grooves, and an elastic band is placed round the arms to keep them in position. D represents a small screen, consisting simply of a piece of white cardboard mounted on a sliding base.

In using an optical bench, care must be taken to ensure alignment between the pieces of apparatus on the bench. The lens or mirror must be mounted so that its optical axis is parallel to the scale on the bench.

2. Radius of Curvature and Focal Length of Spherical Mirror

I. CONCAVE MIRRORS

(1) The simplest method of determining the focal length of a concave spherical mirror is to allow a beam of parallel rays of light to be incident on the mirror in a direction parallel to the principal axis, and then to measure the distance of the focus of the reflected beam from the mirror.

Experiment. Mount the mirror in a stand on the optical bench. At the zero end of the bench, and at right angles to its length, fix a cardboard screen with its centre approximately on the same level as the principal axis of the mirror. Point the arrangement towards the sun, or some other well-illuminated distant object, and adjust the position of the mirror until a clearly defined image of the object chosen is formed on the screen. The distance between the mirror and screen is the focal length of the mirror, for if the object is sufficiently distant the image is at the principal focus of the mirror.

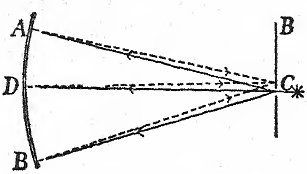


FIG. 107.

(2) When an object is placed at the centre of curvature of a concave spherical mirror, the image is formed also at the centre of curvature, but in an inverted position (see page 51). This provides another method of determination.

Experiments. (*a*) Fix a short polished needle vertically, point upwards, in a suitable stand, and place it in front of the mirror, so that the point of the needle is on the principal axis of the mirror. Unless the needle is too close to the mirror, an inverted image may be seen by an eye placed near the principal axis at some distance from the mirror (as in Fig. 42). By repeated trials, adjust the position of the needle until its point coincides with the point of the image for *several positions of the eye*. The point of the needle is now at the centre of curvature of the mirror, and hence the radius of curvature is obtained by measuring the distance between the point of the needle and the pole of the mirror. The focal length is equal to half this distance.

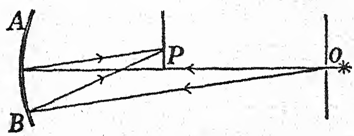


FIG. 108.

If the distance is small, measure it by means of a pair of compasses and a scale; but, if large, fix the needle and mirror in two of the uprights of the optical bench (Art. 1) and use the measuring rod, M.

(*b*) Using an optical bench and the illuminated object described (Art. 1), adjust the mirror until an image of the gauze is focused on the screen alongside the gauze itself (Fig. 107). Since the rays thus return very nearly along their former paths, C is very approximately the centre of curvature of the surface, ADB. Measure CD as in (*a*) above.

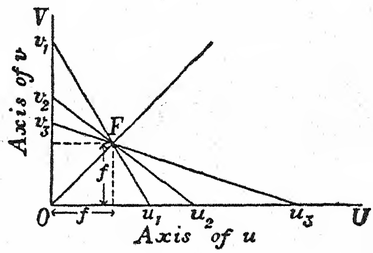


FIG. 109.

(3) The general relation, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$, may also be employed in finding the focal length and radius of curvature.

Experiment. Mount the mirror on an optical bench with its principal axis inclined slightly to the length of the bench, and receive the image, P (Fig. 108), on a screen placed so as not to interfere too much with the rays from the object, O. Measure u and v, and calculate f. Repeat for different values of u and v.

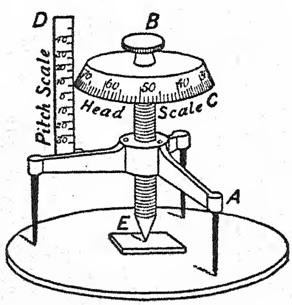


FIG. 110.

Instead of calculating values of f from each pair of values of u and v, these values may be plotted on squared paper and the value of f deduced graphically.

For, since $1/v+1/u=1/f$, it follows that $f/u+f/v=1$, which shows that if the points, (u, O), (O, v), be joined, the line will pass through the point, (f, f), for all values of u and v. Therefore, measure off values of u_1, u_2, u_3 . . . along the horizontal axis, OU (Fig. 109), and corresponding values of v_1, v_2, v_3 . . . along the vertical axis, OV. The lines, u_1v_1, u_2v_2, u_3v_3, . . . , should all intersect on the line, OF, bisecting the angle, UOV. Find the mean point of intersection; its coordinates are equal to each other and to the focal length, f.

(4) The radius of curvature of a spherical surface may also be measured mechanically by means of a spherometer (Fig. 110).

This consists of a metal frame, A, carrying four pointed legs, three of which are fixed and form the corners of an equilateral triangle, while the fourth, E, which is central, screws through the frame. In use, the feet of all four legs are made to rest first on a flat surface and then on the spherical surface. From a knowledge of the distance through which the central leg has been elevated or depressed, for the accurate measurement of which a circular scale, C, and a vertical scale, D, are provided, and the distance between the fixed legs, the radius of curvature of the surface can be calculated from the relation,

$$R = \frac{l^2}{6h} + \frac{h}{2},$$

in which h is the distance the central leg is raised or lowered, and l the distance between the fixed legs (see *Practical Physics*, Bower and Satterly, Arts. 27, 28, 30).

II. CONVEX MIRRORS

(1) The radius of curvature may be determined by means of a parallax method similar to I., (3) above for a concave mirror.

Experiment. Mount the mirror on an optical bench as usual, and place in front of it a well-defined object, such as a white thermometer tube. A *virtual* image can be seen. Place a large pin on a stand behind the mirror, so that it is just visible over the top, and adjust its distance from the mirror until it stands as nearly as possible over the image of the thermometer tube for all positions of the eye. Measure u, the distance of the tube from the mirror, and v, the distance of the pin from the mirror. Calculate f from the general relation. Repeat for several values of u and v.

This method can also be used for a concave mirror, the thermometer tube being placed so near it as to produce a virtual image. Unless, however, the mirror has a very large radius of curvature, this method is not recommended.

(2) Other optical methods require the use of lenses, and a description of the methods will be deferred (see Art. 5).

(3) The radius of curvature may be determined mechanically by means of the spherometer, as described in I., (4), above.

3. Discrimination Between Lenses and Plates of Glass

If the focal length of a lens is short, the curvature of the surfaces is very apparent, and the lens can be easily classified as convex or concave by the simple process of handling. If the focal length is

long, however, the surfaces are very nearly flat, and it may be difficult to distinguish the lens from a plate of glass, and, if a lens, to decide whether it is convex or concave.

Experiments. (*a*) *Detection of plane and spherical surfaces.*—To detect a plane surface, hold it in a horizontal position near to and just below the level of the eye, and observe in it the image of the horizontal bars of a window frame reflected at nearly grazing incidence. If the images are straight, the surface is plane; if distorted, the surface is curved. If, further, the extremities of the image are bent towards the observer, the surface is convex; if bent away, the surface is concave.

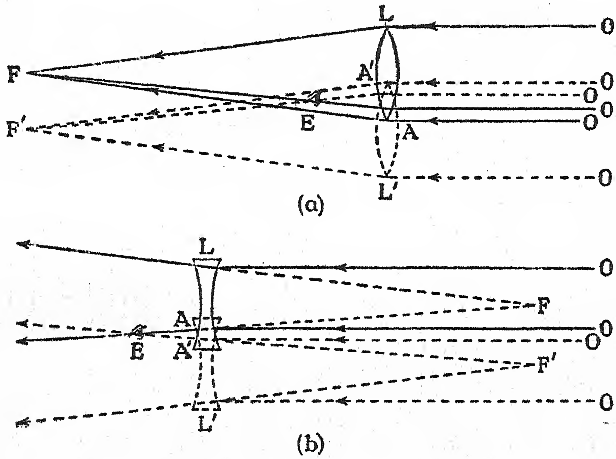


FIG. 111.

(*b*) *Detection of the nature of a lens.*—To detect the nature of a lens, hold it just in front of the eye, and view a distant object through it. Move the lens to and fro across the line of sight. If the image appears to travel across the lens in the same direction as the lens is moving, the lens is concave; if the directions of motion are different, the lens is convex.

In the case of a convex lens, L [Fig. 111 (*a*)], the rays coming from the distant object, O, are sensibly parallel, and, after refraction by the convex lens, converge to the principal focus, F. The focal length is long, and the eye, which may be considered to be at E, is therefore well inside F. E sees the object by means of the rays,

OAEF. When the lens is moved to L′, the object is seen by means of the rays, OA′EF′, and from the diagram it is evident that the motion of A to A′ is in an opposite direction to that of L to L′. Thus, the image appears to travel across the lens in an opposite direction to that in which the lens is moving.

In the case of a concave lens [Fig. 111 (*b*)], it is evident that the lens and image move in the same direction.

This method is applicable, and indeed need be used, only for convex lenses when the focal length is large and the eye can be placed between the lens and its principal focus. If the real image of an object be viewed through a convex lens, it will be found that lens and image move in the same direction.

4. Determination of Focal Length of a Lens

The experimental determination of the focal length of a lens is of great importance. The methods adopted depend upon the nature of the lens, and upon the degree of accuracy required. A few of the simpler approximate methods for each type of lens will now be considered.

I. CONVEX LENSES

(1) The simplest method of determining the focal length of a convex lens is to allow a beam of parallel rays of light to be incident on the lens in a direction parallel to the principal axis, and then to measure the distance of the focus of the refracted pencil from the lens.

Experiment. Mount the lens in a suitable stand on an optical bench, with its axis parallel to the length of the bench. At one end of the bench, and at right angles to its length, fix a white cardboard screen with its centre approximately on the same level as the principal axis of the lens. Point this arrangement to the sun or other well-defined distant object, and adjust the position of the lens until a clearly defined image of the sun, or other object chosen, is formed on the screen. The distance between the lens and the screen gives the required focal length.

(2) This method is based on the fact that if a beam of parallel rays of light leaves the lens, the source of the rays must be at the principal focus of the lens.

Experiment. Focus a telescope, T (Fig. 112), on a distant object. Fix the lens, L, to the telescope, in front of and coaxial with the object glass of the telescope. Now place a sheet of printed matter,

P, in front of the lens, and move it to and fro along the line of sight until the print is seen clearly by an eye looking through the telescope. Since the telescope is set for parallel rays (see page 231), P must be at the principal focus of L.

(3) A source is placed at the principal focus of the lens, and the parallel beam formed by the lens is reflected normally by a plane mirror, passes again through the lens, and is brought to a focus alongside the source, the rays retracing their paths because the reflection is normal.

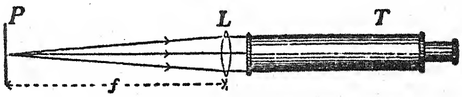


FIG. 112.

Experiment. The object is an upright pin, strongly illuminated by a lamp (Fig. 113). The distance of the pin is adjusted until its point is coincident with its (inverted) image. It may be necessary to tilt the mirror to obtain this coincidence. If a second, fainter, image is seen it is probably due to reflection at the back surface of the lens [see Art. 5, II. (2)], but the correct image disappears if the mirror is removed. Once the image is located, the distance is adjusted until there is no parallax between the points of the pin and of its image, *i.e.* until they remain coincident for all positions of the eye. A very common fault is to place the eye too close to the pin, it should be looked at from a *comfortable reading distance.* This method of locating images is distinctly more accurate than that of receiving the image on a screen owing to the difficulty of deciding when such an image is sharply focused. It can be applied, *e.g.* in method (4) below or methods (2) and (3) for the concave lens.

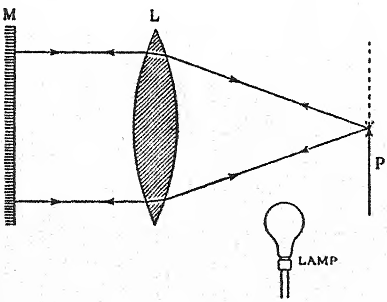


FIG. 113.

(4) Use the relation, $1/v - 1/u = 1/f$,† placing an object and lens to obtain a real image, measure u and v, and calculate f.

Experiment. Mount the lens in one of the uprights of the optical bench with its principal axis parallel to the length of the bench. In two other uprights, placed one on each side of the

† $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ if using the Real is Positive convention.

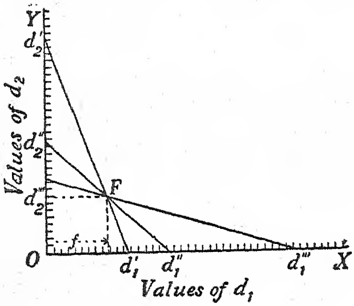


FIG. 114.

lens, fix an illuminated object and a screen, the object and the centre of the screen being adjusted to the level of the principal axis of the lens. By properly adjusting the positions of these two uprights, relative to that carrying the lens, focus a sharply defined image of the object on the screen. Measure the distances, u and v, by noting on the scale of the bench the distances between the illuminated object and the lens, and between the lens and the screen. Substitute these values in the above relation, taking care that v is represented by a *negative* number.† Repeat for several values of u, taking the mean of the results as the mean value of f.

If, in the general relation above, u is replaced by d_1, v by $-d_2$, and f by $-f_1$ where d_1, d_2, f_1, are simply the numerical values of u, v, and f,† then

$$\frac{1}{d_1} + \frac{1}{d_2} = \frac{1}{f_1},$$

which is simpler for purposes of calculation. The arithmetic is still further simplified if tables of reciprocals are employed.

Plot the results of the experiment on squared paper, as in Art. 2, I., (3). Take values of d_1 along OX, and values of d_2 along OY (Fig. 114). Join up corresponding points. All these lines should intersect one another at the point whose coordinates are equal to the focal length of the lens.

(5) *The Displacement Method.*—This method is of special importance. Let A and B (Fig. 115) represent respectively the positions of a luminous object and a screen. Then, if a *magnified* image of the object, A, be formed on the screen at B by a lens placed at C, a *diminished* image can also be obtained on the screen by placing the lens at a point, C′, such that BC′ = AC. For, if AC and BC are conjugate focal distances, then the equal distances, AC′ and BC′, are also conjugate.

Let AB be denoted by l, and CC′ by a. Then, if AC and C′B be each denoted by d_1, and CB and AC′ be each denoted by d_2,

$$AB = AC + CB, \text{ or } l = d_1 + d_2 \quad \ldots\ldots\ldots\ldots (1)$$

† v and f are positive if the Real is Positive convention is in use.

Also, $CC' = AC' - AC$, i.e. $a = d_2 - d_1$ . . . . (2)

So, from (1) and (2), $d_1 = \dfrac{l-a}{2}$, and $d_2 = \dfrac{l+a}{2}$.

Using the relation, $\dfrac{1}{d_1} + \dfrac{1}{d_2} = \dfrac{1}{f_1}$,† from (4) above,

$$\frac{2}{l-a} + \frac{2}{l+a} = \frac{1}{f_1};$$

$$\therefore f_1 = \frac{l^2 - a^2}{4l} \quad \ldots\ldots\ldots\ldots (3)$$

Hence, by measuring l and a, f may be readily determined. This method does not involve any error due to inexact knowledge of the position of the optical centre of the lens.

Experiment. Using an optical bench, find the focal length of a convex lens by this method. Measure, with a pair of dividers and a fine scale, the dimensions of the object, either the diameter of the circle in B (Fig. 106) or the width of a convenient number of wires of the gauze, and image in each case.

Note that the image is as much magnified for one position of the lens as it is diminished for the other position. The proof of this is simple. Let the corresponding dimensions of the object and the two images be denoted by O, I_1, and I_2 respectively. Then, when the lens is at C, $\dfrac{I_1}{O} = \dfrac{d_2}{d_1}$, and when the lens is at C′, $\dfrac{I_2}{O} = \dfrac{d_1}{d_2}$. Hence,

$$\frac{I_1 I_2}{O^2} = \frac{d_2}{d_1} \cdot \frac{d_1}{d_2} = 1; \text{ that is, } O = \sqrt{I_1 I_2}.$$

Thus, if I_1 and I_2 are measured, O can be calculated. This method proves very useful if the object cannot be measured directly.

If a, in equation (3) above, becomes zero, then, neglecting signs,

$$f = \frac{l^2}{4l} = \frac{l}{4}.$$

When this is the case, the points, C and C′ (Fig. 115) are evidently coincident, and AC=CB. Then, object and image are equidistant from the lens, and are therefore equal in size (see page 117).

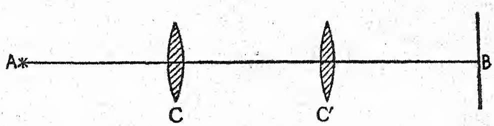


FIG. 115.

† This relation also holds if the Real is Positive convention is used, because only the *numerical* values are involved.

This particular case of this method is applied in *Silbermann's* focometer (Fig. 116). This focometer consists of a fixed scale carrying three slides, A, B, C. Mounted in tubes on A and B are two glass scales photographed from the same negative, the graduations being uncovered and facing the lens, whose focal length is to be determined, carried by the slide, C. The positions of these slides are adjusted until the image of the scale in A is seen to coincide exactly with that in B. It will then be found that A and B are equidistant from C, and the distance, AB, read off on the scale, gives l, from which f is calculated, since $f = \frac{l}{4}$.

*(6) *The Magnification Method.*—This method is of importance because it is applicable to thick lenses and combinations of lenses, such as the photographic lens, as well as to thin lenses.

Let a lens of focal length, f, at a distance, u_1, from the object

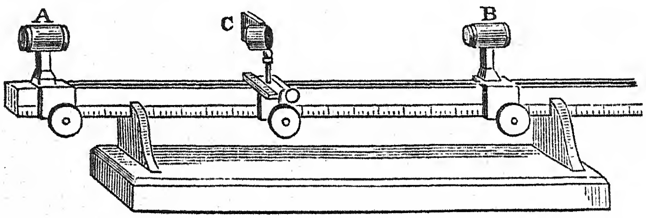


FIG. 116.

produce an image of magnification, m_1, at a distance, v_1. Also, let the same lens and object at a distance, u_2, produce an image of magnification, m_2, at a distance, v_2. Then,

$$\frac{1}{f} = \frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{v_2} - \frac{1}{u_2}, \quad m_1 = \frac{v_1}{u_1}, \text{ and } m_2 = \frac{v_2}{u_2};$$

$$\therefore \frac{v_1}{f} = 1 - \frac{v_1}{u_1} = 1 - m_1 \text{ and } v_1 = (1 - m_1) f.$$

$$\text{Similarly, } v_2 = (1 - m_2) f;$$

$$\therefore v_2 - v_1 = f(m_2 - m_1),$$

$$\text{and } f = \frac{v_2 - v_1}{m_2 - m_1}.$$

The apparatus illustrated above (Fig. 116) is very convenient for experiments on this method.

Experiment. Find the focal length of a thick convex lens.—Use as an object a pair of dividers, the legs being opened so that the points are some exact distance apart, say 1 cm. Focus a magnified image of the points on a fine scale, and note the number, n_1, of scale divisions bridged by the tips. Now move the scale towards the lens through a measured distance, d, say 2 cm. for a short focus lens, to 15 to 20 cm. for a long focus lens. Adjust the position of the dividers, the lens must not be moved, until an image of them again rests on the scale. Note the number, n_2, of scale divisions bridged by them now. Then find, by direct application, the number, n, of divisions of the scale bridged by the points of the dividers themselves. In the first position the magnification, $m_1 = \frac{n_1}{n}$, and in the second, $m_2 = \frac{n_2}{n}$. Then the focal length $= \frac{d}{m_2 - m_1}$.*

II. CONCAVE LENSES

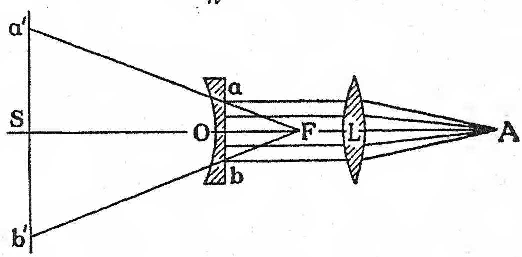


FIG. 117.

It has already been seen that with a concave lens the image of a real object is virtual, and so cannot be received upon a screen. This renders it difficult to determine the focal length of a concave lens, but the methods described below may be adopted with fairly accurate results

(1) One face of the concave lens is covered with a circular piece of black paper, through which two large pinholes have been made on a diameter of the circle, at points equidistant from the centre. A beam of parallel rays of light is directed on the lens in a direction parallel to the principal axis. All the incident rays, except those passing through the pinholes, are stopped by the black paper, and, if a screen be placed behind the lens, two bright spots are formed upon it where the rays passing through the pinholes meet its surface.

Let a and b (Fig. 117) represent the positions of the pinholes. Then, the incident light being parallel, the rays refracted at a and b diverge from the principal focus, F, and bright spots are formed at a' and b' on a screen placed at any point, S, behind the lens. From the figure,

$$\frac{ab}{a'b'} = \frac{FO}{FS} = \frac{FO}{FO + OS}.$$

Hence, if the focal length of the lens be denoted by f,

$$\frac{ab}{a'b'} = \frac{f}{f + OS}.$$

Experiment. Place a source of light, A, at the principal focus of a convex lens on an optical bench, and then place the concave lens and screen in position. Measure ab and $a'b'$ with a pair of dividers and a fine scale, and read off the distance, OS, on the bench. Calculate f from the relation above.

Very rough results only can be obtained by this method.

(2) A second method is as follows:—Let P (Fig. 118) denote the position of an object, and P′ the position of its image, formed on the screen at S by the convex lens, L. If now the *concave* lens be placed at L′ in such a position that L′P′ is less than its focal length, then the rays converging to P′ become less convergent and

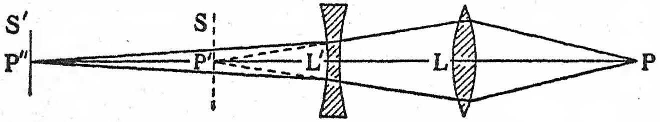


FIG. 118.

thus meet at a more distant point, P″. If the screen be placed at S′, an image of the object at P is formed on it. This image may be considered as the real image of the *virtual* object, P′ (and u is thus *negative*), and, if L′P′ and L′P″ be measured, the focal length of the concave lens may be calculated from the relation, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$.†

Experiment. Mount the convex lens, L, on the optical bench, and locate the image, P′, by means of the screen, S. Note its position on the scale. Mount the concave lens, L′, so as to be on the same level as L, and place it in position. The image is now thrown back to P″. Locate it by the screen as before. If the distance, L′P″, is very great, there will be a considerable range for which the image is in focus. To overcome this difficulty, place L′ nearer P′, when P″ will be considerably nearer to P′.

(3) A third method is as follows:—If the lens, L′ (Fig. 118), is so placed that L′P′ is equal to its focal length, the rays leaving L′ will be parallel to one another, and may be made to retrace their paths by means of a plane mirror placed at right angles to them,

† $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ if the Real is Positive convention is used. u is negative because the object is virtual, v is positive because the image is real.

Experiment. Place a plane mirror, M (Fig. 119), behind the concave lens, L′, and move the lens along the bench until an image of the object at P is formed alongside P. Note the position of L′ and then remove it and the mirror, M. The rays now come to a focus at P′, and will form an image of P on a screen placed there. P′L′ is the focal length of the concave lens.

(4) A fourth method consists in putting a convex lens in combination with the concave lens. It has been shown above (see page 121) that if two thin lenses of focal lengths, f_1 and f_2, be placed in contact so as to act as one compound lens, then the focal length, F, of the combination is given by the relation,

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}.$$ V.P. Singh

If a concave lens of focal length, f_1, be combined in this way with a convex lens of *shorter* focal length, f_2, the combination is evidently equivalent to a convex lens, and its focal length, F, may be determined by any of the methods described above. Similarly, f_2 may be determined, and the required focal length, f_1, may then be calculated from the relation. Great care must be taken over the signs in using the relation.

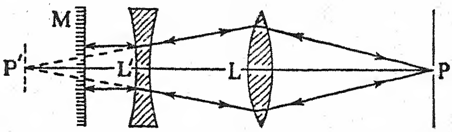


FIG. 119.

5. Radius of Curvature of the Surface of a Lens

The spherical surface of a lens may always be treated as a spherical reflecting surface, and, unless the radius of curvature is very great, the methods used for the determination of the radius of curvature of a spherical mirror may generally be applied to the surface of a lens.

I. CONCAVE SURFACES

All the methods described above (Art. 2, I.) may be used to determine the radius of curvature of the concave surface of a lens, when the surface under consideration is an *external* surface of the lens. When, however, the surface under consideration is an *internal* surface of the lens, it may be either considered as an external convex surface, or, treated as described below in II., (2).

II. CONVEX SURFACES

Two methods of measurement are described above [Art. 2, II., (1) and (3)]. The following are also in general use:—

(1) If a convex spherical surface is so placed in a converging beam of light that the focus of the beam is the centre of curvature of the surface, the rays are reflected back upon themselves and retrace their former paths.

Experiment. Place the convex spherical surface, M (Fig. 120), to be measured behind a convex lens, L, of short focal length, and adjust their positions until a sharp image of the object, P, is formed on the screen alongside the object by rays returned after reflection at the convex surface. Note the position of M, and then remove it. The rays now travel on, and an image of P is formed at P′, and can be focused on a screen. Since the rays were incident normally on M, they were converging to its centre of curvature. Hence, MP′ is the radius of curvature.

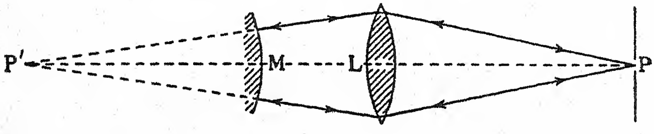


FIG. 120.

This method may be used to find the radius of curvature of the convex surface of a lens; the second surface will not interfere.

(2) This method is applicable only to the convex surfaces of lenses, but for them it is the simplest and best method, and is easily carried out at the same time as the determination of focal length (see page 131).

Experiment. Mount the lens, L (Fig. 121), on the optical bench, with the surface under consideration turned *away* from the illuminated object, P. Some light is reflected from the surfaces of the lens. Starting with the lens close up to the object, gradually withdraw it, until one of the reflected beams forms an image of the object alongside the object itself. If necessary, tilt the lens slightly to one side, in order to throw the image on the screen and thus allow it to be seen clearly. When the image is sharply focused, measure the distance, LP, from lens to screen, and denote it by C.

*Since the rays of light, originally diverging from P, return along their original paths, it is clear that they must be incident normally on

the back *interior* surface of L, and hence those rays which penetrate L diverge from O, the centre of curvature of this surface. Therefore P and O are conjugate foci. Denoting the focal length of the lens by f, and the radius of curvature of the back surface by R, then

$$\frac{1}{R} - \frac{1}{C} = \frac{1}{f}, \text{ and, } R = \frac{Cf}{C+f}.$$

R and C are of course positive, but f is positive or negative according as the lens is concave or convex. *

The method is applicable to the surfaces of double convex lenses, concavo-convex lenses, and to convexo-concave lenses, in which the radius of curvature of the convex surface is *numerically* smaller than the focal length of the lens.

In the case of a concavo-convex lens, the first focused image of the object on the screen, as the lens is gradually withdrawn from the illuminated object, is due to reflection at the back surface of the lens, and gives C. As the lens is still further withdrawn, another image is focused on the screen. This image is formed by reflection at the concave surface of the lens, and the distance of the lens from the screen gives the radius of curvature of that surface [Art. 2, I., (2) (*b*)].

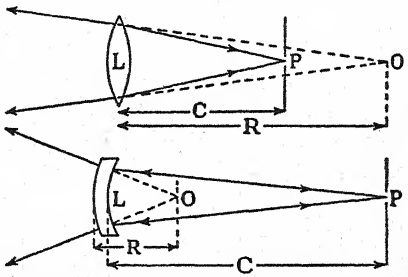


FIG. 121.

In the case of a convexo-concave lens, the distance of the first image, as the lens is gradually withdrawn, gives the radius of curvature of the concave surface. The distance of the second image gives C (Fig. 121).

6. Refractive Index of the Material of a Lens

The focal length and the radii of curvature of the two surfaces of a lens having been found by the experiments described above, the refractive index, μ, of the material of the lens can be determined from the relation,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right),$$

in which r is the radius of curvature of the surface upon which the light first falls.

Great care must be taken over the signs of the various terms. It should be remembered that the focal length of a convex lens is negative and that of a concave lens positive.† It should also be noted that in the case of a double convex lens, or a double concave lens, the radii of curvature of the surfaces are of opposite sign, while in the case of a concavo-convex lens, or a convexo-concave lens, the two radii of curvature of the surfaces have the same sign.

7. Deviation Produced by a Prism

The phenomenon of deviation of light by a prism was illustrated (p. 89) by the displacement of the *virtual* image of a slit seen through the prism. Deviation can be illustrated, however, by noticing the displacement of a *real* image obtained by means of a lens and prism. The apparatus described below may be employed for this purpose, and will also serve to give measurements of the deviation produced.

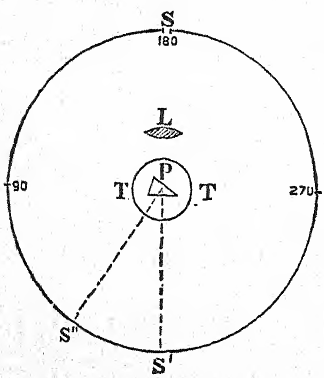


FIG. 122.

Experiment. A small accurately circular table, about 30 cm. in diameter, is fitted round its edge with a strip of thin tough paper in such a way that the upper edge of the strip projects about 5 cm. above the face of the table. On this projecting edge, about half-way up, a scale in degrees is marked all round the strip. At the 180th division on the scale, a narrow vertical slit, S (Fig. 122), about 2 cm. long and 0·5 mm. wide, is so cut in the paper that one edge coincides accurately with this division.

Take the apparatus into a dark room, and illuminate the slit by a properly shaded sodium flame. Fix a mounted convex lens, L, of about 5 cm. focal length, on the table between the slit and the centre of the table, in such a position that an image of the slit, having one edge coincident with the zero on the scale, is obtained on the paper strip at S′ (see also Fig. 123). At the centre of the table fix a small stand, TT, which can be rotated about a central vertical axis. On this stand place a prism so that its refracting

† Using the Real is Positive sign convention, the equation still holds, but a surface convex to the incident light is reckoned to have a positive radius of curvature, and the focal length of a converging lens is thus positive.

edge is vertical, and the plane bisecting its refracting angle passes through the axis of rotation of the stand.

Rotate the stand until the rays of light coming from the slit, through the lens, are refracted through the prism. Observe that the position of the image of the slit is changed, and that the change of position indicates that the rays are deviated by the prism in a direction away from the refracting edge. Note also that as the rotation of the prism continues, the position of the image changes, indicating that the magnitude of the deviation produced depends upon the position of the prism relative to the incident light.

If, for any position of the prism, the image is formed at S″, then the magnitude of the deviation is measured by the angle, S′PS″, which may be read off on the scale.

Now rotate the prism so as to cause the deviation to diminish. As the prism is rotated, always in the same direction, the image will travel at a gradually decreasing rate towards S′, and at a certain point will become stationary, and then turn back in the opposite direction. The deviation at the instant at which the image is stationary is the *minimum deviation*, which can thus be read off on the scale by noting the division at which the image ceases to advance towards S′ and begins to turn back.

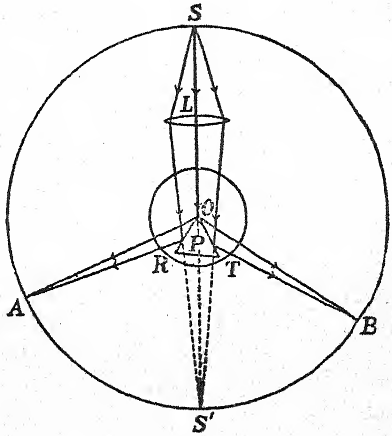


FIG. 123.

The image obtained on the scale, after the interposition of the prism, is not clearly defined except at, or near, the position of minimum deviation, and consequently measurements made near this position can be made more accurately than for other positions. Very accurate measurements of deviation are made by means of the spectrometer (see page 205).

8. The Angle of a Prism and the Refractive Index of its Material

The experiment described above (Art. 7) may be extended to determine the refracting angle of a prism.

Experiment. Using the same apparatus as in the experiment described above (Art. 7), and having adjusted L in position, place

P on the stand so that the refracting edge is over the axis of rotation of the stand, and the faces of the prism, OR, OT (Fig. 123), are nearly symmetrical about SO. The converging beam of light falling on P is now reflected in two portions, and the reflected beams converge to points, A and B, on the scale. Read off on the scale the magnitude of the angle, AOB. This angle is twice that of the refracting angle of the prism; for (page 24)

$$\text{Angle AOS}' = 2 \text{ angle ROS}', \text{ and angle BOS} = 2 \text{ angle TOS}';$$

$$\therefore \text{ angle AOB} = 2 \text{ angle ROT}.$$

Having now determined the refracting angle of the prism and the angle of minimum deviation, all the data necessary for the calculation of the refractive index of the material of the prism for sodium yellow light are available. Thus, if A is the refracting angle, and D the angle of minimum deviation, then (see page 88) the refractive index, μ, of the material is given by the relation,

$$\mu = \frac{\sin \frac{1}{2}(D + A)}{\sin \frac{1}{2}A}.$$

The refractive index of a liquid can be determined by this method if the liquid is enclosed in a hollow prism whose walls are made of thin parallel plates of glass suitably cemented together at their edges.

The refractive indices of metals have been measured by using very acute angled prisms of glass, having the metals deposited on them by electric discharge or by chemical decomposition. Some metals behave as if they had a refractive index of less than unity referred to a vacuum (see Table I.). A prism of such a metal deviates a ray *towards* the refracting edge, *i.e.* O is negative.

APPENDIX TO CHAPTER VIII

ALTERNATIVE TREATMENT OF CERTAIN SECTIONS IN TERMS OF THE REAL IS POSITIVE CONVENTION

4. Determination of the Focal Length of a Lens

(6) *The Magnification Method.*—This method is of importance because it is applicable to thick lenses and combinations of lenses, such as the photographic lens, as well as to thin lenses.

Let a lens of focal length, f, at a distance u_1, from the object produce an image of magnification, m, at a distance, v_1. Also, let the same lens at a distance, u_2, from the same object produce an image of magnification, m_2, at a distance, v_2. Then,

$$\frac{1}{f}=\frac{1}{v_1}+\frac{1}{u_1}=\frac{1}{v_2}+\frac{1}{u_2},\ m_1=-\frac{v_1}{u_1},\ \text{and } m_2=-\frac{v_2}{u_2};$$

$$\therefore \frac{v_1}{f}=1+\frac{v_1}{u_1}=1-m_1,\ \text{and } v_1=(1-m_1)f.$$

Similarly $$v_2=(1-m_2)f;$$

$$\therefore v_2-v_1=f(m_2-m_1)\ \text{and } f=\frac{v_2-v_1}{m_2-m_1}.$$

The apparatus illustrated above (Fig. 116) is very convenient for experiments on this method.

Experiment. Find the focal length of a thick convex lens.—Use as an object a pair of dividers, the legs being opened so that the points are some exact distance apart, say 1 cm. Focus a magnified image of the points on a fine scale, and note the number, n_1, of scale divisions bridged by the tips. Now move the scale towards the lens through a measured distance, d, say 2 cm. for a short focus lens, to 15 to 20 cm. for a long focus lens. Adjust the position of the dividers, the lens must not be moved, until an image of them again rests on the scale. Note the number, n_2, of scale divisions bridged by them now. Then find, by direct application, the number, n, of divisions of the scale bridged by the points of the dividers themselves. In the first position the magnification, $m_1=\frac{n_1}{n}$, and in the second, $m_2=\frac{n_2}{n}$. Then the focal length $=\dfrac{d}{m_2-m_1}$.

5. Radius of Curvature of a Surface of a Lens

II. CONVEX SURFACES

Since the rays of light, originally diverging from P, return along their original paths, it is clear that they must be incident normally on the back *interior* surface of L, and hence those rays which penetrate L diverge from O, the centre of curvature of this surface. Therefore P and O are conjugate foci. Denote the focal length by f, and the radius of the back surface by R. Adopting the usual convention, R is to be reckoned negative if the surface is concave to the direction of the incident light. In this case, the light only apparently diverges from O, which is a virtual image, and v is negative. In the case of a new surface convex to the direction of the incident light, O is a real image and v is positive. Thus, we have $v = -R$. Hence,

$$\frac{1}{C} - \frac{1}{R} = \frac{1}{f}, \text{ and, } R = \frac{Cf}{f - C}.$$

C is always positive because the object is real, but f is positive or negative according as the lens as a whole is diverging or converging.

CHAPTER IX

DISPERSION. ACHROMATISM

IN dealing with the phenomenon of refraction of light by means of prisms and lenses in the preceding chapters, no account has been taken of the *nature* of light, or the possible consequences of its nature upon the phenomena considered and upon the effects produced. It is essential, however, to know something of this nature in order to understand completely the behaviour of prisms and lenses. At the same time, this nature may produce undesirable effects in certain cases, and it may be necessary, therefore, to know how to eliminate such effects. In what follows, these points will be discussed in detail.

1. Homogeneous and Compound Light

It has been indicated already (page 2) that there is reason to believe that the physical cause of light is a species of transverse vibratory motion in the ether. When this motion is made up of a series of waves, all of the same wave-length, then the light is said to be **homogeneous** or **monochromatic.** It is more generally the case, however, that the wave motion is made up of an infinite number of waves of different wave-length. The light is then said to be **non-homogeneous** or **compound.**

Monochromatic light is of a definite colour, corresponding to its wave-length, and difference in wave-length is always indicated by a difference in colour.

Compound light may be of any colour, or may be *white* or colourless, but its colour is no indication of its composition; two compound lights of almost identical colour may be made up of very different constituents, and may even match exactly the colour of any of the monochromatic lights. In the case of white light, however, it can always be stated that it is compound, for all monochromatic lights are coloured; but, without experiment, the constituents of any given source of white light cannot be stated.

Solar light, and the other most familiar white lights—such as gaslight, electric light—are found to be very similar in composition, and to include almost all possible shades of monochromatic light. The reason for this is evident when it is remembered that white light of this nature always results from incandescence, and that an

incandescent or white-hot body has passed through all the phases of change of colour attendant on rise of temperature. It is, therefore, giving out light of all wave-lengths, from dark red, which are the longest waves and which first appear when it begins to get red-hot, to violet, which are the shortest waves and which are added when it first appears to be perfectly white.

2. Newton's Experiments on Solar Light

The light coming from the sun was first shown by Newton in 1676 to be of a composite character. The experiments by which this fact was demonstrated are worthy of special notice, both on account of the historical interest attached to them, and because of the great importance of the facts which they illustrate.

In its simplest form, Newton's first experiment may be performed in the following way:—A beam of sunlight is admitted into a dark room through a small circular aperture, A (Fig. 124), in a shutter or blind. The beam will be seen in the room as a small pencil of light diverging from A, and, if allowed to fall on a vertical screen at S, forms a small elliptical bright spot which is a rough image of the sun. If now a prism, P, with its edge horizontal, be placed edge downwards in the path of the beam, the latter will be deviated from its original course, and deflected upwards so as to form an image at S′S″. This image differs from that first formed at S in several important particulars; *the vertical diameter is much longer, and, instead of appearing as a bright patch on the screen, it is made up of several coloured bands, arranged horizontally.* In fact, the image is made up of several overlapping images, similar in shape to that originally seen at S, but each of a different colour.

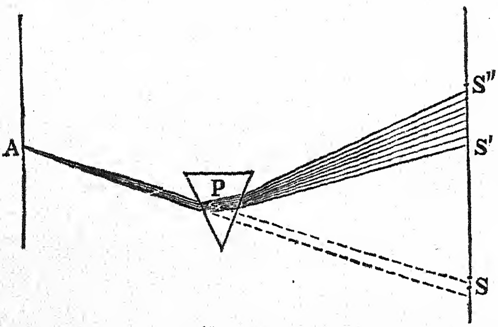


FIG. 124.

By selecting one of the constituents after passing through the first prism, and allowing this to be deviated by a second prism, Newton found that no further colouring was produced.

Again, by inverting the second prism and allowing the whole

coloured image, S′S″, to fall upon it, the sunlight was brought to its original state.

It should be noted that Newton did not use a narrow slit, but a small circular aperture, and further that no lenses were used in the experiments (see Art. 4).

These experiments show that the beam of *white light* incident on the prism is, on refraction through the prism, separated into its different coloured constituents, each of which forms its own image on the screen, and thus the multi-coloured compound image at S′S″ is formed. Such a compound image is called a **spectrum.** When a spectrum is formed by the decomposition of solar light, as in the case just considered, it is called a **solar spectrum,** and, on a first analysis, may be taken as made up of *six* colours—**red, orange, yellow, green, blue,** and **violet.** Of these, the red rays are the least deviated, and therefore appear at S′, the bottom of the image, S′S″. The violet rays are the most deviated, and therefore appear at S″, the top of the same image. The intermediate rays are arranged in the order given, from below upwards, between S′ and S″.

Incidentally, Newton described *seven* principal colours, including **indigo** between blue and violet, and others have followed him; but no such colour is to be seen by a normal eye in a spectrum, and if it were, indigo is merely a kind of blue, and there is no more reason to subdivide the blue than the green or any other colour.

The student must beware of thinking that the spectrum can be divided into six distinct blocks of different colours, and that solar light is made up of only six different constituents, corresponding to the six colours. This is not the case; the number of constituents of solar light is *infinite*, but, considered with reference to their action on the eye, they may be divided into six groups, each of which corresponds to a definite *colour sensation*, and comprises an infinite number of rays, each corresponding to a certain *shade* of the colour which characterises the group to which it belongs.

White light is now believed to be made up of irregular disturbances or pulses. The student who is familiar with Fourier analysis will be aware that any irregular disturbance can be represented by the superposition of continuous waves of all possible frequencies, just as any *noise*, however irregular and discordant, can be analysed into its component musical tones. The prism merely acts as a device for sorting out these frequencies, that are already present in any irregular pulse, which it does by virtue of the fact that the speed of light varies with frequency in all media except a vacuum.

3. Refrangibility

It has been seen that when the rays of a compound beam of light are refracted through a prism, as in Newton's experiment above, each constituent undergoes refraction to a different extent. This is sometimes expressed by saying that the constituents of the compound beam have different **refrangibility**. The *most refrangible* rays are those which undergo the greatest deviation, while the *least refrangible* are those which undergo the smallest deviation. In the solar spectrum the red rays are the least refrangible, while the violet rays are the most refrangible. The intermediate rays increase in refrangibility from red through orange, yellow, green, and blue to violet.

From what has been said above, it will be seen that difference in refrangibility corresponds to difference in wave-length. Light of high refrangibility is of *short* wave-length, and the corresponding refractive index for any given medium is relatively *high*, while light of low refrangibility is of *long* wave-length, and the corresponding refractive index for any given medium is relatively *low*.

This different refrangibility is due to the different frequency of the various coloured rays. In fact, colour in light corresponds with pitch in sound. The violet waves are the shortest and their frequency therefore highest, the red are the longest and their frequency therefore least. Hence, a violet light corresponds to a high note, and a red light to a low note. Incidentally, it will be remembered that—velocity = wave-length × frequency, and, when velocity is constant, a large wave-length means a low frequency, and a small wave-length a high frequency.

4. Pure Spectrum

The spectrum obtained by using the simple apparatus of Newton's experiment (Art. 2) is indistinct and badly defined because of the overlapping of the images of which it is composed. Such a spectrum is said to be *impure*. To obtain a pure spectrum a very narrow slit must take the place of the aperture in the shutter, and some means must be adopted to obtain a spectrum made up of a series of adjacent, but not overlapping, images of this slit.

Let S (Fig. 125) denote the position of the slit. The pencil of rays diverging from S forms a broad band, *ab*, on the screen, S′S″. If now the prism, P, be interposed *in the position of minimum deviation*, with its edge parallel to the length of the slit, the rays of the pencil are deviated and dispersed in such a way that the red

light appears to come from r', a virtual image of the slit, S, and forms a red band, rr, on the screen. Similarly, the violet light appears to come from v', and gives the violet band, vv; and so on, for each colour of the spectrum. In general, oblique refraction does

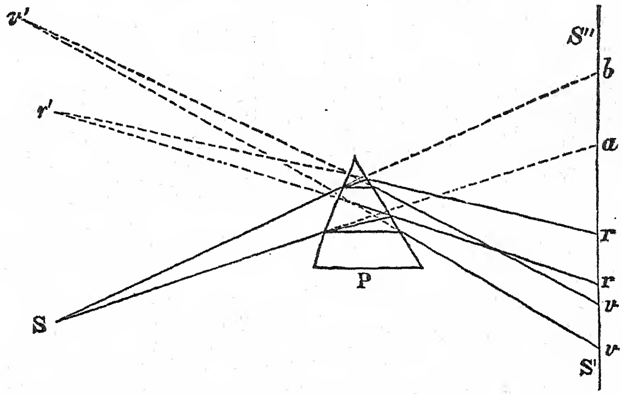


FIG. 125.

not produce a point image of a point source (see pages 77 and 89), but in the special case when the prism is symmetrically placed with respect to the incident and emergent rays, the emergent pencil does diverge from a point, and hence r' and v' can be considered as definite images. It is evident that the spectrum thus obtained on the screen is composed of a series of overlapping bands, and is therefore *impure*.

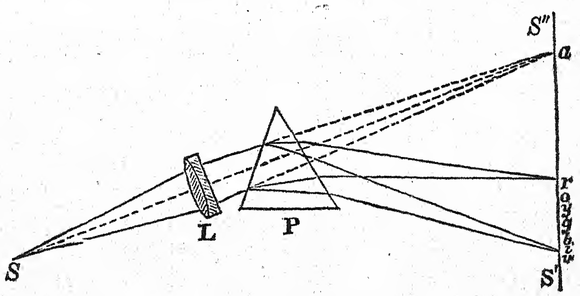


FIG. 126.

If, however, a suitable lens, L (Fig. 126), be placed so as to give, when P is removed, a distinct image of the slit, S, at a, and the

prism then interposed in the position of minimum deviation for the mean rays, the pencil of rays converging to a will, after refraction through the prism, be dispersed and give rise to a series of pencils converging to the points, r, o, y, g, b, v. Real images of the slit of light of each colour are thus formed at these points, and, as each image is narrow and distinct, like that at a, there is no overlapping and a *pure* spectrum is obtained. If the slit itself is not sufficiently narrow, the images may be broad enough to overlap and thus give an impure spectrum.

Instead of placing the lens at L (Fig. 126), it may be placed on the other side of the prism in such a position that (Fig. 127) real images of the virtual foci lying between r' and v' (Fig. 125) are formed on the screen.

From the above, it follows that to obtain a real, pure spectrum, the essential requirements are:—

(1) A very narrow slit.

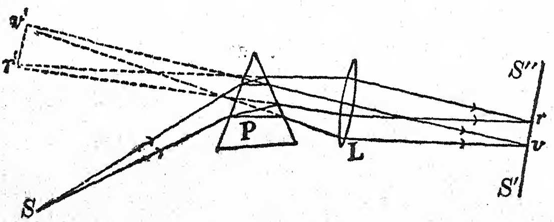


FIG. 127.

(2) The prism in the position of minimum deviation for the mean rays and therefore approximately for all rays.

(3) A lens so placed as to form a *clearly defined* spectrum on the screen.

The second condition is of importance, for it is only when the prism is in the position of minimum deviation that clearly defined images can be obtained. In practice, it will be found convenient to illuminate the slit by means of a lamp, and to employ, instead of the large screen shown in the diagrams, a smaller one, placed first at a (Fig. 126) to receive the direct image of S, and then at $r \ldots v$ to receive the spectrum. The prism is placed in the position of minimum deviation by rotating it until the position of the spectrum is as near as possible to a.

It should be noticed here that the above arrangement of apparatus is necessary to obtain a *real*, pure spectrum, which can

be received on a screen. A *virtual*, pure spectrum can be seen merely by looking through a prism at a narrow slit. An eye placed near the prism, so as to receive the emergent pencil, sees a small but very bright and pure spectrum at the virtual foci of the different constituents of the pencil entering the eye. The violet end of this spectrum is seen nearest the refracting edge, because the violet rays are most refracted. This is evident from Fig. 127, the lens and screen there shown being replaced by the lens and the retina of the eye. The red of the spectrum is seen at r', and the violet at v'.

A third method of obtaining a real, pure spectrum will be described later under the *spectroscope* (page 230).

A fourth method—a modification of Figs. 126 and 127—is shown (Fig. 128). The lens, L_1, is so placed that the rays of light leaving it are parallel. The rays of the different colours are deviated through different angles by P, the rays of each colour emerge as a parallel beam, and the lens, L_2, brings each colour to a different focus on the screen.

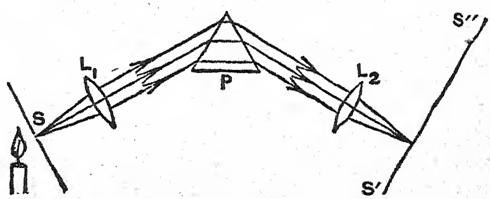


FIG. 128.

Experiments. (*a*) Cut a narrow slit, about 1 mm. in width, in a piece of cardboard or metal foil, and place it vertically in front of a bright white light, such as the sun, a bright cloud, an electric arc, or an electric lamp. Take a glass prism, and stand it in a vertical position in the path of the light issuing from the slit. At some distance away place a white screen. Note the spectrum formed, its deviation, and the fact that the violet is deviated more than the red, the centre being very nearly white. Set the prism in the position of minimum deviation.

(*b*) Close behind the prism stand another similar prism, also in the position of minimum deviation. Note that the deviation of the spectrum is increased and that it is further drawn out.

(*c*) Take this second prism and now place it horizontally with its refracting edge downwards, close behind the first prism. Note that the spectrum is now elevated, the violet more so than the red, so that the spectrum is now on the slant. This is Newton's celebrated *crossed prisms* experiment.

(*d*) Repeat (*a*), but instead of using the screen, place the eye to receive the emergent beam. The spectrum seen is almost *pure*.

(*e*) Now take a lens of about 1 ft. focal length, and set up the prism and lens as in Figs. 126 or 127. A real, pure spectrum is now formed on the screen. Note the purity of the colours compared with those produced in (*a*).

5. Dispersion

It has been seen that when a beam of compound light is refracted through a prism, each constituent of the beam undergoes deviation to a different degree. The light of shortest wave-length is deviated most, and that of longest wave-length least, and thus the different constituents of the incident beam are separated, as it were, each travelling in a definite direction determined by the deviation it has experienced.

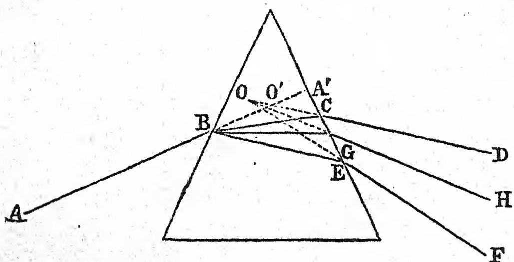


FIG. 129.

This separation of the different constituents of a compound beam of light by refraction through a prism is called dispersion. Quantitatively, dispersion is measured, for any two rays of the refracted pencil, by the angle between these rays. Thus, if AB (Fig. 129) represent a ray incident on a prism at B, and split up by refraction through the prism into a pencil of rays bounded by BCD and BEF, then the angle, DOF, measures the dispersion for the extreme rays of the emergent pencil.

Now, the angle, DOF, is the difference between the deviations of the extreme rays, ABCD and ABEF. Hence, if μ_v denote the refractive index for ABEF, and μ_r the refractive index for ABCD, the deviation for ABEF is given approximately for a prism of small angle, A (see page 88), by

$$D_v = (\mu_v - 1)\,A,$$

and the deviation for ABCD is given approximately by

$$D_r = (\mu_r - 1)\,A.$$

Thus, the angle, DOF $= D_v - D_r \doteq (\mu_v - 1)\,A - (\mu_r - 1)\,A$;

$$\therefore \text{ Dispersion} = (\mu_v - \mu_r)\,A.$$

It should be noted that the angle of the prism in the diagram (Fig. 129) is made large intentionally for the sake of clearness.

6. Dispersive Power

The dispersive power of a refracting medium is determined by *the ratio of the extreme dispersion produced by a prism of that medium, of small refracting angle, to the mean deviation produced by the same prism, when a beam of white light is refracted through it when in the position of minimum deviation.*

Thus, the dispersion is measured by the angle, DOF (Fig. 129), as explained above (Art. 5), and the *mean* deviation by the angle, A′O′H. Therefore, in accordance with the definition,

$$\text{Dispersive power} = \frac{\text{Angle DOF}}{\text{Angle A'O'H}}.$$

Now, the dispersion, represented by the angle, DOF, is given by

$$(\mu_v - \mu_r)\,\text{A},$$

where μ_v, μ_r are the refractive indices of the medium for the extreme rays, and A is the angle of the prism. Similarly, if μ denote the refractive index for the mean ray, ABGH, of the refracted pencil, then the mean deviation is given by $\text{D} = (\mu - 1)\,\text{A}$;

$$\therefore\ \text{Dispersive power} = \frac{(\mu_v - \mu_r)\,\text{A}}{(\mu - 1)\,\text{A}} = \frac{\mu_v - \mu_r}{\mu - 1}.$$

Dispersive power is usually denoted by the symbol, ω, so that the above relation is generally written

$$\omega = \frac{\mu_v - \mu_r}{\mu - 1}.$$

This is regarded as the strict definition of ω, the written definition given above being only approximate.

Experiment shows that in general the dispersive powers of different media are different. Water has little dispersive power, crown glass has about the same, flint glass has about twice that of crown glass, and carbon disulphide has still more. For this reason, hollow prisms filled with carbon disulphide are usually employed in spectrum work for lecture illustration. (See Table II., page 353.)

7. Achromatic Prisms

To realise the full significance of dispersion, the following experiments should be considered:—

(1) Let two exactly similar prisms, P and P′ (Fig. 130), of the *same material*, be placed as shown with their refracting edges turned

in opposite directions and their adjacent faces parallel. Let a beam of solar light, AB, be incident on P. On refraction through P, this beam is dispersed and the refracted pencil, CD, after emergence from P, is incident on P′. Now P′, being exactly similar to P, but having its refracting edge turned in the opposite direction, will produce an equal and opposite effect to that produced by P. The pencil, EF, after emergence from P′, will be parallel to AB, and the dispersion produced by P will be destroyed by P′. Thus, the beam, EF, will be exactly similar to AB in all respects. In fact, the action of the combined prisms is exactly the same as a parallel *plate* of the same medium (see page 63).

This experiment, which may be employed to illustrate the recomposition of white light (see page 168), shows that whenever, by the actions of two similar prisms of the *same material,* dispersion is destroyed, the deviation is also destroyed. *Newton,* after some research in this direction, came to the conclusion that this result was true generally, whether the prisms were of the same material or not, and that it was impossible to obtain deviation without dispersion, or dispersion without deviation. This conclusion is now known to be wrong.

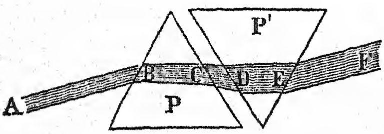


FIG. 130.

For, let d and D denote respectively the extreme dispersion and mean deviation produced by P, and d' and D′ the dispersion and deviation produced by P′. Then, if, on refraction through the combined prisms, the dispersion and deviation are simultaneously destroyed,

$$d = d', \text{ and } D = D'. \quad \text{Hence, } \frac{d}{D} = \frac{d'}{D'}, \text{ or } \omega = \omega.$$

In other words, if Newton's conclusion were true generally, it would mean that all media have the same dispersive power. Experiment has shown that this is not the case (Art. 6).

(2) Let the prisms, C, F (Fig. 131, (i) and (ii)), made of media of different dispersive powers, but of equal refracting angle, α, yield spectra, rv, r_1v_1, and let D, D_1 be the middle points of these spectra. If F has the greater dispersive power, rv is not as long nor as deviated as r_1v_1.

It is possible now to cut down the refracting angle of F until its spectrum is of the same length as that of C. Let F_1 [Fig. 131 (iii)]

be the new prism producing the spectrum, r_2v_2, equal in length to rv. Let O_2D_2 be now less than OD. Then, on combining C and F_1 [Fig. 131 (iv)], a combination is obtained *which deviates, but does not disperse.* The deviation is given by

$$O_3D_3 = OD - O_2D_2.$$

This fact was discovered by *Hall* in 1730, and first used by *Dollond,* a London optician, in 1757.

Example.—Suppose that C and F are made of crown and flint glass respectively, and that the angle, a, is small.

The dispersion produced by
$C = (\mu_H - \mu_A)\ a = 0{\cdot}023 a$
(Table II., page 353).

If β is the value of the angle of F_1, the dispersion it produces $= 0{\cdot}044\beta$.

Since these dispersions are equal,

$$\beta = \frac{0{\cdot}023}{0{\cdot}044}a = 0{\cdot}52 a.$$

Therefore, as combined in Fig. 131 (iv), the deviation produced is:—

$$= (\mu_D - 1)\ a - (\mu'_D - 1)\ \beta$$
$$= 0{\cdot}534a - 0{\cdot}619\beta$$
$$= 0{\cdot}534a - 0{\cdot}322a = 0{\cdot}21 a.$$

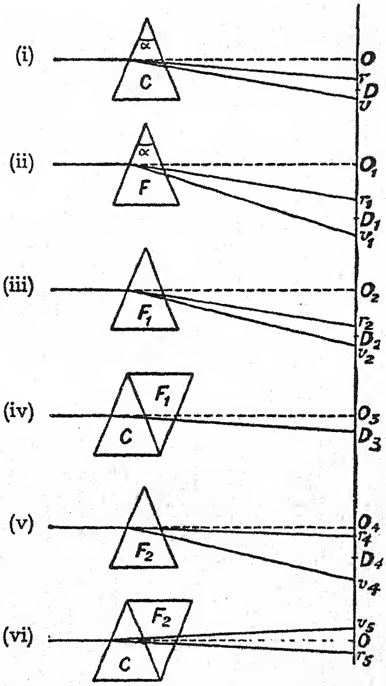


FIG. 131.

(3) Instead of cutting down the angle of F as described above, it may be cut down until the mean deviation which it produces is equal to that produced by C. This gives the prism, F_2 [Fig. 131 (v)], where $O_4D_4 = OD$. The spectrum, r_4v_4, is still greater than rv, and hence, on combining F_2 with C [Fig. 131 (vi)], a combination is obtained *which disperses but does not deviate.* The dispersion is given by $r_5v_5 = r_4v_4 - rv$. In other words, the beam of light, after emergence from the combined prisms, will continue approximately in its original direction, but the beam itself will be dispersed, and if allowed to fall on a screen, will show a spectrum. This is the principle of the *direct vision*

spectroscope (see page 236). With crown and flint glass prisms of small angle, the angle of F_2 would have to be about $\frac{9}{10}$ that of C.

Example.—Using the data supplied in the previous example above, if γ be the angle of F_2, and since the deviations are equal,

$$(\mu_D - 1)\,\alpha = (\mu'_D - 1)\,\gamma,$$
$$\text{or } 0{\cdot}534\alpha = 0{\cdot}619\gamma.$$
$$\text{Thus, } \gamma = \frac{0{\cdot}534}{0{\cdot}619}\alpha = 0{\cdot}86\alpha.$$

Hence, as combined [Fig. 131 (vi)], the dispersion produced

$$= 0{\cdot}023\alpha - 0{\cdot}044\gamma = 0{\cdot}023\alpha - 0{\cdot}038\alpha$$
$$= -\,0{\cdot}015\alpha.$$

The negative sign shows that the dispersion of the combination is in an opposite direction to that produced by C.

8. Irrationality of Dispersion

If the dispersive powers of two materials be calculated for several pairs of selected rays, it will be found that the ratio of the dispersive powers varies with the rays selected. This **irrationality of dispersion** is sometimes very apparent, some media compressing the red portion of the spectrum and extending the violet portion, others doing the reverse, while a few others give spectra whose colours are not in the usual order. This last phenomenon is called *anomalous dispersion.*

Hence, in general, if the spectra produced by two prisms be so arranged that the extreme rays are equally distant from each other, the intermediate rays of one spectrum will not correspond exactly in position with the intermediate rays of the other. Hence, also, when the dispersion is destroyed in a prism or lens combination for a given pair of rays, there is still left a residual dispersion of some colours. The coloured images which are formed by these residual rays are known as *secondary spectra.*

9. Dispersion in a Lens

When a pencil of compound light is refracted through a lens, it undergoes dispersion just as in refraction through a prism. Thus, if a diverging pencil of solar light, P*ab* (Fig. 132), be incident on the convex lens, L, then the red rays, being the least refrangible, are brought to a focus at R, while the violet rays converge to a focus, V, nearer the lens. The orange, yellow, green, and blue rays converge to points intermediate between R and V, and thus, instead of the refracted rays all meeting in one focus, the rays of

each colour converge to their own focus, and the image formed on a screen placed anywhere near V or R will be coloured at its edges. If the screen be placed anywhere between the line, *vr*, and the lens, then the outer edges of the image will be red, but if placed beyond *vr*, the outer edges will be violet.

This fact is taken advantage of in focusing an image on a screen. The points, V and R, are very close together, and the best *definition* of the image is obtained when the screen is at *vr*. This adjustment is made readily by gradually changing the position of the screen until the colour showing at the outer edges of the image changes from red to violet. When this change of colour takes place, the screen is in the position indicated by the line, *vr*.

This effect of the dispersion of light, when refracted through a lens, is called **chromatic aberration,** and was a great source of trouble in the construction of optical instruments, until it was shown that it was possible to obtain deviation without dispersion

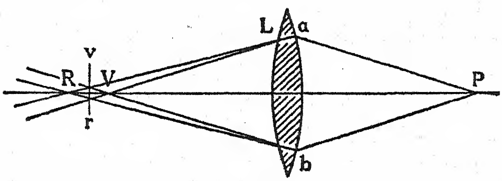


FIG. 132.

—that is, that it was possible to make the rays converge to a focus without obtaining a coloured image. This result, as in the case of prisms, is achieved by combining two lenses of materials of different dispersive power, and such that the chromatic aberrations which each produces singly are equal and opposite. Thus, for example, if a convex lens of crown glass, of 20 cm. focal length, is combined with a concave lens of flint glass, of 34 cm. focal length, the combination is equivalent to a convex lens of about 49 cm. focal length, and the image produced by it is almost entirely free from all colour defects. Such a combination is said to be an **achromatic combination.** The problem will be dealt with fully later (see page 219).

10. The Prismatic Spectrum

The **prismatic spectrum** is that which is obtained by the *decomposition of white light* on refraction through a prism.

All radiant waves are capable of refraction and dispersion, and thus, when a beam of white light is refracted through a prism, the

emergent beam is made up of a series of rays, separated and arranged in order of *continuously* increasing refrangibility.

Beginning at the least refrangible portion of the spectrum determined by this emergent beam, and proceeding in the direction of increasing refrangibility, a group of rays, known as the **dark heat rays**, exists before coming to the red. These rays *do not excite the sensation of sight.*

Then another group is reached, the rays of the **visible spectrum** ranging through the colours red, orange, yellow, green, blue, and violet. This group of rays, in addition to possessing heating properties, has the peculiar property of exciting the optic nerve (see page 186), and thus producing the sensation of sight. In this visible spectrum the intensity of the light is different in different parts, being a maximum in the yellow and gradually diminishing on both sides towards the red and the violet, as shown by the ordinates of the light intensity curve (Fig. 133).

Beyond the visible spectrum is found another group of rays, the *dark chemical rays* or **actinic rays**, which again do not excite the

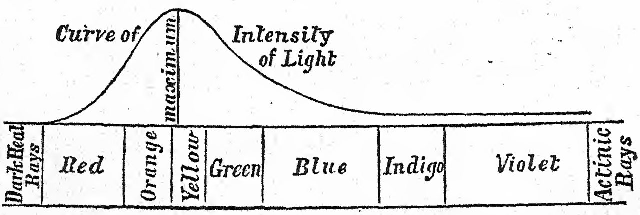


FIG. 133.

sensation of sight. These rays extend considerably beyond the violet, and are characterised by their power of producing chemical action in a certain class of substances.

The complete spectrum may thus, at first, be regarded as made up of *dark heat rays*, *light* or *luminous rays*, and *actinic rays*. The only essential difference between these different groups of rays is that of wave-length, which *decreases* continuously from the first to the last. The vibration frequency, of course, *increases* from the first to the last.

11. The Solar Spectrum

If the slit employed in the production of a pure spectrum (Art. 4) be illuminated by sunlight, a bright and apparently continuous spectrum will be thrown upon the screen, but with a

sufficiently narrow slit it will be found to be crossed by a number of dark lines, some well defined and easily seen, others extremely thin and only visible after very careful focusing. The light is probably not absolutely wanting in these *dark* lines, but is so faint as to appear dark by contrast.

These lines were first observed by *Wollaston*, but *Fraunhofer* in 1804 was the first to map their positions accurately in the spectrum, and hence they are usually called **Fraunhofer lines.** In position, and in distinctness also, these lines correspond in almost every case to *bright* lines found in the spectra obtained from one or other of the elementary bodies.

To understand fully the formation of these dark lines, the following experiment should be considered:—An intensely white-hot substance is obtained and its spectrum produced on a screen. Between it and the prism is now placed a Bunsen burner into the flame of which a piece of common salt is inserted. The flame by itself emits light of a greenish-yellow colour whose wave-length has been found to be approximately 5893 tenth-metres, or 0·00005893 cm. When, however, both sources are in action, the continuous spectrum due to the white-hot substance is found to be crossed in the yellow by a well-defined dark line, in a position corresponding to the same wave-length. The inference is that the amount of light proceeding from the flame is relatively so small that it may be neglected, and the only effect to be considered is that which the presence of the vapour in the flame may have upon the rays proceeding from the white-hot body.

It follows from thermodynamic considerations that, if the white-hot source be hotter than the vapour in the Bunsen flame, the waves proceeding from the white-hot body will pass readily through the vapour, *except those vibrations whose wave-lengths correspond to those of the screen of vapour.* These are in a large measure absorbed or quenched in the screen of vapour, and the dark line results. In the same way, if the vapour contains also lithium, there will be an absorption at wave-length 6705 tenth-metres, and so on for each substance whose vapour is present.

In the case of the sun, the white-hot radiating surface is the body of the sun itself, the **photosphere,** and the absorbing layer is an envelope or atmosphere of the cooler vapours emitted from the body of the sun, the **chromosphere.** *Under such circumstances, the spectrum of white light is interspersed with dark lines corresponding to all the substances so present in this layer of vapour.* This is the explanation, due to *Kirchhoff* in 1859, of the existence of the dark

lines in the solar spectrum, and the coincidence of these lines in position with those given by various terrestrial elements is convincing evidence of the existence in the sun of a large number of the elements identical with those which form part of the earth's crust.

The positions of the most clearly defined Fraunhofer lines have been determined (Fig. 134). There are three well-marked lines, A, B, and C, in the red; one, D, in the orange; another, E, in the green; another, F, in the blue; another, G, on the borderline of the blue and violet; and two, H and K, in the extreme violet. Of these A and B are due to absorption by oxygen in the earth's atmosphere, the remainder being caused by absorption in the chromosphere as follows:—C and F by hydrogen; D by sodium vapour; E, H, and K by calcium vapour; and G by the vapour of iron.

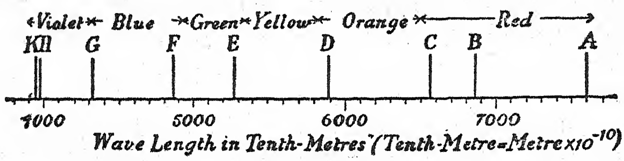


FIG. 134.

12. The Ultra-Violet and Infra-Red

(1) THE ACTINIC OR ULTRA-VIOLET RAYS.—If an ordinary photographic plate be exposed in the prismatic spectrum, it will be found on development that the red light has had little effect upon the plate. It is, however, strongly affected by the blue and violet light, and also to a large extent by the rays beyond the violet, these *ultra-violet* rays being particularly able to decompose the sensitive silver salt, silver bromide, though not able to excite the sensation of vision. Incidentally, it may be mentioned in passing that in ordinary photographic work the inability of the ordinary plate to record colours in their correct luminosities is a serious drawback in all work other than mere black-and-white. Special plates, called orthochromatic, have been manufactured, however, which, in conjunction with a specially prepared coloured screen, reproduce approximately the correct luminosities.

Glass prisms and lenses absorb ultra-violet rays to a great extent, but if quartz prisms and lenses be used, it will be found that the ultra-violet portion of the spectrum of an incandescent body extends to an enormous extent beyond the violet portion of the visible spectrum, and when solar light is employed, it will be found

also to be crossed by dark lines, just as the visible spectrum is crossed by the Fraunhofer lines. Its extent may be measured to some degree by means of a fluorescent body (see Art. 14).

Ultra-violet rays have intense chemical and electrical effects. The ultra-violet constituents of the radiation from an electric arc or spark, falling on a negatively charged zinc plate, cause it to lose its charge rapidly, and the radiation from a mercury vapour lamp, which is very strongly ultra-violet, rapidly charges the air through which it passes with ozone. The rays are used extensively in medicine. It is of interest to note, however, that the decomposition of atmospheric carbon dioxide, which takes place in the leaf cells of plants with the liberation of oxygen, is effected mainly by yellowish-green light.

(2) THE DARK HEAT OR INFRA-RED RAYS.—In 1810 *Herschel* found that, as a small thermometer was moved through the solar spectrum from violet to red, it showed only a small rise of temperature in the blue portion of the spectrum, a little more in the green, a large rise in the red portion, and even for some distance beyond the red portion the thermometer was very sensibly heated. Since then, *Langley* has done much work on the infra-red portion of the spectrum. As glass absorbs these dark heat rays, prisms and lenses made of rock-salt or fluorspar were used, and the thermometer was replaced by a lamp-blacked linear thermopile (see *Advanced Textbook of Electricity and Magnetism*, Hutchinson), or a bolometer. Lamp-black absorbs all the radiation which falls upon it, and hence the energy measured at any point in the spectrum is the *total* energy sent to that portion.

If solar light be used, the position of maximum energy is found within the visible spectrum, but if the electric arc or an incandescent lamp be used the maximum is found some distance in the infra-red, the distance being greater the lower the temperature of the source. An exception to this occurs in the case of the spectrum of electric sparks produced between metal electrodes, when the position of maximum energy is in the ultra-violet. The results of Langley's work are shown (Fig. 135), the energy spectrum of solar radiation being given by the irregular curve and that of the radiation emitted from an electric arc by the smooth curve. The extent of the visible spectrum is shown, VBYR. The large depressions in the solar curve are due to absorption by the atmospheres of the sun and earth; if no absorption had occurred, the dotted line would probably have represented the solar radiation.

Experiment. Pass an electric current of gradually increasing strength through a platinum wire in a dark room. The wire becomes perceptibly warm to the touch very quickly, then too hot for the hand to bear, and soon so hot that the heat radiated from it may be felt at a distance of several inches. It is still invisible, however; all the rays emitted are obscure rays.

After a time, as the temperature increases, the wire becomes faintly visible, first as a peculiar flickering grey glow, followed as the temperature rises by an emission of the extreme red rays of the spectrum. With further rise of temperature, the yellow and orange rays are added to the red and obscure rays, then follow the green, blue, and violet rays. The wire being then white hot and very near the melting point of platinum, the experiment must come to an end.

This, considered together with the experiment with the thermometer in the various spectrum rays, shows that the varying

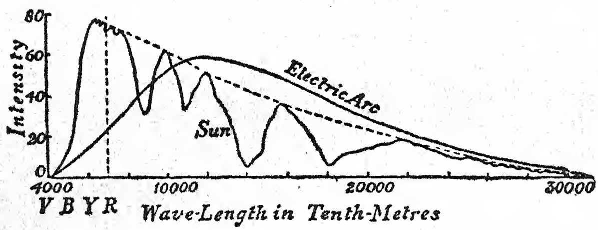


FIG. 135.

thermal and luminous effects depend upon the frequency or wave-length of the emitted rays, and that the greatest heating effects are due to waves of low frequency and comparatively large wave-length.

It is thus possible to estimate the temperature of an incandescent body, the light of which is due to heat, by carefully noting its colour. As a body is heated, the first colour, a dull red, appears at 525° C. This turns to cherry colour at 800° C., and to a bright cherry at 1000° C. Bright orange appears at 1200° C., white at 1300° C., and dazzling white at 1500° C. and above. Intermediate temperatures may be observed by means of an instrument based on this scheme, the Optical Pyrometer (see *Textbook of Heat*, Archer).

13. Transmission and Absorption of Radiation

Experiments show that, when radiation of a definite kind is transmitted through a substance, the amount transmitted decreases

in geometrical progression as the thickness increases in arithmetical progression—that is, each layer of the substance, of a given thickness, transmits the same proportion of the radiation which enters it. Thus, if I_0 represents the quantity of radiation entering the substance, the amount which emerges after traversing unit thickness is given by $I_0 a$, where a is a constant. Similarly, on transmission through another layer of unit thickness, it is reduced to $I_0 a^2$, and therefore, after transmission through a layer of thickness, n, the quantity of transmitted radiation is given by

$$I = I_0 a^n.$$

The constant, a, has been called the **coefficient of transmission.** It is independent of the intensity of the incident beam and depends only on the nature of the substance and the wave-length of the radiation employed. When radiation of a compound nature is transmitted by any substance, its various constituents are absorbed in different degrees, and thus the nature of the transmitted radiation is subject to continuous change. The character of this change, however, is such that the nature of the transmitted radiation tends to become constant and capable of further transmission without absorption. For this reason, radiation which has passed through a thick plate of any substance passes readily with little loss through another plate of the same substance.

Hence, absorption is the prime factor in the production of colour. If white light falls upon a plate which absorbs unequally the rays of different wave-lengths, the emergent light will be coloured. For considerable thicknesses, the colour remains the same for different thicknesses, the shade becoming darker, but with thin enough layers, the colour gradually changes. Thus, thin plates of cobalt glass transmit chiefly blue light, while thick plates transmit a preponderance of red.

The light reflected from a body is also very often coloured. This is due to the reflection not being wholly superficial. A portion of the incident light penetrates the body for some distance, undergoes internal reflection, and returns to the front surface, from which it emerges in line with the reflected light. This portion has undergone absorption, and hence is usually coloured.

Experiment. Admit a beam of white light into a dark room. Reflect it by a coloured surface and catch the reflected beam on a white screen. Observe that the screen now appears of the same colour as the reflecting surface. Note also the reflections of coloured shop-signs.

A large crystal of copper sulphate is transparent and deep blue, because it absorbs all but the blue components of white light before the light has travelled very far through it. However, when the crystal is crushed to a fine powder, not only does it become opaque, but its deep colour is reduced to a pale blue. The light cannot then penetrate far enough into the crushed mass for any great absorption to take place. So it is with other coloured crystals, which when crushed assume a much paler tint.

14. Fluorescence

If a test-tube containing a solution of quinine sulphate be moved through a spectrum which is cast on a screen, it will be observed that in the red, yellow, and green it appears red, yellow, and green respectively. In the blue and violet, however, a change appears, the solution glowing with a bluish light, and this glow exists even when the test-tube is some distance in the ultra-violet. This phenomenon is called **fluorescence**, the name being derived from *fluorspar*, the natural occurring form of calcium fluoride. If the same solution be examined in sunlight, it will be found to exhibit this fluorescence at its edges. The phenomenon was investigated by *Stokes*, who showed that the alteration was caused by the quinine sulphate absorbing light energy of one wave-length and emitting a part of it as light of longer wave-length. The quinine sulphate absorbs ultra-violet light and renders it violet. Similarly, chlorophyll will appear red in the blue part of the spectrum, and uranium glass, yellow in the green portion.

The fluorescence is confined invariably to the surface layers, the reason being that all the light which the substance is able to attack is disposed of in the region near the surface, and that which passes on is therefore rendered inactive.

If a spectrum be thrown on a screen painted with a solution of quinine sulphate, the spectrum will be found to be extended in the violet portion, and, if solar light be used, dark bands will be found at various places, just as the Fraunhofer lines are found in the ordinary visible spectrum. Thus the ultra-violet region may be mapped.

Many common substances afford examples of fluorescence. Among them may be especially noted ordinary paraffin oil, eosin or red ink, fluorspar, and an infusion made from fresh horse-chestnut bark. The fluorescence in these cases is blue, red, blue, and blue respectively. The yellow salt, barium platinocyanide, is used largely in X-ray work, since it fluoresces brilliantly in these rays.

Thus, if a dense object be interposed between a point source of X-rays and a prepared screen, a shadow of the object is thrown upon the screen, and, if the object vary in thickness, corresponding portions of the shadow will vary in intensity.

Experiment. Chip some fresh horse-chestnut bark into a beaker of warm water. Note the blue colour of the solution. Take it out into the sunlight, and concentrate light on it by means of a large convex lens. Note the blue shimmer where the cone of light enters the solution.

15. Phosphorescence

In the cases of substances just considered, the fluorescence ceases as soon as the substances are withdrawn from the light. In the case of certain substances, however, notably the sulphides of barium, calcium, and strontium, the emission of light will continue for some hours after the exciting light has been cut off. *Balmain's luminous paint* consists of a mixture of these sulphides, and if a card coated with this substance is exposed to a bright white light, or even ultra-violet light, and then taken into a dark room, it will emit a peculiar violet-coloured light, the rate of output of which is intensified by heating. The phenomenon is known as phosphorescence.

The name phosphorescence is rather misleading, because the glow of slowly oxidising phosphorus is due to a chemical change, while the phenomenon here dealt with is purely physical. Phosphorescence is now believed to be due to imperfections in the crystal lattice, called *electron traps*, which are normally unoccupied, but are possible temporary resting places for electrons. Electrons are given energy by the incident light, enter the traps, leaving them later on, the potential energy being converted back into light.

To study the duration of the period during which substances are luminous, *Becquerel* devised a *phosphoroscope*, which consists of sectors revolving at the ends of a cylindrical darkened chamber. By means of this device, a substance can be exposed to the light; then the light is cut off, and the substance is viewed after any required interval. It was shown that all substances are more or less phosphorescent. More recently, *Dewar* has shown that such bodies as feathers, egg-shells, etc., phosphoresce brilliantly when cooled to the temperature of liquid air.

16. The Emission of Light. Calorescence and Luminescence

(1) The most common method of causing a body to emit light is to raise it to a high temperature. In a flame the high temperature

is due to the result of chemical action, and in an incandescent electric lamp it is caused by the passage of an electric current through a high resistance.

The phenomenon of **calorescence** is a variety of this method of some historical interest. *Tyndall* found that, if the radiation from a hot body is passed through a solution of iodine in carbon disulphide and the waves of large wave-length, which are the only ones to penetrate the solution, are focused upon a piece of thin blackened platinum foil, the latter is heated to redness. The invisible infra-red radiations are thus converted, in part at least, into luminous radiations of much shorter wave-lengths. This was thought to be the converse of fluorescence, and hence the name calorescence. It is obvious, however, that the effect is a true temperature effect, for the infra-red radiations are poured into the foil faster than it can radiate them at low temperatures, and a balance is obtained only when the temperature of the foil has risen so high that it becomes incandescent.

(2) The cases of the production of light other than that by high temperature sources are collected under the general term **luminescence.** The different kinds of luminescence may be summarised:—

(*a*) *Photo-luminescence.* This is caused by the action of light. Fluorescence and phosphorescence come under this head. The emission of light by a Welsbach mantle is supposed to be partly a true heat radiation, and partly phosphorescence.

(*b*) *Tribo-luminescence.* This is due to mechanical effects such as friction, percussion, and cleavage. Simple instances occur when quartz crystals are rubbed together, a lump of sugar is crushed, and mica is cleaved.

(*c*) *Electro-luminescence.* This occurs in a vacuum discharge tube (see page 321). The glow in the body of the tube is probably produced by the impact of negative corpuscles or electrons (see page 327) against the gaseous molecules. At low pressures, the parts of the walls of the tube struck by the corpuscles also glow. In the same way, many naturally occurring crystals glow when exposed to the radiations from radium and other radioactive substances.

(*d*) *Chemi-luminescence.* This is usually due to oxidation, the most noteworthy cases being that of phosphorus, and the decay of animal and vegetable matter.

(*e*) *Thermo-luminescence.* This occurs when certain bodies are warmed slightly, the temperature being far too low to produce a

red heat. The effect is noticeable with diamonds, and with fluorspar, particularly the variety called chlorophane.

(*f*) *Anima-luminescence.* This is observed chiefly with the glow-worm, marine infusoria, and the firefly. The glow is probably due to the action of an oxidising ferment on a chemical continually renewed by the animal.

17. Colours of Bodies

If a piece of red cloth, or a red poppy, be held in the red portion of a spectrum, it appears red. Held in the green or blue portion of the spectrum, it appears black. Again, a green leaf is distinctly green in the green portion of the spectrum, but is black elsewhere. Similarly, a piece of cloth exhibits the colour which it has in sunlight only in that part of the spectrum which is coloured like itself. These simple experiments show that a body which is red in daylight is able to reflect red rays only. As it appears black in the green or blue light of the spectrum, it reflects no green or blue rays. The same reasoning applies to other colours; a green surface reflects only green, blue only blue, and so on.

With the help of Art. 12, it is now possible to understand the meaning of the colours which bodies are seen to exhibit in white light. White light is made up of many colours. When it falls upon a red surface, only that part of it which is red is reflected. The other spectrum colours are absorbed by the surface. When a body appears yellow, it is to be understood that all the colours except yellow are absorbed.

So far it has been assumed that the colours of bodies are simple, and not made up of a mixture of two or more different colours. It is difficult to obtain a pure green, or a pure yellow or blue, and hence it often happens that when a coloured cloth is held in other parts of the spectrum than that which matches its colour, it appears coloured. A piece of green baize appears bluish in a blue light and yellowish in a yellow light, because most greens contain some blue and yellow in their composition.

If a piece of cloth of different tints be looked at in a light which is deficient in one or more of the colours of the spectrum, or in a light in which one of the colours predominates, it does not always *look* the same as when viewed in daylight. If the light lacks one of the tints which the cloth exhibits in daylight, then that particular tint cannot be reflected by the cloth. In gas light yellows are brightest, because a gas-flame is chiefly a yellow light. But, since a white body seen by yellow light appears yellow, the difference

between white and yellow by gas light is much less distinct than by the white light of day.

There is now no difficulty in explaining the colours of transparent plates. A plate of red glass allows only red rays to pass through, a green plate transmits the green rays, and so on. A blue plate does not allow red rays to pass through it. Hence, if a beam of sunlight be made to fall upon a piece of red glass, and if a piece of blue glass be held in the course of the red light, it follows that, as none of the red light can pass through the blue glass, the two plates together cut off all the light, and the source of light, if it is visible at all, appears black. This is found to be very nearly the case.

Experiment. Look at a gas flame through two plates of different colours, say blue and red, red and green, yellow and blue. The flame is invisible, or nearly so, if the tints are sufficiently deep. It is difficult to obtain glasses of pure colour, so that ordinary red glass allows other rays than red to pass through. Some of these may be able to pass through the second plate of glass.

Observe that a white cloth appears red in red light, because it reflects red, blue, green, or any other colour. In fact, it appears white in daylight because it reflects all the colours.

18. Primary and Complementary Colours

A *primary* colour is defined as one which cannot be imitated to the eye by the mixture of any other colours. *Maxwell* showed that there are three primary colours—red, green, and violet—and that any other colour can be produced by mixing suitable proportions of these, which may be done thus:—A spectrum is thrown upon a screen which is provided with adjustable slits. The slits can be opened to varying extents and also shifted to occupy different places in the spectrum. In this way beams of different colours are allowed to pass through, and these can be combined by a judicious arrangement of mirrors and lenses. Another method is afforded by Newton's disc (Art. 20). Any two colours which produce the sensation of white when they are mixed together are said to be *complementary*. If, in the arrangement described above, a slit is opened in the yellow, and another slit is moved up and down the spectrum, it will be found that when the second slit is in the blue, the mixture of the two beams produces white. Thus, yellow and blue are complementary; so also are red and greenish-blue, and green and purple. If, however, yellow and blue *pigments* are mixed, the result is not white, but green. Pigments are far from spectrally pure

colours and reflect light over a wide range of wave-lengths. Thus yellow pigment usually reflects red, yellow and green light and *absorbs* the blue end of the spectrum, while blue pigment usually reflects blue and green light and *absorbs* the red end. A mixture of the two pigments thus absorbs practically the whole spectrum except the green, and accordingly appears green. For this reason one can have "additive" mixing, *e.g.* Newton's disc, and "subtractive," *e.g.* by mixing pigments or by superposing colour-filters.

Experiment. Throw a continuous spectrum on a screen. Cut away part of the screen, so that all the colours except red pass through. By means of a reversed prism [see Art. 7, (1)] and a convex lens, combine all the colours which pass through the opening. Then the combined colours form a colour—greenish-blue—which is exactly complementary to the red.

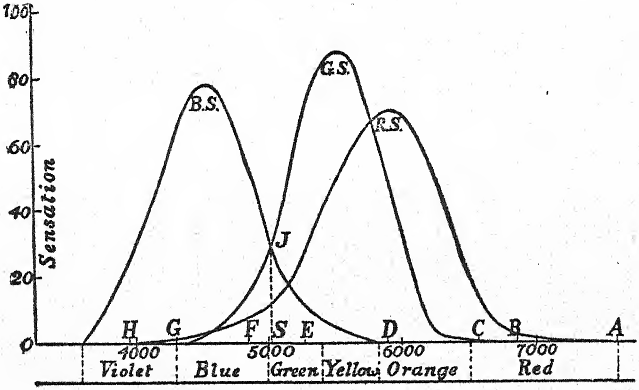


FIG. 136.

19. Colour Vision. Colour Blindness

There is no theory of colour vision which can be regarded as completely satisfactory. The commonly accepted theory is that due to *Young* and *Helmholtz*, which postulates that, just as there are three primary colours, so there are three sets of nerves in the eye by which colours are appreciated. The three colours to which these nerves respond are red, green, and blue, and the colour of a body simply depends upon the proportion in which the three sets of nerves are excited. *Abney* experimented with the primary colours and obtained colour equations which gave the curves shown (Fig. 136). These curves show that each region of

the spectrum exercises all the sensations to varying degrees, and if a number of points in the spectrum are found such that the sum of the red-sensation ordinates is equal to the sum of the green-sensation ordinates, and also equal to the sum of the blue-sensation ordinates, the combination of the colours at these points would result in white light.

If one set of nerves is absent or inactive in a person, the person is said to be *colour-blind*. Suppose, for example, that a man is red-blind—this is the most common form of colour blindness—then he has no sensation corresponding to the curve, RS (Fig. 136), and therefore the red constituent of all colours is unappreciated. To such an observer, the point, S, in the spectrum appears white, for the ordinate, JS, is the same both for blue and green sensations, and, according to such an observer, the spectrum is composed of two colours, yellow and blue, and other colours which are more or less shades of these. Red to this observer is only dark yellow, and very often the spectrum is shortened considerably in the red portion. Green is rather muddy yellow, and the middle of the spectrum, near S, is white or grey, while the violet is preferably called dark blue.

Of course, colour-blind persons may learn by experience to give many colours their proper names, but in some occupations, such as engine-driving and signalling, colour-blindness is such a great defect that candidates for such posts are weeded out carefully by tests both with the spectrum and with coloured skeins of wool.

20. Recomposition of White Light

The experiments on the production of the spectrum described above (Art. 4) were analytical. The composition of white light has been determined by decomposing it into its constituent coloured rays. There are many ways, however, by which the process can be reversed—that is, starting with the separate colours, they may be recombined into a beam of white light, thus effecting a synthesis. This recombination may be effected by the following methods:—

(1) By the use of a second prism exactly like the first, but with its refracting edge turned in the opposite direction (see Fig. 130).

(2) By receiving the spectrum on a line of plane mirrors so that a separate colour falls on each, and then inclining the mirrors so that all the coloured rays are reflected to the same spot on a screen.

(3) By interposing in the course of the spectrum rays an achromatic lens. A cylindrical lens (see page 201), with its

geometrical axis parallel to the slit used in producing the spectrum, gives the best results.

(4) By *Newton's* disc. This is a circular disc of cardboard, divided into sectors painted with the colours of the spectrum, and attached to a whirling table. When the disc is rotated rapidly, it appears nearly white, not because there is any real mixture of colours, but because of the fact that luminous impressions on the retina of the eye persist only for a small fraction, about $\frac{1}{10}$, of a second, so that, before the impression due to any one colour has died away, it is succeeded by all the other colours. Thus, there is a physiological though not a physical blending of the colours.

(5) By blending complementary colours (Art. 18).

21. The Rainbow

When the sun is shining upon a rain cloud or upon the spray from a waterfall or fountain, an observer standing with his back to the sun, and facing the rain or spray, often sees a circular arc of colour, apparently in the midst of the water drops. Such an arc is called a rainbow, and is due to the reflection and refraction of the light falling upon the drops of water.

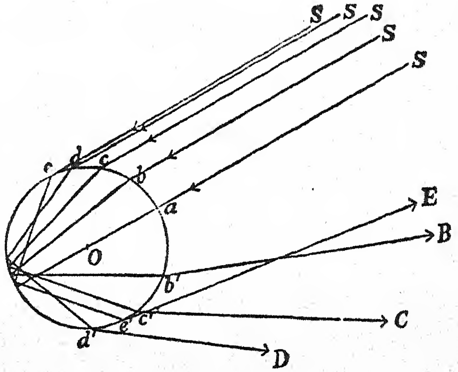


FIG. 137.

Let us consider what happens when a parallel beam of light, SS (Fig. 137), coming from a distant point falls upon a spherical drop of water whose centre is O. The ray, S*a*, incident along the normal will be reflected back along its former path. The rays, S*b*, S*c*, S*d*, S*e*, . . . will undergo refraction on entering the drop, total internal reflection at the rear surface, and refraction on leaving, and will emerge in varying directions, *b*'B, *c*'C, *d*'D, *e*'E, . . . The deviation of the ray, S*a*, is equal to 180°. Considering the other rays, it can be shown by accurate construction, or by mathematics, that the deviation of the rays decreases in passing through *abcde* until a minimum is reached; after that the deviation again increases.

Now, the deviations of the rays on each side of the ray of minimum deviation are approximately equal to the minimum

deviation so that an approximately parallel beam emerges in the direction, $c'C$, $d'D$, and if an observer is situated along these directions, he will receive a large quantity of light.

Let $SdRd'D$ (Fig. 138) be the ray undergoing minimum deviation, ϕ. Then,

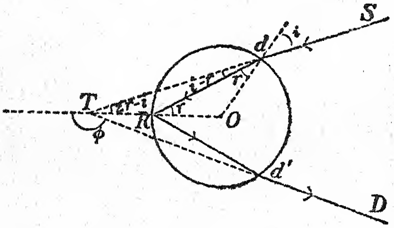


FIG. 138.

$$\phi = 180^\circ - 2 \text{ angle } RTd$$
$$= 180^\circ - 2\,(2r - i);$$
$$\therefore \phi = 180^\circ + 2i - 4r.$$

Plot a graph between ϕ and i. The curve obtained is convex towards the axis of i, the minimum value of ϕ being about 138°.

All other drops which yield this minimum deviation will lie on a circle, MOR (Fig. 139), which forms the cross-section of a cone whose semivertical angle is θ, where $\theta = 180^\circ - \phi$. Thus, a circular arc of light will be seen, the centre, S′, of which is the point in the heavens exactly opposite to the sun. A complete semicircle only is seen therefore by an observer on the earth when the sun is on the horizon, but from a balloon a complete circle may often be observed.

So far, only monochromatic light has been considered. Since, however, refrangibility depends on wave-length, the angles of

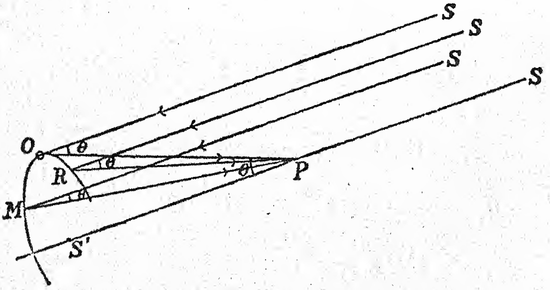


FIG. 139.

minimum deviation will be different for different colours, so that a spectrum-coloured circular arc is seen. Violet light is deviated more than red, so that $\phi_v > \phi_r$, and hence $\theta_r > \theta_v$, and the radius of the red arc is greater than that of the violet. Calculation gives

the values of θ_r and θ_v as 43° and 41° respectively, which are substantiated by actual measurement.

Light may also reach the eye after two reflections inside the drop (Fig. 140). The deviation in this case is greater than 120°, and, as before, a position of minimum deviation occurs, making the light a maximum along the direction which it determines. This yields a bow concentric with the primary bow, known as a secondary bow. The violet deviation being greater than the red, as before, the angular radius of the violet arc is greater than that of the red, the numerical values being 54° and 51° respectively. The space between the two bows is darker than the spaces within the primary bow and beyond the secondary bow.

In the same way as described above, bows arising from three, four, five, etc., internal reflections within the drop may be formed, but they are very faint and are rarely seen. Since the sun is not a point source of light, the rainbow is not a pure spectrum, and if it should happen that the sun is shining through a thin cloud, which has the effect of increasing the diameter of the source of light, the rainbow is nearly white.

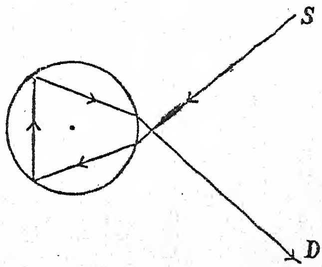


FIG. 140.

Experiment. By means of a tank and a piece of glass tubing of about 1 mm. bore, obtain a vertical downward flowing smooth jet of water. Place a source of light, a candle-flame will do, about 4 ft. from the jet, and place the eye close to the jet with the back of the head towards the light, taking care not to obstruct the light. The primary and secondary rainbows will be seen, together with a large number of spurious bows. By means of pins and paper, map their directions, and confirm the values of the angular radii quoted above.

22. Lunar Rainbow

Lunar rainbows are also seen occasionally. Owing to their faintness, however, they usually appear destitute of colour.

The large circular rings, or **halos**, which are sometimes seen around the moon, and more rarely around the sun, are due to refraction through tiny hexagonal ice crystals. The ice crystals being in the shape of hexagonal prisms, and the refractive index of

ice being 1·31, light which enters at one prismatic face cannot emerge at the next, but it may emerge at the next but one, and, of course, at the face opposite and parallel to it. Considering two alternate faces, it is evident that the crystal will act like a 60° prism, and, applying the relation,

$$\mu = \frac{\sin \frac{1}{2}(D + A)}{\sin \frac{1}{2}A},$$

for light passing through at minimum deviation, D is found to be 22°. If, therefore, a thin cloud of such crystals exists, the axes of many of the crystals being perpendicular to the line joining the position of an observer to the moon, the observer will receive scarcely any light refracted through these prisms, except at a point about 22° from the moon. Hence, a bright circle of 22° radius will be seen. The theory would make the circle of light coloured as in the case of the rainbow, but as a rule the only colour seen is a red tinge on the inside of the circle.

Experiment. Measure the angular diameter of a lunar halo.—When a good halo around the moon appears, take a foot-rule and a set-square of about 6 in. side. Place one end of the scale close to the eye, and sight the edge straight at the moon. Place the set-square so that one of the sides enclosing the right angle may slide along the edge of the scale, and move it backwards and forwards until the angular point not in contact with the scale just reaches out to the halo. Make sure that the scale is still pointing straight at the moon, and then take the reading of the point of the set-square at the right angle on the scale. If this reading is y, and the length of the side of the set-square standing out from the scale is x, the angular radius of the halo is given by $\tan^{-1}\frac{x}{y}$, and the angular diameter is $2\tan^{-1}\frac{x}{y}$. If x is about 5 in., y will be about 12 in.

The smaller brightly coloured circles, or coronas, seen close round the moon, especially when it is full, are due to diffraction effects (see page 295) produced by very small water drops in high clouds.

23. The Scattering of Light

It can be proved experimentally in the case of sound waves, and mathematically for all wave motion, that the smaller the wave-length of any radiation the more perfectly will an obstacle

stop the waves and scatter them. Red light has a wave-length double that of violet light. Hence, when white light is travelling through a medium containing very fine particles in suspension, the scattered light would be expected to have a blue tint, and the transmitted light a red tint. This is borne out by the facts that street lamps look very red in a fog, and that the setting sun appears red as it approaches the horizon. Also, the smoke from a burning cigarette or wood fire, a distant haze, and a reservoir of water containing very fine particles in suspension look blue.

The colour of the sky may be explained in the same way, scattering in this case being due to either fine salt particles, fine metallic dust from meteorites, or the gaseous molecules themselves.

The brilliant blue colours of some insects are due to scattering of light by very small opaque particles, and not to the presence of a true pigment.

Experiment. A pretty experiment to illustrate the colour of the setting sun can be performed by illuminating a screen by means of a parallel beam of white light from an optical lantern, and then inserting in the path of the beam a glass cell containing a freshly made dilute solution of sodium thiosulphate, or "hypo," to which a little dilute hydrochloric acid has been added. Precipitation of sulphur occurs gradually, and the image on the screen gradually changes colour first to orange, then to red, and finally is obscured, while the blue light is scattered out of the beam, and can be seen if the cell is looked at from the side. The path of light in the cell will be seen to be a bright blue, very like that in the sky.

CHAPTER X

SIMPLE OPTICAL INSTRUMENTS

IN this chapter, the various optical instruments of the simpler type in every day use will be described, and an explanation of the principles involved in the use of each instrument will be given. It will be seen that mirrors, plane and spherical, prisms, and lenses, are used in instruments designed for different purposes, and each instrument will be found to utilise one or more of the phenomena which have been dealt with in the preceding chapters.

The human eye, which may be described as *nature's optical instrument,* together with vision and its more common defects, will also be dealt with.

1. Artificial Horizon

The altitude of a star is the angle between the direction of the star and its horizontal projection, and it is frequently determined by a method based on the laws of reflection (see page 17). The accuracy of the results obtained by this method furnishes an indirect but rigorous proof of the truth of these laws.

A divided circle, adjusted in a vertical plane, carries a telescope, TT (Fig. 141), which can be rotated about an horizontal axis passing through the centre of the circle. In making an observation, the telescope is first pointed to a particular star, and the reading on the circular scale for this position of the telescope is noted. The telescope is then turned into the position, T′T′, so as to view the image of the star formed by reflection in the horizontal surface of mercury, *the artificial horizon,* in the vessel, M. The reading on the scale corresponding to this position is also noted, and the difference between the two readings—that is, the angle, S′P′R—gives twice the altitude of the star. Thus, assuming the laws of reflection to be true:—

$$\text{Angle SPN} = \text{angle NPR}; \quad \therefore \text{angle SPH} = \text{angle RPH}.$$

$$\text{But, angle RP}'\text{H}'' = \text{angle RPH}';$$

$$\therefore \text{angle RP}'\text{H}'' = \text{angle SPH}.$$

Since SP and S′P′ are parallel,

$$\text{Angle S}'\text{P}'\text{H}'' = \text{angle SPH};$$

$$\therefore \text{ angle } S'P'R = \text{angle } S'P'H'' + \text{angle } H''P'R = 2 \text{ angle } SPH.$$

$$\text{But, angle } SPH = \text{altitude of star;}$$

$$\therefore \text{ angle } S'P'R = \text{twice altitude of star.}$$

2. Hadley's Sextant

The sextant is an instrument employed for measuring the angle between two distant objects, as seen from the position of the observer. The principle of its action has already been explained (see page 24), and the essential parts of the instrument are shown diagrammatically (Fig. 142).

The frame is made up of the circular arc, SS′, and the two arms, SC and S′C. These two arms, which are radii of the circle of

V.P. Singh

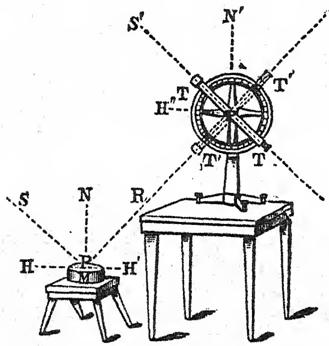


FIG. 141.

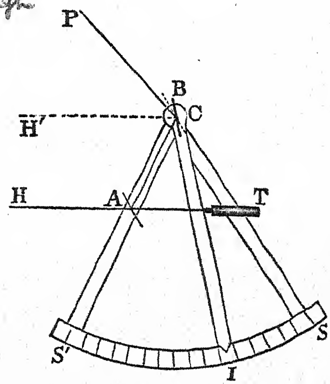


FIG. 142.

which SS′ is an arc, intersect at C, the centre of the circle, and CI is an index arm which can be rotated about an axis passing through C. Two plane mirrors, A and B, are attached to this arrangement, A being fixed on the arm, S′C, and B attached at C on the index arm, CI. Both mirrors are perpendicular to the plane of the diagram. The mirror, A, is unsilvered, or only partially silvered, so that an observer looking through the telescope, T, which is directed towards A, can see objects in the direction, TH. When I is at S, the planes of the mirrors, A and B, are parallel, so that any ray, H′C, incident on B parallel to HT, is reflected along CA to A, and thence along AT to T. The observer looking through T thus sees objects in the direction, TH, or CH′, both directly through A and by successive reflection from B and A respectively.

On moving the index arm, CI, towards S′, other objects, in addition to those seen directly through A, are brought into view, and if, when any particular object in the direction, CP, is brought into view, the arm, CI, has been turned through an angle, θ, then the angle, PCH′, is equal to 2θ (see page 24). In other words, the angle between an object seen in the direction, CH′, and another object in the direction, CP, is equal to twice the angle, SCI.

Hence, in determining the angle between any two given distant objects, the instrument is first adjusted until one of the objects is seen both directly through A, and also by reflections from B and A. The index, I, will then be at the zero of the scale on SS′. The arm, CI, is then moved until the other object, seen by reflections from B and A, appears to coincide with the first object, still seen directly through the unsilvered part of A. The required angle is then obtained by *doubling* the angle, SCI, which is given by the reading of the scale. Usually the scale is graduated on the principle of marking half-degrees as whole degrees, so that the direct reading gives the required angle.

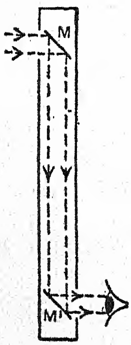


FIG. 143.

3. Heliograph, Heliostat, and Periscope

A **heliograph** is a plane mirror, suitably mounted so that by its means sunlight can be reflected from one station to another several miles away. It is used for the transmission of messages. For this purpose, the mirror is tilted alternately away from, and back to, its correct position according to a given code, and the observer at the distant station notes the duration and regularity of the flashes, from which the message is constructed.

The **heliostat** is simply a heliograph mirror from which by suitable means the reflected beam is sent in the same direction all day long. This is done by mounting the mirror on a frame driven by clockwork, the mirror being moved so that its normal always bisects the angle between the direction of the sun and the direction in which the light is to be sent.

A **periscope** is an instrument by means of which objects can be seen when direct vision is made impossible by an intervening obstacle. The instrument has been developed principally in connexion with submarines to enable observations to be made of objects on or above the surface of the water while the vessel

remains submerged. It is also employed in trench warfare to watch the movements of the enemy while the observer remains in the shelter of the trench.

A simple type of the instrument is illustrated (Fig. 143). It consists essentially of a tube, two or three feet long, having a mirror fixed near each end, and an opening in the wall of the tube opposite each mirror. The two mirrors are parallel, and each is inclined at an angle of 45° to the axis of the tube. Rays of light entering the upper opening are reflected down the tube by the upper mirror, M, and on reaching the lower mirror, M′, are reflected through the lower opening. In use the periscope is held so that the upper mirror is above the obstacle over which it is desired to see. An observer, looking into the lower opening, will see images of the objects opposite the upper opening, the images being formed by the two reflections at M and M′.

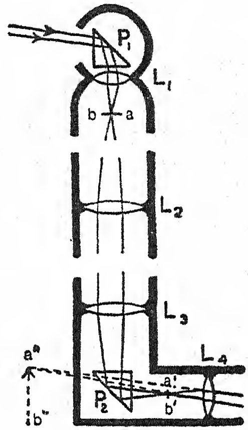


FIG. 144.

The submarine periscope is a more complex instrument than the above, but is similar in principle. It has a tube sufficiently long to reach from the interior of the vessel to twenty feet or more above the deck. Totally reflecting prisms (see Art. 6) are used instead of mirrors (Fig. 144), and these, together with a system of lenses, reflect images of external objects into the object glass of a telescope at the lower end of the tube. The tube can be rotated, and the bearings of external objects read off on a graduated circular scale.

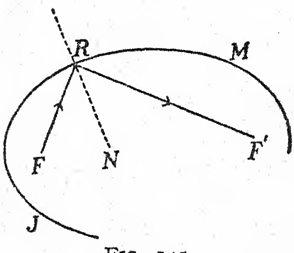


FIG. 145.

4. Ellipsoidal Mirrors

These afford the most accurate method for concentrating, by a single reflection, the light proceeding from one point to another.

If F and F′ (Fig. 145) are the geometrical foci of the ellipse, JRM, and R is any point on it, the angles which FR and F′R make with the normal, RN, at R are equal. This is due to the geometrical properties of the ellipse. Hence, if the luminous source be placed at F, all the reflected

rays will pass through F′. If the ellipse be revolved about the line, FF′, any portion of the surface thus generated will constitute an *ellipsoidal mirror.*

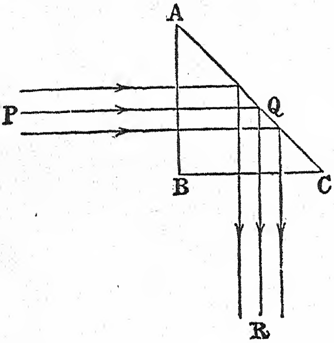


FIG. 146.

5. Cylindrical Mirrors and Lenses

Cylindrical mirrors behave like plane mirrors for dimensions of objects parallel to their lengths, and as concave or convex spherical mirrors for dimensions perpendicular to their axes. They are used largely for shop-window illumination.

Cylindrical lenses act similarly. They are often used in the laboratory to produce images of illuminated slits; they are also used largely by spectacle-makers as a corrective to astigmatism [see Art. 12 (4)].

6. Total Reflection Prisms

If a plane mirror of glass, silvered at the back, is employed to deviate a beam of light through a right angle, confusion is often caused by the succession of images which are formed (see page 79). This confusion can be eliminated by silvering the front surface, but when it is remembered that even the most highly polished silver surface reflects regularly considerably less than the whole of the incident light, and that there is great difficulty in keeping a silver surface in good condition, it can be easily seen that a device which has none of these defects is desirable. Totally reflecting prisms are very frequently used in optical instruments instead of plane mirrors.

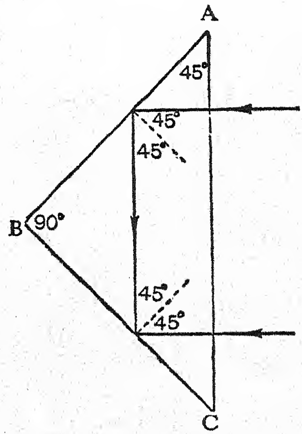


FIG. 147.

Let ABC (Fig. 146) represent a section of a glass prism with angles of 90°, 45°, and 45°. If an incident beam of parallel light, PQ, falls normally on the face, AB, it undergoes no refraction and there is very little loss by reflection. It then meets the hypotenuse, AC, at an angle of

45°, which is greater than the critical angle, 41°, for glass (see page 66). Consequently the beam is *totally* reflected in the direction, QR, and, reaching the face, BC, normally, it emerges without refraction and with very little loss by reflection from the surface, BC. The beam is thus deviated through a right angle with very little loss of light.

Such prisms can be used only to reflect light at an angle of incidence greater than the critical angle for the glass of the prism. Since the rays are intended to enter and leave normally, the prism should be isosceles with its angle, B, equal to the angle of deviation required. If used for other angles, there is more loss of light by reflection at BA and BC. The emergent beam is not identical with the incident beam, but undergoes the same kind of reversal, right and left interchanging, as with reflection from a plane mirror.

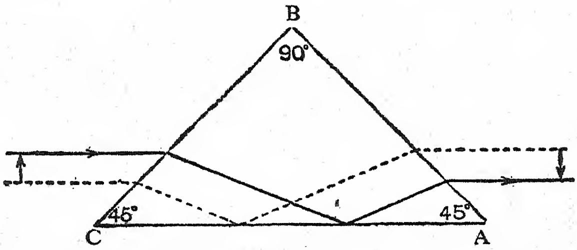


FIG. 148.

If the incident light falls normally on the hypotenuse face, AC (Fig. 147), then it passes into the prism, is reflected at the face, AB, and again at the face, BC, so that the beam is turned through 180°.

The same prism may also be used as an erecting prism, to invert a beam of light. If an inverted image is formed by any apparatus, such as an *optical lantern* (see Art. 9), the image can be made erect by passing the light through the prism as shown (Fig. 148). The light is refracted at the face, BC, and meets the hypotenuse face, AC, at an angle of incidence greater than the critical angle. Total reflection occurs, and the light is then refracted at the face, AB. Thus inversion of the beam takes place, but no deviation occurs.

One example of the use of total reflection prisms has been given above (Art. 3). They are also used largely in lighthouses. In this case, a large compound plano-convex lens is placed exactly in front of the light, and around the lens, arranged in circles, are fixed a number of total reflection prisms of varying angle, such that all the light emerges in a parallel beam. Other important applications

of such prisms in optical instruments will be described later (see pages 206 and 227).

A special form of prism known as Wollaston's prism, sometimes called the *camera lucida*, is a totally reflecting prism with four angles, generally used as an aid to sketching. In this prism, the angle, ABC (Fig. 149), is a right angle, ADC is 135°, and the other two angles each $67\frac{1}{2}$°. Light incident normally on the face, BC, in the direction, PQ, is totally reflected from the face, DC, to the face, DA, whence it is totally reflected along RS normally to the face, AB. To an eye looking in the direction, SR, objects in the direction, QP, are seen in the direction, SRP′, and the image thus seen may be traced on a sheet of paper placed at P′, vertically below S. The sheet of paper is seen past the edge, A, of the prism, while the image is seen by reflection from the face, AD.

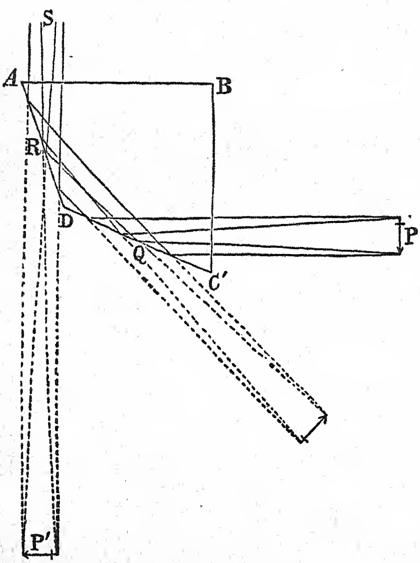


FIG. 149.

It is important that the image should be in the plane of the paper, because then paper, pencil, and image are seen with the same focusing of the eye. For this reason, a concave lens of short focal length is placed in front of the face, BC, when the object to be sketched is very distant. By adjusting the height of the prism in its stand (Fig. 150), the image can then be made to coincide with the plane of the paper. The two reflections at the faces, CD and DA, are necessary to give an erect image.

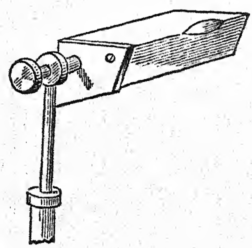


FIG. 150.

This form of prism is also used in one form of spectroscope (see page 322), as a *constant deviation* prism, for the examination of spectrum lines at minimum deviation.

7. Camera Obscura

The principle of the usual arrangement is illustrated diagrammatically (Fig. 151). At the top of a small tent or suitable structure is a small cylindrical or cubical box, which contains a mirror, R, and a lens, L, arranged as shown. The mirror is inclined at 45° to the horizontal, and reflects the rays from any external object, AB, on to the lens, which forms an image, A′B′, on a white table or screen placed vertically below. The tent is perfectly dark and the inside carefully blackened, so that the image thus formed on the screen or table may be seen clearly. The box containing the mirror and lens can be rotated about a vertical axis, and thus images of all objects surrounding the tent may be formed in turn on the screen.

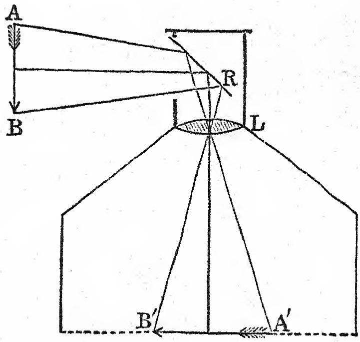


FIG. 151.

Instead of the mirror and lens, it is better to employ a totally reflecting prism with the faces which are turned towards AB and A′B′, convex and concave respectively. The curvature of the concave surface is less than that of the convex surface, and the arrangement thus acts as a convex lens and a mirror combined. Since the objects under observation are relatively distant, all the images on the screen are in focus at the same time. The sizes of the images are the same as if a simple aperture were used instead of the lens and mirror.

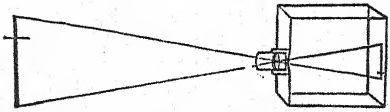


FIG. 152.

8. Simple Camera

If all unessential details are disregarded, this instrument may be regarded as a box (Fig. 152) with a convex lens, usually an achromatic combination (see page 155), in front, and at the back a ground glass screen, which can be replaced by a slide carrying a sensitised plate. When this instrument, with the lens uncovered, is towards any object to be photographed, an inverted image of that object will be formed, and, by sliding the lens or the back of the box in or out, the image may be focused accurately on the ground glass screen. If a suitably prepared photographic plate—

the first photographs were taken by Daguerre in 1839—be then substituted for the screen, the action of the light on this plate is such that, when subjected to proper chemical treatment, a *negative* is obtained, from which the ordinary photographs can be printed on sensitised paper.

The pinhole camera (see page 12) is very useful for some kinds of photographic work, such as that of photographing buildings, as it produces no distortion, whereas an ordinary lens gives images with strongly curved edges. In fact, many of the cheap cameras on the market are simply pinhole cameras. A disadvantage of such cameras is that the light can enter only through a very small hole, and thus long exposures are required, though this difficulty can be overcome to a certain extent by using extra rapid plates.

Experiment. To make a pinhole camera.—Cut a strip of brown paper 8 in. by 30 in. Paste one side of the paper, and roll it, with the pasted side inwards, on a roller about 3 in. in diameter, and about 9 in. in length. In this way a serviceable cardboard tube can be made. Over this roll a single layer of dry paper, and over this again a pasted sheet of brown paper as before. Thus, two cardboard tubes about 8 in. long and 3 in. in diameter will be made, so that one may slide stiffly in the other.

Close one end of the wider tube by a piece of thin card, in the centre of which a small pinhole is made. Close the opposite end of the smaller tube with a piece of tracing paper or ground glass. Direct the pinhole to any brightly illuminated object. A small inverted image of the object will be seen on the tracing paper or ground glass. Push the smaller tube farther in the wide tube—the image is reduced in size, but increased in brightness. Pull it farther out—the image is increased in size, but diminished in brightness. Enlarge the pinhole—the size of the image is unaltered, its brightness is increased, but it is less distinct.

The more elaborate photographic camera will be dealt with in the next chapter (see page 237). It may be stated here, however, that cameras and other optical instruments are always painted dead-black inside in order to prevent internal reflections. In the camera, for instance, when the light reaches the sensitised plate, a considerable proportion is scattered in all directions, and, falling on the walls of the camera, would, if these were light in colour, be mostly reflected back to the plate and would fog it. In other instruments, such internal reflections would similarly confuse the effect produced by the direct rays.

9. Optical Lantern

Anyone attending a course of lectures will see this instrument in use for a variety of purposes, and the student should have some idea of its construction and mode of action. Essentially, it is a light-tight box (Fig. 153) enclosing a powerful source of light, S. For scientific experiments it is important also that the source should be very small, the smaller the better. The lime light and electric arc are satisfactory in both these respects. In front of the box is the *condenser*, L, usually a pair of plano-convex lenses about 4 in. in diameter. The source is so mounted that it can be moved backwards and forwards in the optic axis of the condenser. When the source is at the principal focus of the condenser, a powerful parallel beam of light emerges, but if the source is moved further back, the

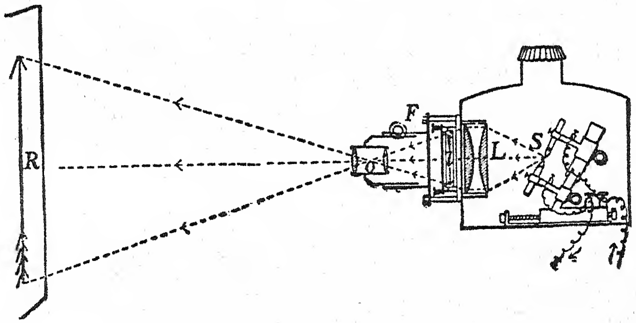


FIG. 153.

beam can be made to converge to any required point. When one is projecting images of various pieces of apparatus or of pictures, an achromatic lens combination (see page 155) of about 6 in. focal length, called the *objective*, O, is mounted in front of the condenser, and the source is placed so that the whole of the convergent beam issuing from the condenser enters the objective.

The lantern slide, *l*, which is a transparent photograph, or other suitably prepared representation of the object to be shown, is placed between O and L, as close as possible to the latter, and the tube carrying the former is moved backwards and forwards until a clearly defined image is focused on the screen, R. The image is inverted, and magnified in the ratio RO : O*l*. Since the illumination of the image varies inversely as the square of RO (see page 258), the best magnification that can be obtained depends ultimately on the

luminous intensity (see page 255) of S. To get an erect image, the lantern slide is put in upside down, but in the projection of a piece of apparatus on the screen, inversion is usually undesirable, and in such cases the beam is re-inverted by means of an erecting prism (Art. 6).

The condenser is used simply to concentrate, or condense, the light on the object. It plays no part in the focusing, so need not be corrected for spherical or chromatic aberration (see page 216).

The **cinema projector** is similar to the optical lantern, except that it includes a mechanism for exchanging the pictures very rapidly, and covering the aperture while the actual change is taking place. The result is that about 30-40 slightly different still pictures appear on the screen per second, separated by very brief intervals of darkness. Owing to the persistence of vision, the eye blends

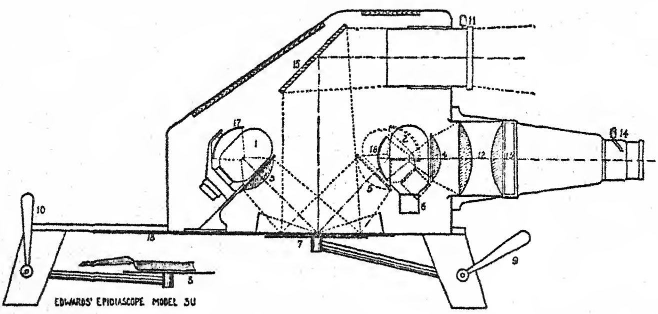


FIG. 154.

all these pictures together and the illusion of movement is produced by a succession of pictures taken at different instants.

The **epidiascope** is a modern development of the optical lantern which may be used not only with transparent lantern slides, but also to project pictures of opaque objects. For the latter purpose the illumination of the object must be very intense, since some of the light is absorbed by the object and much of the reflected light is scattered in such directions that it does not pass through the objective. The heat produced is considerable, so rotating fans are employed and the object is covered by heat-resisting glass.

A modern form of epidiascope is shown diagrammatically (Fig. 154). The source of illumination, 1, 2, are 500 watt lamps. The lenses, 3, 4, and the highly reflecting spherical mirrors, 16, 17, concentrate the light on the object, 7, which is placed on a stage

movable by means of the lever, 9. The plane mirror, 15, reflects the light through the objective, 11. The condenser, 12, the lantern slide holder, 13, and the objective, 14, are used when the apparatus is employed as an ordinary optical lantern.

10. The Eye

The human eye is essentially an optical instrument, similar in principle to the photographic camera described above (Art. 7).

A vertical section of the eye from front to back is shown diagrammatically (Fig. 155). In front is the *cornea*, C, behind which is the anterior chamber of the eye, bounded by the *crystalline lens*, L, and the *ciliary processes*, *cc*, to which the lens is attached. This chamber is filled with a watery fluid, A, called the *aqueous humour*, and in front of the crystalline lens lies the *iris*, II, a circular curtain with a central aperture, *p*, called the *pupil*. The iris is seen in the eye as the coloured ring surrounding the pupil, and is a muscular structure made up of circular and radial fibres, so arranged that the size of the pupil can be increased or diminished at will. Behind the crystalline lens, in the posterior chamber, is the *vitreous humour*, V, a watery fluid very similar to the aqueous humour.

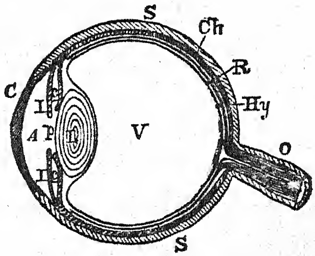


FIG. 155.

The crystalline lens, L, is the lens of the eye. It consists of a soft transparent substance enclosed in a thin transparent membrane, and is held in position by the ciliary processes which are attached round its circumference. In structure it is somewhat complex; the posterior face is more convex than the anterior, and it is built up of a large number of concentric shells, increasing in density as the centre is approached, the outer shell having the same density as the surrounding medium. By this arrangement, the optical action of the lens is more powerful than if it were composed of a homogeneous medium of the same density as the nucleus, and also the loss of light by reflection at the surfaces of the lens is diminished.

The walls of the eyeball are made up of three coatings. The outer, S, called the *sclerotic*, is a tough white skin, giving consistency to the eyeball. The cornea, C, in the front, is fitted into this coating like a watch-glass into the case of a watch. The middle coating, C*h*, the *choroid*, is a thin pigmented layer which divides in front

into two layers, the anterior layer forming the iris, and the posterior layer constituting the collar of ciliary processes which carry the crystalline lens. Adjacent to the ciliary processes is a muscular collar attached to the sclerotic, and inserted at the circumference of the crystalline lens. This collar is the *ciliary muscle,* and serves by its action to vary the curvature of the surfaces of the lens. The inner coating, R, is the *retina,* a delicate membrane which is almost a fine network expansion of the *optic nerve,* O. It covers the whole of the inner posterior surface of the eyeball as far as the ciliary collar, and is lined by a very delicate membrane, H*y*, called the *hyaloid membrane.*

At the centre of the retina is the *yellow spot,* a small slightly raised yellow spot which has a minute depression, called the *fovea centralis,* at its summit. This yellow spot, about $\frac{1}{20}$ of an inch in diameter, is the region of *most distinct vision,* and the fovea centralis is the most sensitive spot of the retina. About $\frac{1}{10}$ of an inch on the inner side of the yellow spot is the *blind spot,* the point at which the optic nerve enters the eyeball. This spot is not sensitive to light.

The structure of the retina is very complicated. The surface next to the vitreous humour consists of thin connective tissue, the hyaloid membrane. Below that extend the ramifications of both the optic nerve and also the artery which enters the eyeball with the nerve. The nerve filaments end in ganglion cells, and fresh processes proceed from these through the rest of the retinal thickness and, penetrating the external layer of connective tissue, end in a layer called the *bacillary layer*, or *Jacob's membrane.* This layer or membrane consists of elongated bodies, some shaped like rods and some like cones. Their extremities are embedded in a layer of pigment cells. The rods and cones form the sensitive part of the retina. An element of the retina, consisting of rods and cones, is about 0·004 mm. in diameter, and the eye cannot distinguish between two objects unless the retinal images are separated by a distance greater than this. Experimental evidence indicates that the rods are most sensitive to faint light, while the colour sensations are produced by the cones.

Considered optically, the eye thus consists of a double convex lens, the crystalline lens protected in front by a circular diaphragm, the iris, and having a sensitive screen, the retina, on which the images of external objects are formed. The impressions, conveyed to the brain by these images, give rise to the sensation of sight.

11. Vision

The condition of distinct vision of any object is that a clearly defined image of it is formed on the retina. Such a case is illustrated diagrammatically (Fig. 156). When light enters the eye, refraction occurs at the surfaces of the cornea and crystalline lens. The centres of these surfaces lie on a straight line, the optic axis, which meets the retina between the yellow and blind spots. Since no change of refractive index occurs from the cornea to the aqueous humour, the aqueous humour may be considered to extend to the anterior surface of the cornea.

If the object is large the image is distorted, due partly to obliquity of the extreme rays and partly to spherical aberration. However, the distortion is greatly corrected by the heterogeneity of the crystalline lens and by the spherical shape of the eyeball. In all cases in which real objects are seen distinctly, real inverted images are formed on the retina. That objects are seen erect is

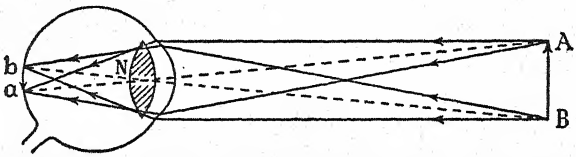


FIG. 156.

due to the interpretation which the brain puts upon the stimuli received by it.

Under ordinary conditions it is evident that, with an eye as described above, objects only at a certain definite distance from the eye can be seen distinctly, because, the distance between the lens and the image being fixed, the distance between the lens and the object must also be fixed. However, it is known from experience that objects at all distances greater than a certain minimum limit, known as *the least distance of distinct vision*, can be seen distinctly by the normal eye. This is due to the power of *accommodation* possessed by the eye. The ciliary muscle is able to alter the radii of curvature of the surfaces of the crystalline lens, making the front surface much more convex, and bringing the lens as a whole nearer to the cornea. Thus the focal length is *accommodated* to the distance of the object on which the eye is focused. For a normal or *emmetropic* eye, the limits of distinct vision are from a point distant about 10 in. from the eye to infinity. When the eye is at

rest, it is supposed to be adjusted for parallel light—that is, for distinct vision of very distant objects.

The eyes of various individuals vary much in their accommodating power. Young children can see distinctly objects placed two or three inches in front of their eyes, and ordinary adults can see objects as near as 10 in. As the age of a person advances, however, the power of accommodation of the eye decreases, probably because of a loss of elasticity in the outer layers of the crystalline lens. This defect of vision is called *presbyopia* (Art. 12), and it causes the nearest point of distinct vision to recede gradually from the eye. Thus, in order to read a book an old person often is compelled to hold the book at arm's length.

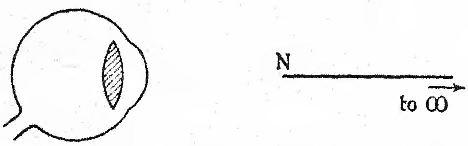


FIG. 157.

12. Defects of Vision. Spectacles

The most common defects of vision and their remedies will be dealt with. A **normal** or **emmetropic** eye brings parallel light to a focus on the retina. By means of its power of accommodation, the eye can also focus light from nearer points. Let N (Fig. 157) be the nearest point of distinct vision. Then images of all points on the line from N to $+\infty$ can be focused on the retina by the unaided eye.

(1) MYOPIA, OR SHORT-SIGHTEDNESS.—In *myopic* eyes, either the axis of the eye is too long or the crystalline lens is too convergent. Light from a distant object is brought to a focus in front of the retina (Fig. 158), and thus the object either is not seen at all or is seen very indistinctly. As the object approaches the eye, the image travels backwards from the focus of the lens, and, when the object reaches a point, P, a certain distance away, the image falls exactly on the retina. This point, P, is the point of farthest distinct vision which, if the eye were normal, would be at infinity. If the accommodating mechanism is perfect, the eye will be able, up to a certain limit, to adjust itself so as to give distinct vision for objects

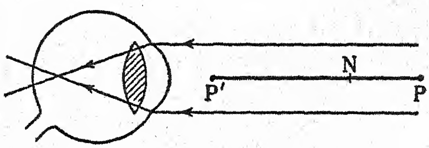


FIG. 158.

nearer to the eye than P, and even nearer than N, the near point for normal eyes.

Let P′ be the nearest point of distinct vision for a myopic eye. By the unaided eye, only those points which lie between P and P′ can be seen distinctly. Rays from a distant point are brought to a focus in front of the retina, even when the ciliary muscle makes it as little convex as possible. This defect can be remedied by the use of spectacles. To determine the nature of the lenses required, it is evident that the necessary condition for remedying the defect is that rays diverging from a point on the *normal* range of vision, between infinity and N, should apparently diverge, after refraction through the lens, from a point within the range, PP′, of the short-sighted eye. Suppose this latter range to be from 3 to 8 in. from the eye. This cannot be made to coincide with the normal range at *both* ends, and therefore coincidence with the nearest or with the farthest point of distinct vision must be decided upon. It is usual to choose the latter, because it corresponds to the quiescent state of the eye. Hence, in this case, a lens is required such that rays coming from infinity—that is, parallel rays—will diverge, after refraction through it, from a point, F (Fig. 159), about 8 in. from the eye. The required lens is thus a *concave* lens of 8 in. focal length, for with such a lens rays from infinity will diverge from its principal focus, a point about 8 in. from the eye. Then, if x denote the least distance of distinct vision with this lens,

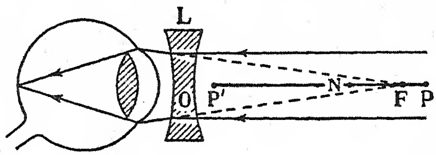


FIG. 159.

$$\frac{1}{3}-\frac{1}{x}=\frac{1}{8}, \text{ from which } x = 4{\cdot}8 \text{ in.}\dagger$$

This is determined by the condition that rays diverging from a point x in. in front of the lens must appear to diverge, after refraction, from a point 3 in. in front of the eye. From this, the range of vision *with* the spectacles is from 4·8 in. to infinity, instead of from 3 in. to 8 in. in front of the eye as is the case *without* spectacles.

When a concave lens is used the retinal images are diminished, and thus objects appear smaller than to a normal eye.

† If using the Real is Positive convention, we have $f = -8$, $d = x$, $v = -3$ in. (virtual image). Substituting in the relation $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$, gives $x = 4{\cdot}8$ in.

(2) HYPERMETROPIA, OR LONG-SIGHTEDNESS.—In *hypermetropic* eyes, either the axis of the eye is too short, or the lens is not sufficiently convergent. When unaccommodated, the only light which can be focused on the retina is that which is converging to a point, P (Fig. 160), *behind* the eye. By means of accommodation, rays can be focused which are converging to points on the line from P to infinity behind the eye, and rays which are diverging from between infinity in front of the eye and P′, the nearest point of distinct vision. P′ is at a greater distance from the eye than N, the nearest point of distinct vision for the normal eye.

Rays diverging from points nearer than P′ are focused behind the retina. The power of the crystalline lens to converge the rays is too small, and so a *convex* lens must be used to help it. Suppose, for example, that P′ is 30 in. in front of the eye, and P is 10 in. behind the eye. Then, to render the farthest points of distinct

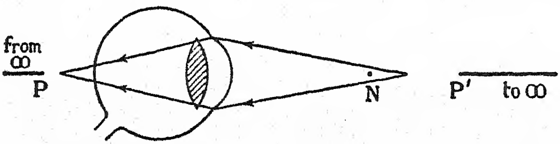


FIG. 160.

vision for the unaided normal eye and for the aided hypermetropic eye coincident, a lens must be employed such that rays from infinity will converge, after refraction through it, to the point, P, 10 in. behind the lens. The required lens is thus a *convex* lens of 10 in. focal length, and the distance, x, of the nearest point of distinct vision is given by†

$$\frac{1}{30} - \frac{1}{x} = -\frac{1}{10}, \text{ from which } x = 7\tfrac{1}{2} \text{ in.}$$

This means that the range of vision is now from $7\frac{1}{2}$ in. to infinity in front of the eye. The action of the lens is illustrated diagrammatically (Fig. 161). The effect of the lens, L, is to push P to infinity in front of the eye, and to bring P′ nearer to N. Another effect of the convex lens is to make objects appear larger than they appear to the normal eye.

† If using the Real is Positive convention, we have $f = 10$, $u = x$, $v = +30$ in., because the image would be real in the absence of the eye. The relation $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$, then gives $x = 7\frac{1}{2}$ in.

Example.—*A person's range of distinct vision is from 4 in. to 8 in. in front of the eye. Find the focal length of the lenses to be worn, and the range of vision with those lenses.*

Here the lenses required are such that rays coming from infinity in front of the eye will be caused to diverge from a point 8 in. in front of the eye. Thus, *concave* lenses of 8 in. focal length are required.

If x denote the least distance of distinct vision, then for these lenses,

$$\frac{1}{4} - \frac{1}{x} = \frac{1}{8}, \dagger \text{from which } x = 8 \text{ in.}$$

Hence, the range of vision with the lenses is from 8 in. to infinity.

(3) PRESBYOPIA.—In the case of *presbyopic* eyes, distant objects can be seen distinctly, but light from near objects cannot be focused on the retina—that is, the nearest point of distinct vision has receded. This defect is found usually in old people (Art. 11).

To remedy this defect by means of spectacles, the focal length of the lenses must be adapted to the purpose for which the lenses

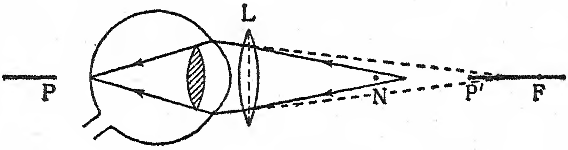


FIG. 161.

are required. For reading purposes, the lenses must be chosen so as to cause rays, diverging from the point of *normal* nearest distinct vision, to appear to diverge, after refraction through them, from the nearest point of distinct vision of the defective eye. Thus, for example, suppose the nearest point at which a person can see distinctly is 30 in. Then, if f denote the focal length of the lenses required,

$$\frac{1}{30} - \frac{1}{10} = \frac{1}{f}, \text{ from which } f = -15 \text{ in.}\ddagger$$

This means that *convex* lenses of 15 in. focal length are required.

When the accommodating mechanism is not perfect, there may be practically only one point of distinct vision, and the defect can be remedied only for particular cases. Thus, for example, suppose a person is able to see distinctly only at a point about 4 in. from the

† If using the Real is Positive convention, this relation is replaced by $-\frac{1}{4} + \frac{1}{x} = -\frac{1}{8}$, giving $x = 8$ in.

‡ If using the Real is Positive convention, this relation is replaced by $-\frac{1}{30} + \frac{1}{10} = \frac{1}{f}$, from which $f = 5$ in. and the lenses are convex.

eye. Then, if spectacles are required to enable distant objects to be seen distinctly, *concave* lenses of about 4 in. focal length must be used. If, however, the same person requires spectacles for reading purposes, so that the book can be held in the same position as by a normal-sighted person, then the focal length of the lenses is determined from the fact that light coming from objects at the normal least distance of distinct vision, 10 in., should appear, after refraction, to come from a point 4 in. from the eye. In this case,

$$\frac{1}{4} - \frac{1}{10} = \frac{1}{f},\dagger \text{from which } f = 6\tfrac{2}{3} \text{ in.}$$

This means that *concave* lenses of $6\frac{2}{3}$ in. focal length are required.

(4) ASTIGMATISM.—In the case of *astigmatic* eyes, lines inclined in one direction can be seen much more distinctly than lines in a direction perpendicular to this. The defect is due mainly to non-sphericity of the cornea, a vertical section being usually more curved than a horizontal section. There are very few eyes which do not suffer from this defect, horizontal lines usually being brought to a focus in front of the focus of vertical lines. A test may be made by drawing four or five parallel lines close together on a sheet of paper, which is then placed facing the observer about 5 yd. away and rotated slowly. The observer with one eye open watches the lines, and in general it will be found that for quite a large range of rotation the lines appear very indistinct.

To remedy this defect, a cylindrical lens (Art. 5) is required, its position being arranged so that the refraction which it produces is in the same direction as the weaker refraction of the cornea. If the eye is myopic or hypermetropic, as well as astigmatic, a lens which is cylindrical on one side and spherical, convex or concave, on the other side, is required.

13. Experiments and Observations with the Eye

The following experiments will emphasise many important facts in connexion with the eye and vision.

(1) *To show that the eye is over-corrected for spherical aberration* (see page 122).—Bring a page of print so close to the eye that the print is indistinct. Now interpose a sheet of paper with a pinhole in it between the page and the eye, and just in front of the latter.

† If using the Real is Positive convention, we have $u = 10$ in., $v = -4$ in. (virtual image). Whence $f = -6\frac{2}{3}$ in.

The print seen through the pinhole is quite distinct. This shows that rays passing through the centre of the crystalline lens are converged more than those passing through the peripheral portions. The opposite occurs with ordinary lenses (see page 122).

(2) *To show that the eye is not achromatic* (*see page* 155) *for the extreme rays though very nearly so for intermediate rays.*—Look at a window frame with a bright background. Hold a finger close in front of the eye, and move it slowly across the field of view. As the finger approaches a bar of the window frame, the near edge of that bar will bear a blue fringe, and the far edge a red fringe. In this case it must be remembered that the image on the retina is inverted.

(3) *To show that images of real objects are inverted.*—Experience shows that objects are the right way up. If therefore an erect shadow could be thrown on the retina, it should appear inverted. To verify this, make a pinhole in a piece of paper, and, holding it about an inch in front of the eye, view through it a brightly illuminated surface such as a white lamp globe. Now take a pin, and, holding it head upwards, introduce it between the eye and the pinhole. The pin appears inverted (Fig. 162).

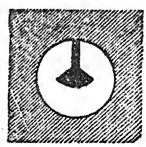


FIG. 162.

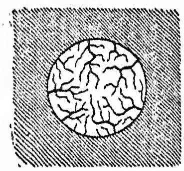


FIG. 163.

(4) *Shadows of the blood vessels of the eye.*—It is evident that since the blood vessels of the retina are in front of the sensitive layer, light entering the eye should cast shadows of them upon this layer. That these shadows are not always evident is due to the fact that ordinarily they are formed by diffused light, and hence are indistinct. In the following simple experiment, parallel or nearly parallel light is used so that the shadows are distinct and therefore easily seen.

View a very bright white surface through a pinhole in a sheet of paper held just in front of the eye. Move the pinhole about. The blood vessels will be seen as black shadows on the bright surface viewed. Now give a quick circular motion to the pinhole, and observe that the blood vessels seem to extend from the periphery to the centre, getting smaller and more ramified as they approach it (Fig. 163), the centre, however, being free from them. Now hold the paper still. The appearance vanishes, due to the rods and cones becoming fatigued.

The above is a subjective method of observation. The interior of the eye of another person may be examined, however, by means of the *ophthalmoscope*, devised by Helmholtz, which consists essentially of a concave mirror, about $1\frac{1}{2}$ in. in diameter, pierced through the centre by a small hole. Light from a lamp is reflected by this mirror into the eye, and the observer, looking through the small hole, is able to examine minutely the various parts of the retina.

(5) *The yellow spot.*—Although the yellow spot is most sensitive to ordinary light, it is not as sensitive to very faint light as the surrounding portion of the retina. This is due to the fact that the bacillary layer at this point is composed entirely of cones (Art. 10). For this reason, faint stars, when looked at a little obliquely, appear brighter than if viewed directly.

(6) *To prove the existence of the blind spot.*—Close the right eye, and, keeping the left fixed on the spot, B (Fig. 164), move the book to and from the eye. When the book is about 12 in. from the face, the spot, A, will be invisible, but will come into view again for

FIG. 164.

greater or smaller distances. Repeat with only the right eye open and fixed on A.

(7) *Persistence of impressions.*—The retina continues to feel the effects of light after the exciting agent has been removed. This phenomenon is called the *persistence of impressions*, the impressions lasting about $\frac{1}{10}$ of a second. Thus, the glowing end of a match-stick yields a bright circle when the match is swung round, and not a bright point changing its position. Again, the colours of a rapidly rotating colour disc (see page 169) blend into one, and an alternating electric current gives a steady light, the fluctuations of the light being too rapid to be noticed, unless it be used to illuminate a rapidly moving object.

(8) *Positive and negative after-images.*—Look at a bright object, such as a white lamp globe, and then close the eyes, and, in addition, cover them. An image of the globe will appear; this is the *positive after-image*. After a time this image disappears, and may be followed by a very dark image of the globe on the dark background; this is the *negative after-image*. The positive image is due to

continued nerve irritation, similar to the persistence of impressions, whilst the negative image is due to the fatigue of those nerves upon which the bright image fell. This prevents them from being excited to the same extent, as the unaffected nerves are, by the dull light penetrating the eyelids; hence the dark image. Repeat the experiment with a strongly illuminated colour form, such as a church window. In this case the positive after-image is similar in colour to the window; it then fades away, and the negative after-image appears in the complementary colours (see page 166).

Next, look steadily at a bright red spot, and then shift the eyes to a dull grey surface; a greenish-yellow spot will be seen on it. Other contrast results may be obtained by placing a small disc of white paper upon larger pieces of coloured paper. In each case, the white disc seems to be illuminated by the colour complementary to that of the background. Thus, on yellow it appears blue, and on green it takes a ruddy hue. Again, if a hole be made in a piece of bluish-green glass, and the glass is then held between the sun and a white screen, the shadow of the hole will look dark pink.

(9) *Stimulating eye nerves by pressure.*—Close the eyes, and press with the fingers into one of the hollows on the upper sides of the eyes next to the nose. A circle of light will appear at the opposite side of the eye ball, the nerves having been stimulated by mere pressure from the outside.

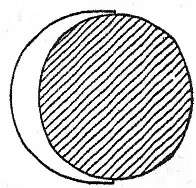


FIG. 165.

(10) *Size and brightness. Irradiation.*—If there are two bodies of the same size, one bright and the other dark, the former will appear the bigger. This is caused by the rods and cones which are being excited, causing their neighbours to be excited. The most striking example of irradiation occurs when the moon is in the first quarter, and is called *the old moon in the young one's arms*. The illuminated crescent (Fig. 165) then appears to be part of a much larger circle than that of the faintly illuminated remainder.

(11) A peculiar wave-like motion is often observed in a row of close-set railings when one is walking near by and looking through them at a farther row. At certain places bars in the two rows are behind each other, and thus the maximum amount of light is able to penetrate them. At other places, bars in the farther row are behind spaces in the front row, and the illumination is smaller, varying to a minimum when the bars are behind the centres of the

spaces. The same phenomenon is also very noticeable when a meat safe of perforated zinc is under observation, and also when a piece of wire gauze is lying upon another. The appearance is very much like that of *watered silk*.

(12) *Judging the distance of objects.*—The distance of objects is judged partly by the amount of accommodation it is necessary to impress upon the eyes in order to see them distinctly, and partly by the amount of convergence between the optic axes of the two eyes. When the distance exceeds a certain amount, however, the accommodation is constant, and the optic axes are sensibly parallel. Hence, other methods of judgment must be used. The usual method is that of comparing the size of a known object at the far distance with the apparent size it would have when close to the observer. In judging distances, therefore, practice counts a lot. On a very clear day, distant objects appear to be nearer than they really are, and hence their magnitude is judged to be smaller. In a fog, vision is indistinct, and, as indistinction is associated with distance, the object is estimated unconsciously to be some distance away, and therefore larger than it really is.

Again, the sun and moon usually look larger when low on the horizon than when high in the sky. This false impression is not in accordance with measurements of the angular diameters. When near the horizon, the eye is apt to estimate the size and distance of the sun and moon by comparing them with neighbouring terrestrial objects, such as trees, hills, etc. When the sun and moon are at a considerable altitude, however, no such comparison is possible, and a different estimate of their size is formed instinctively.

(13) *To find the least distance of distinct vision.*—Make two pinholes $\frac{1}{16}$ in. apart in a sheet of paper. Hold the paper close to the eye, the holes being in a horizontal line, and look through the holes at a vertical pin held just in front. The pin appears double and indistinct. Remove the pin gradually farther away. The images become more distinct, and approach each other, coinciding at a certain distance, the least distance of distinct vision, and afterwards remaining in coincidence. Two narrow pencils of light from the pin pass through the pinholes, and, after refraction in the eye, converge to a common focus. If the pin is at or beyond the least distance of distinct vision, the eye accommodates itself so that this common focus is on the retina. If, however, the pin is within this least distance, the pencils reach the retina before meeting, and thus two images are seen, both indistinct. Repeat the experi-

ment, blocking out, say, the left-hand pinhole. The right-hand image disappears, and vice versa. It must be remembered that the brain inverts the images.

14. The Magnifying Glass and the Simple Eye-piece

The magnifying glass is simply a convex lens furnished with a handle or mounted on a small tripod, underneath which the object to be examined is placed. The object is usually placed at a distance slightly less than the focal length of the lens, and, under these conditions, a magnified virtual image is seen. The mode of action will be clear from Fig. 166.

In practice, the lens is usually placed roughly half-way between eye and object. If the lens is mounted on a tripod, the legs of the tripod are cut to a length slightly less than the focal length, so that an object placed on the table beneath it is automatically at

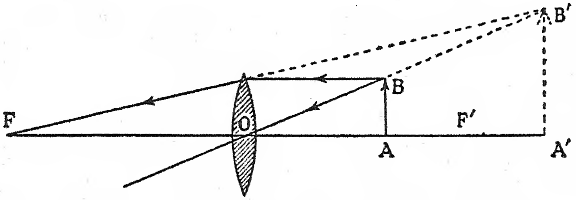


FIG. 166.

a suitable distance. A magnifying glass often used by watchmakers is fitted into a frame that can be held in the eye, thus holding the lens at a suitable distance from the eye and leaving both hands free.

In order to obtain a *magnifying* effect in the strict sense of the word it is necessary for the eye to be away from the lens. The reason for this is that the eye has only one means of estimating the apparent size of an object, namely by taking account of the fraction of the retina that is covered by the image. Thus a cloud a great distance away may appear to be "no bigger than a man's hand," or a distant building may appear smaller than a sheet of paper seen close at hand. What determines the apparent size of an object as judged by the eye? Reference to Fig. 167 shows that the answer is simple. Two objects will be of the same apparent length if they subtend the same angle at the pupil of the eye, or strictly at the optical centre of the lens of the eye. Since rays of light passing through the optical centre of a lens are undeviated,

it is evident that objects X and Y will form images of the same length on the retina and will be judged to be the same size. The sun and moon appear the same size because they subtend practically equal angles at the eye.

Referring back to Fig. 166, it is evident that, if the eye is placed very near the lens, the object and image will subtend the same angle at the eye, so one might say that there will be no appreciable magnifying effect. Yet lenses are very often used in just this fashion, for example, in the eye-piece of a telescope or microscope. What is their function, if it is not to magnify? From Fig. 166 it will be evident that the image is not only larger than the object but is also further away. Now it will be remembered (Art. 11) that the eye is not capable of seeing distinctly objects nearer to the eye than a certain minimum distance (10 in. for a normal eye). An attempt to examine a small object at a distance of 2 in. merely causes eyestrain without enabling the details to be seen more clearly.

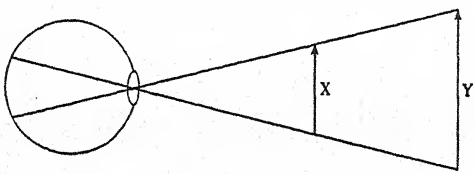


FIG. 167.

A lens may now be placed close to the eye so that the rays of light appear to come from a virtual image 10 in. away which, nevertheless, subtends the same angle at the eye as the object does at 2 in. The details are now clearly visible, yet the size of the retinal image is unaltered. A convex lens used in this fashion is called an eye-piece. The elaborate and expensive eye-pieces of telescopes and microscopes all act in this manner, though they usually consist of at least two lenses, to enable corrections for chromatic and spherical aberrations to be made. The mode of action of an eye-piece is as follows: The optical system of the telescope or microscope forms an image near the eye-piece which then forms another image at a comfortable distance, subtending the same angle at the eye as the first image would do if the eye-piece were removed without moving the eye.

The *effective magnification* of an eye-piece, in the sense of the extent to which it increases the angle the object may subtend at the eye without causing eyestrain, can easily be calculated from the theory of lenses.

Example.—*An eye-piece has a focal length of* 1 *in. Calculate the effective magnification* (a) *If the image is formed at infinity;* (b) *If it is formed at a distance of* 10 *in. which is to be taken as the least distance of distinct vision.*

(*a*) The object is placed at 1 in. whereas without the eye-piece it would have to be at 10 in. The effective magnification is therefore 10.

(*b*) The distance of the object is given by $\frac{1}{10} - \frac{1}{u} = -1$;†

$\therefore \quad u = \frac{10}{11}$ in. Without the eye-piece it would have to be at 10 in. The effective magnification is therefore 11.

It will thus be seen that a slightly greater effective magnification can be obtained in case (*b*). In practice, those who use optical instruments for long periods at a time prefer to keep their eyes at rest, *i.e.* accommodated for infinity, because keeping them accommodated for 10 in. keeps the ciliary muscles constantly in action and tends to induce fatigue, and the slightly greater effective magnification that is obtainable does not justify this.

15. The Magnifying Power of an Eye-Piece, Telescope or Microscope

The above discussion of the magnifying power of a simple lens shows that it can have no definite meaning to talk of the magnifying power of an eye-piece or optical instrument unless the conditions under which this quantity is to be measured are precisely specified. However, these instruments are used in clearly defined ways, and it is possible to lay down such a definition.

Definition. *The magnifying power of an eye-piece, telescope or microscope is the ratio of the angle subtended at the eye by the final image to the angle that would be subtended at the eye if the object were being examined unaided.*

It will be seen that this definition covers both possible uses of the simple lens as a magnifying glass, or as an eye-piece, but it is still insufficiently precise to cover the telescope and microscope, because, in order to decide what would be the angle subtended at the eye by the object we have to consider how the object would be examined if the instrument were not available. For the telescope this presents no difficulty. It is used for examining distant objects which there is, in general, no possibility of moving, so that the magnifying power is simply the ratio of the angles subtended at the eye by the final image, and by the object seen directly. For

† Using the Real is Positive convention, $-\frac{1}{10} + \frac{1}{u} = 1; \quad \therefore u = \frac{10}{11}$ in.

the eye-piece and microscope the object to be viewed is near at hand, and it would appear at its largest (without eyestrain) if brought to the nearest distance of distinct vision. The magnifying power of a microscope is thus the ratio of the angle subtended at the eye by the final image to the angle the object would subtend at 10 in. from the eye. For the eye-piece we have an exactly similar definition except that the "object" in this case is a real image formed by the rest of the optical instrument, and the magnifying power is defined in the same way, on the assumption that, without the eye-piece, this image would be examined at 10 in. from the eye.

When a set of eye-pieces is supplied with an instrument the magnifying power of each is often stamped on its side. This may either refer to the magnifying power of the *whole instrument* with that particular eye-piece in use, or the magnifying power of the *eye-piece itself*, defined as above. It is important, in using such a set of eye-pieces, to be quite clear which is meant, as some makers supply eye-pieces that only suit one particular instrument, whereas others make eye-pieces that can be used with several instruments. For the reason already explained, magnifying powers are usually calculated on the basis that the *final* image is at infinity, but the calculations can easily be modified to cover the case of the final image being at 10 in. (see Chapter XI.).

CHAPTER XI

MORE COMPLEX OPTICAL INSTRUMENTS

IN this chapter the more complicated optical instruments, such as telescopes, microscopes, spectroscopes, etc., will be described, and the principles involved in their action will be explained. Each instrument will first be dealt with in its simplest form, and the various devices, by means of which spherical and chromatic aberrations are either reduced to a minimum or eliminated, used in the more complex instruments will be described and explained. It will be seen that the principles utilised in the construction of such optical instruments are applications of the phenomena of reflection and refraction dealt with in the preceding chapters.

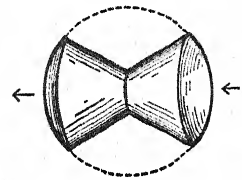


FIG. 168.

1. Pocket Microscopes

The use of a single convex lens as a magnifying glass has been discussed above (see page 197). A lens of high power used in this way is called a simple microscope, and is most efficient if made plano-convex, the plane side being towards the eye. *The magnifying power is inversely proportional to the focal length of the lens.* In practice, it is found that as the focal length of the lens is decreased, distortion and chromatic effects creep in, and a single lens acts well only if its focal length is not less than one inch, so that for greater magnifying power recourse must be had to combinations of lenses (see Art. 14).

There are several simple forms of pocket magnifiers, of which the following are examples:—

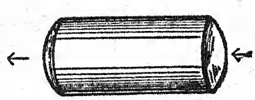


FIG. 169.

(1) THE CODDINGTON LENS.—This was invented by Wollaston, and is simply a sphere of glass (Fig. 168) in which a deep groove has been cut all the way round, leaving only a small central aperture through which the rays of light may pass (see Art. 2).

(2) THE STANHOPE LENS.—This consists of a cylinder of glass (Fig. 169) whose ends are ground to spherical surfaces of unequal radii. The dimensions are so chosen that when a small object is placed near the end of smaller curvature, an eye placed near the other end sees a magnified and well-defined image.

(3) THE WOLLASTON DOUBLET.—This was the first combination of two lenses devised for magnifying, and in appearance is very similar to an inverted Huyghens eye-piece (see Art. 12). The deviation of the rays passing through the arrangement is borne equally by the two lenses, and thus aberration and chromatic defects are minimised.

2. Thick Lenses

The lenses dealt with so far (see Chapter VII.) were *thin*, the curved surfaces being very small portions of comparatively large spheres. Refraction through a *complete spherical lens* is of some importance, and a simple treatment will be useful to the student. The Coddington lens (Fig. 168) is really a common practical form of spherical lens.

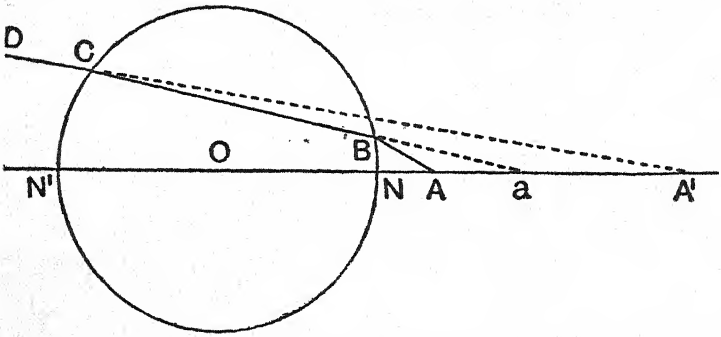


FIG. 170.

* Refraction at a single spherical surface has already been dealt with (see page 93), and the relation

$$\frac{\mu}{v} - \frac{1}{u} = \frac{\mu - 1}{r}$$

was obtained. In Fig. 170 let a be the image of A in the first surface BN. For this surface u and v are to be measured from N. Now consider refraction of the pencil apparently diverging from a in the surface CN′, A′ being the image of a in this surface. For this refraction the relation is

$$\frac{\frac{1}{\mu}}{v} - \frac{1}{u} = \frac{\frac{1}{\mu} - 1}{r},$$

since the light is now travelling from glass to air. The distances

are now to be measured from N′. Let U be the distance from A to O, and V the distance from A′ to O, while X is the distance of the intermediate image a from O, the sign convention described on page 40 being used. (The reason for measuring distances from O will appear later.) Then for the surface BN, we have, if R is the *numerical* value of the radius of the sphere,

$$u = \text{AN} = \text{AO} - \text{ON} = \text{U} - \text{R},$$
$$v = a\text{N} = a\text{O} - \text{ON} = \text{X} - \text{R},$$
$r = -\text{R}$ since r has to be measured from N to O in the negative direction.

Thus:— $$\frac{\mu}{\text{X} - \text{R}} - \frac{1}{\text{U} - \text{R}} = -\frac{\mu - 1}{\text{R}} \qquad (1)$$

For the surface CN′ we have:—

$$u = a\text{N}' = a\text{O} + \text{ON}' = \text{X} + \text{R},$$
$$v = \text{A}'\text{N}' = \text{A}'\text{O} + \text{ON}' = \text{V} + \text{R},$$
$r = \text{R}$ since r has to be measured from N′ to O in the positive direction. V.P. Singh

Thus:— $$\frac{\frac{1}{\mu}}{\text{V} + \text{R}} - \frac{1}{\text{X} + \text{R}} = \frac{\frac{1}{\mu} - 1}{\text{R}} \qquad (2)$$

Clearing equation (1) of fractions.

$$\mu\text{R}(\text{U} - \text{R}) - \text{R}(\text{X} - \text{R}) = -(\mu - 1)(\text{X} - \text{R})(\text{U} - \text{R})$$
or $$\text{X}[-\text{R} + (\mu - 1)(\text{U} - \text{R})] = -\mu\text{R}(\text{U} - \text{R}) - \text{R}^2 + (\mu - 1)\text{R}(\text{U} - \text{R}),$$
i.e. $$\text{X}[\mu\text{U} - \text{U} - \mu\text{R}] = -\text{RU}.$$

Treating equation (2) similarly.

$$\text{R}(\text{X} + \text{R}) - \mu\text{R}(\text{V} + \text{R}) = (1 - \mu)(\text{V} + \text{R})(\text{X} + \text{R})$$
or $$\text{X}[\text{R} + (\mu - 1)(\text{V} + \text{R})] = \mu\text{R}(\text{V} + \text{R}) - \text{R}^2 + (1 - \mu)(\text{V} + \text{R})\text{R},$$
i.e. $$\text{X}[\mu\text{V} - \text{V} + \mu\text{R}] = \text{RV}.$$

Eliminating X, we have:—

$$\text{U}(\mu\text{V} - \text{V} + \mu\text{R}) + \text{V}(\mu\text{U} - \text{U} - \mu\text{R}) = 0$$
or $$2\text{UV}(\mu - 1) + \mu\text{RU} - \mu\text{RV} = 0.$$

Dividing by μRUV:—

$$\frac{1}{\text{V}} - \frac{1}{\text{U}} = -\frac{2(\mu - 1)}{\mu\text{R}}.$$

Thus, a sphere of glass behaves exactly like a thin convex lens of focal length $\frac{-\mu R}{2(\mu - 1)}$ situated at its centre (with the restriction that the angular width of the pencils of rays is small). *

This method of considering each refraction in turn can be applied to any thick lens whatever, but the results are considerably more complicated than for the thin lens or sphere. For example, for a thick lens, or system of lenses, there is, in general, no point corresponding to the optical centre of a thin lens. Instead there are *two* points on the principal axis, called the **principal points** of the lens system, and one can still apply the ordinary lens formula to the whole system provided that u and v are each measured from the nearest principal point, the sign conventions being the same as for a thin lens. A proof of this statement is beyond the scope of this book. There is a second pair of points, the **nodal points,** which have the property that any ray directed towards one of them (at not too great an inclination to the axis), will be unchanged in *direction*, but will be displaced in position so that it passes through the other nodal point. In general, the nodal points are distinct from the principal points.

For the sphere, the above investigation shows that the principal points are both at the centre of the sphere and the fact that any ray proceeding along a radius is underrated shows that both nodal points must be there too. The Coddington lens (Art. 1) is a practical form of the spherical lens, the effective aperture being cut down in order to reduce spherical aberration.

3. The Telescope

Telescopes are employed for the purpose of obtaining distinct vision of distant objects, especially stars and other celestial bodies. They are of two kinds, *refracting* and *reflecting*, but the same general principle underlies them all. This general statement does not include, however, the opera or field glass. It is possible also to make use of the properties of prisms. The principle involved may be stated briefly as follows:—

A real image of the object is formed by a convex lens, the *object glass*, or a concave mirror, the *speculum*, and is examined by means of an *eye-piece* (see Chapter X., Art. 14).

There are several different *forms* of telescope, the details of construction in each case being adapted to the purpose for which that particular form is intended.

The more common forms of telescope will be dealt with below.

4. The Refracting Astronomical Telescope

This instrument was devised by **Kepler**, the astronomer, in 1611, but was first used by **Huyghens** in 1665. In its simplest form it consists of a convex lens, O (Fig. 171), fixed at one end of a brass tube, and another and smaller convex lens, O′, fitted in a tube which slides inside the former. The lens, O, which first receives the rays of light from the distant object is called the *object glass*, and may be a simple convex lens, but in the best instruments it is a compound achromatic lens (see page 155 and Art. 11), having its focus at F. The lens, O′, at the other end of the tube may also be a single lens, but generally consists of a system of two lenses, called an *eye-piece*, the focal length, O′F′, of the system being considerably smaller than that of the object glass. The optical action of the instrument may be explained as follows:—

Let A, A, A denote rays coming from a point, A, on the distant object, AB, not shown in the figure. If the object is very distant,

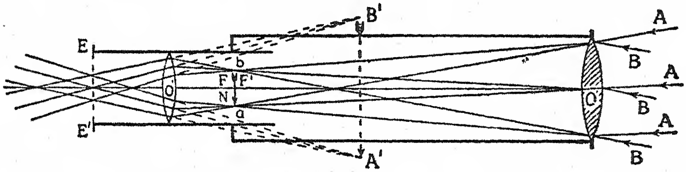


FIG. 171.

then these rays may be considered to be parallel. These rays, after refraction through the object glass, O, are brought to a focus at *a*. Similarly, the rays, B, B, B, coming from a point, B, on the object, are brought to a focus at *b*, and a real inverted image of the distant object is formed at *ab*. By moving the eye-piece tube, the position of the lens, O′, can be adjusted so that the image, *ab*, falls just within its focal length—that is, the distance of *ab* from O′ should be slightly less than the focal length of O′. With this adjustment, an eye looking through O′ sees a virtual magnified image of *ab* at A′B′ (see page 115).

If the instrument is focused so that A′B′ is seen at an infinite distance from the eye, then *ab* is at the focus of O′. Also, if the object, AB, is distant, then *ab* is approximately at the focus of O. Hence, under these conditions, the image, *ab*, is at a point which is the common focus of O and O′, and the length of the telescope is equal to the sum of the focal lengths of the object glass and eye-piece. For more distinct vision, however, the eye-piece is

often focused so that A′B′ is seen at the least distance of distinct vision, the distance of the image, *ab*, from O′ being then less than the focal length of the eye-piece, and the length of the telescope thus slightly less than the sum of the focal lengths of the object glass and eye-piece. If a near object is viewed, the distance of *ab* from O is greater than the focal length of the object glass, and in this case the length of the telescope is greater than the sum of the focal lengths of the object glass and eye-piece.

It will be noticed that the image, *ab*, is inverted, and that, since A′B′ is an erect image of *ab*, the image seen on looking through the instrument is inverted. This is immaterial in astronomical observations, but for terrestrial purposes it is necessary to have an erect image. Therefore, in order to adapt an astronomical telescope to ordinary use, it must be fitted with an *erecting eye-piece* (see Art. 12).

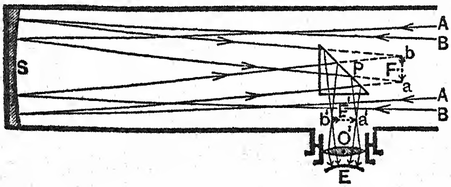


FIG. 172.

5. The Reflecting Telescope

This assumes many forms. The earliest described was devised by Gregory in 1663. In the most common, or **Newtonian,** form, devised in 1668, the end of the tube turned towards the object is open (Fig. 172), and at the other end is a concave mirror, or *speculum*, S, of glass silvered on the front surface. This, if it were allowed to do so, would form at its principal focus a real image, *ab*, of the object to which it is directed. But, before reaching the focus, the rays fall on a total reflection prism (see page 178) which diverts the image to *a′b′*, where it is examined through an eye-piece, E, in the side of the tube.

In another form of the instrument a plane mirror takes the place of the prism, but the latter is the better arrangement.

A great advantage of a reflecting telescope over a refracting telescope is that no chromatic effects are introduced at reflection. Also, if required for observation on celestial bodies, aberration can be eliminated completely by using a paraboloidal mirror (see page 57), so that only the eye-piece needs correction.

6. Why Astronomical Telescopes are Made Large

When an image is highly magnified, it is, of course, reduced proportionately in brilliance. Many terrestrial, and most celestial, objects are themselves comparatively faint. It is desirable, therefore, that the telescope should be able to grasp as much light as possible from the object. This is the reason for the employment of object glasses or specula of large diameter or *aperture*, for the light-grasping power is proportional to the square of the diameter. By this means, many stars which are invisible to the naked eye are revealed. A long focal length, and therefore a long tube, is used to give great magnifying power, and also to decrease aberration by keeping the curvature of the refracting or reflecting surfaces low.

As a rule, astronomical telescopes are provided with several eye-pieces of different focal lengths. This allows the observer a range in magnifying power which is most convenient, as some objects require a greater magnification than others.

Since light is a wave motion, the focus of a beam or the image of an object is simply the place where all the secondary waves reinforce each other (see page 279). It follows, therefore, that the image of a point source of light will always possess a finite size. By increasing the aperture of the object glass or mirror, it may be proved that, quite irrespective of the focal length, the dimensions of the images of point sources are decreased in the inverse ratio—hence the advantage of object glasses and mirrors of large aperture for stellar observations. An increase of magnifying power decreases the brightness of images of extended objects, for the field of view is then occupied by a smaller portion of the object. It increases, however, the relative brightness of images of point sources, such as stars, for in this case, although the actual image is not made brighter, the brightness of the surrounding field is decreased.

Refracting telescopes give better results than reflecting telescopes of the same size, but owing to the immense care required in the manufacture of good lenses more than two feet in diameter, most very large telescopes are reflectors. Rosse's large reflector erected at Parsonstown, Ireland, in 1845, has a mirror 6 ft. in diameter and 56 ft. focal length. The 200-inch mirror, recently constructed for the Mount Wilson Observatory, America, is the largest in existence. The largest refracting telescopes are very costly and are to be found chiefly in America. The Lick and Yerkes instruments have apertures of 36 and 40 in. respectively, and focal lengths of 58 and 62 ft. respectively.

7. Reading or Observing Telescope

A small astronomical telescope is very useful for reading distant scales, thermometers, etc. A long tube, A (Fig. 173), carries an achromatic object glass, O, at one end. A tube, B, slides into A, another tube, C, into B, and a Ramsden's eye-piece, D (see Art. 12), into C. At F is a fixed ring, S, carrying cross-wires. The tubes, B, C, D, can be moved by turning the milled head, H, the so-called *focusing screw*. Thus the distance between the objective, O, and the cross-wires can be adjusted without affecting the distance between the eye-piece and the cross-wires.

To use the telescope, the tube, D, is first adjusted by sliding it in or out of C, until the cross-wires appear clear and distinct to an eye placed at E. The instrument is then directed towards the object, and the milled head, H, is turned until the image of the object and the cross-wires appear distinct *simultaneously*. Also, the eye is moved about in front of the eye-piece, as there must be no movement of the image relatively to the cross-wires—that is, no parallax.

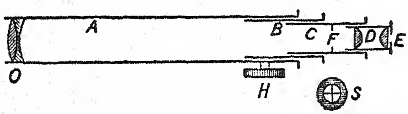


FIG. 173.

8. Galileo's Telescope

This form of telescope was devised by Galileo in 1609. It consists, like the astronomical telescope, of an object glass and eye-piece. The object glass is exactly similar to that of the astronomical telescope, but the eye-piece is simply a double concave achromatic lens placed between the object glass and its principal focus, and at a distance from this focus equal to or slightly greater than its own focal length. The optical action of this arrangement may be explained as follows:—

In Fig. 174, O represents the object glass, having its principal focus at F, and O′ the eye-piece, placed so that the distance, O′F, is equal to or slightly greater than its focal length. Suppose that O′F is equal to the focal length of O′, and that F is therefore the common focus of O and O′. The rays, A, A, A, and B, B, B, coming from points, A and B, of a distant object, would, if uninterrupted, form an image, *ab*, at a point very near to F. These rays, however, falling on O′, are refracted so as to diverge from the points, A′, B′. A virtual erect image of AB is thus seen at A′B′ by

an eye at E. The position of this image depends upon the position of O′ relative to *ab*. If the distance from O′ to *ab* is equal to the focal length of O′, then A′B′ is at infinity, but if this distance is greater than the focal length of O′, then A′B′ is nearer, and, by adjusting the position of O′, may be brought to the nearest point of distinct vision, as in the figure. For the purpose of effecting easily this adjustment, the eye-piece is mounted in a sliding tube, D.T., called a draw tube, which slides in the main tube, M.T., of the telescope.

The chief advantages of this form of telescope over the astronomical are that the construction of the instrument is simplified, its length is reduced, and an erect image is obtained directly without the aid of a special eye-piece. On the other hand, the disadvantages are that the errors of aberration are imperfectly remedied, cross-

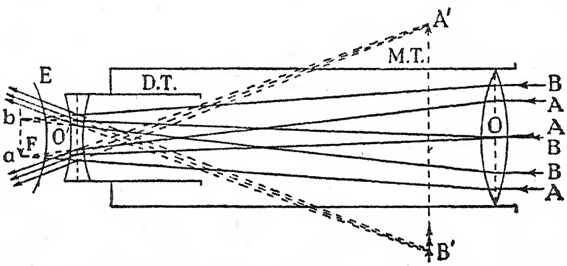


FIG. 174.

wires cannot be used for the reason explained in connexion with Huyghen's eye-piece (see Art. 12), and the magnifying power and field of view are very limited. This last disadvantage arises from the fact that the eye-ring (see page 212) is virtual, and therefore lies inside the instrument, so that only a portion of the rays diverging from it can be received by an eye placed at the eye-piece. For these reasons, Galileo's telescope is best adapted for observation of terrestrial objects, and where only a small magnification is required.

9. The Magnifying Power of a Telescope

In order to determine the magnifying power of a telescope, it is necessary only to obtain the ratio of the visual angle (see page 197) of the image to that of the object, the latter being seen at its actual distance from the eye. If this distance is great compared with the length of the telescope, then AOB (Fig. 171) is very approximately

the angle which the object, AB, subtends at the eye, E. Similarly, A′O′B′ is the angle which the image, A′B′, subtends, and the magnifying power is given by the ratio, $\frac{A'OB'}{AOB}$. But the angle, AOB, is equal to the angle, aOb, and the angle, A′O′B′, is identical with the angle, $aO'b$ (cf. Figs. 100 and 158). Hence, $\frac{A'OB'}{AOB} = \frac{aO'b}{aOb}$, and, the angles involved being small, the magnifying power is approximately equal to

$$\frac{ab}{NO'} \Big/ \frac{ab}{NO} = \frac{NO}{NO'}.$$

Now if the object is very distant, the image, ab, is formed close to the principal focus of the object glass, and, if the position of the eye-piece, O′, is adjusted so that ab is very near to its principal focus, then F and F′ coincide with N, which then becomes the *common* focus of O and O′. Then, **the magnifying power is given by the ratio of the focal length of the object glass to the focal length of the eye-piece.** Thus, if m denote the measure of the magnifying power, F_1 the numerical value of the focal length of the object glass, and f_1 the numerical value of the focal length of the eye-piece, then

$$m = \frac{F_1}{f_1}.$$

Strictly, m is *negative* because the image is *inverted*, for NO $=$ F, and NO′ $= -f$, so that $m = \text{NO}/\text{NO}' = -\text{F}/f$, whichever sign convention is in use. This, it must be remembered, is true only when the object is very distant, and when the eye-piece is placed so that the image, ab, is at its focus, and the virtual image, A′B′, is at infinity.

Similarly, in the case of Galileo's telescope, as in the case of the astronomical telescope above, the magnifying power is given by the ratio, $aO'b/aOb$ (Fig. 174); that is, if the object viewed is very distant, and the image, A′B′, is at infinity,

$$m = \frac{ab}{O'F} \Big/ \frac{ab}{OF} = \frac{OF}{O'F} = -\frac{F}{f},$$

where F and f are respectively the focal lengths of the object glass and eye-piece (using either sign convention).

If either of these telescopes is focused so that A′B′ is at the nearest distance of distinct vision, the numerical value of m is given by $\frac{F_1}{f_1}\left(1 \pm \frac{f_1}{D}\right)$ where F_1 and f_1 represent numerical values

and D is the least distance of distinct vision. For the final image A′B′ is at a distance, D, from O′, and the distance u of the intermediate image ab from O′ is given by

$$\frac{1}{D} + \frac{1}{f_1} = \frac{1}{u} \text{ for the astronomical telescope,}$$

$$-\frac{1}{D} + \frac{1}{f_1} = -\frac{1}{u} \text{ for Galileo's telescope.}$$

It can be verified that these formulae hold whichever sign convention is in use. For the astronomical telescope u is positive because the image ab is actually formed, and thus acts as a real object for the eye-piece, while for Galileo's telescope ab is not actually formed and thus acts as a virtual object for the eye-piece, u thus being negative with either convention. The magnifying power is now determined by the ratio, F/u, by an argument similar to that just used, which is numerically equal to $\frac{F_1}{f_1}\left(1 + \frac{f_1}{D}\right)$ for the astronomical telescope and $\frac{F}{f_1}\left(1 - \frac{f_1}{D}\right)$. for Galileo's telescope. Thus, with Galileo's telescope, one cannot increase the magnification by bringing the final image closer to the eye.

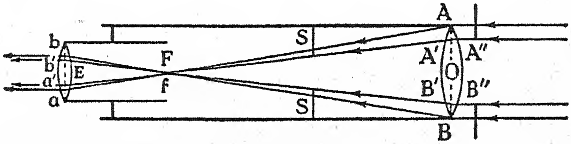


FIG. 175.

Experiment. To find the magnifying power of a telescope.—

(1) Determine F and f by ordinary methods, and substitute in the above relations.

(2) Focus the telescope on a distant object, such as a slate roof or a brick wall. Arrange that the image is formed in the plane of the object, so that all parallax is avoided. With one eye at the telescope and the other unassisted, note how many slates or bricks as seen by the unaided eye occupy the same length as one slate or brick seen through the telescope. This number is the magnifying power.

(3) Focus the telescope for infinity, and then point it towards a bright cloud or a strongly illuminated surface. Observe the bright circle on the eye-piece. Measure its diameter, ab (Fig. 175),

by means of a fine scale, and also the diameter, AB, of the object glass. Then, clearly,

$$m = \frac{F_1}{f_1} = \frac{AB}{ab}.$$

Nearly all telescopes contain stops (see pages 57, 123) whose function is to cut off the marginal rays proceeding from the object glass. Thus the presence of S (see Fig. 175) virtually diminishes the diameter of the object glass from AB to A′B′, with a proportionate decrease from ab to $a'b'$ of the diameter of the circle of light on the eye-piece. To eliminate this error, place a wide adjustable slit in front of the object glass and narrow it down until its shadow begins to encroach on the circle, $a'b'$. Then carefully adjust it so as just not to encroach. The ratio, $A'B'/ab$, is the magnifying power.

(4) Focus the telescope for infinity, and point it towards a bright cloud. Remove the object glass, and obtain a real image of the circular opening in which the object glass fits upon a transparent glass millimetre scale, placed somewhere near E (Fig. 175). Measure the diameter of this circular image, denoted by cd, and measure also AB. Then,

$$\frac{AB}{cd} = \frac{F_1}{f_1} = m.$$

This may be proved as follows:—Let v_1 be the numerical value of the distance of the image from the eye-piece. Then, since the object is at a distance, $F_1 + f_1$, from the lens,

$$\frac{1}{v_1} + \frac{1}{F_1 + f_1} = \frac{1}{f_1}, \text{ from which } v_1 = \frac{f_1(F_1 + f_1)}{F_1}.$$

$$\text{But, } \frac{AB}{cd} = \frac{F_1 + f_1}{v_1}, \text{ and hence } \frac{AB}{cd} = \frac{F_1}{f_1}.$$

In estimating the magnifying power of a telescope fitted with a compound eye-piece, the focal length of the lens equivalent to the eye-piece system (see Art. 10) must be taken.

It will be seen (Fig. 171) that the rays, after emergence from the eye-piece, cross into the plane indicated by E′. The section of the beam in this plane is approximately circular, and, as it marks the proper position of the eye, it is called the *eye-ring*. Telescopes are usually so constructed that the aperture in the cap of the eye-piece indicates the position of the eye-ring, and hence, in looking through a telescope, the eye should be placed close to this aperture. From this figure it will be seen also that the eye-ring, or *bright spot* as it is

sometimes called, is the image of the object glass formed by the eye-piece, and it follows from (4) above that the ratio of the diameter or radius of the object glass to that of the bright spot gives the magnifying power of the instrument. The diameter of the object glass is sometimes called the *aperture* of the telescope.

*Example.—*An astronomical telescope is used to view an object placed at 20 yd. distance, and the eye-piece is adjusted for nearest distinct vision of the image. Find the magnification, given that the focal length of the object glass is 2 ft. and that of the eye-piece 4 in. What will be the magnifying power of the instrument when adjusted for normal vision of a very distant object?*

If N (Fig. 171) denote the position of the real image formed by the object glass,

$$\frac{1}{\mathrm{ON}} - \frac{1}{60} = -\frac{1}{2}, \text{ from which } \mathrm{ON} = -\frac{60}{29} \text{ ft.}$$

Also, if the least distance of distinct vision be taken as 10 in., then

$$\frac{1}{10} - \frac{1}{\mathrm{O'N}} = -\frac{1}{4}, \text{ from which } \mathrm{O'N} = \frac{20}{7} \text{ in.} = \frac{5}{21} \text{ ft.}$$

But the magnifying power is given by the ratio, $\frac{\mathrm{ON}}{\mathrm{O'N}}$, and therefore

$$m = \frac{\mathrm{ON}}{\mathrm{O'N}} = -\frac{60}{29} \times \frac{21}{5} = -\frac{252}{29} = -8\tfrac{20}{29}.$$

That is, the image of the object is inverted, and appears about $8\frac{3}{4}$ times as great as the object itself.

When adjusted for normal vision of a very distant object, the magnifying power is given approximately by—

$$m = \frac{F}{f} = \frac{24}{4} = 6 \text{ numerically.}$$

It should be noticed from this example that the magnifying power of a telescope varies with the distance of the object and with the adjustment of the eye-piece. *

10. Equivalent Lens

The image formed by a single lens is subject to several defects, due to spherical and chromatic aberrations. To remedy these defects, it has been found necessary in optical instruments to employ achromatic lenses, or combinations of two or more ordinary lenses arranged along a common axis.

To understand the actions of such systems of lenses, it is necessary first to understand what is meant by an equivalent lens. A lens is said to be *equivalent* to a combination of lenses when it produces the same *deviation* in a ray incident parallel to its principal axis as

that produced by the combination, the equivalent lens being placed in the position of the first lens on which the light falls.

Let PQ (Fig. 176) represent a ray incident at Q on the lens, L, in a direction parallel to the principal axis. Then, if F represent the principal focus of the lens, this ray is refracted along QF, and the deviation produced is measured by the angle, P′QF —that is, by the angle, QFO. If this angle is small, it is approximately equal to its tangent, OQ/OF. Thus, *if a ray parallel to the axis of a lens of focal length, f, be incident on it at a distance, x, from the axis, the deviation produced is given approximately by the ratio,* x/f. This is also approximately true when the inclination of the rays to the axis is small.

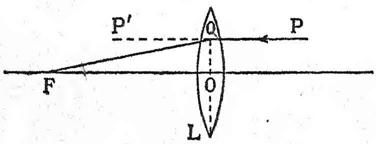


FIG. 176.

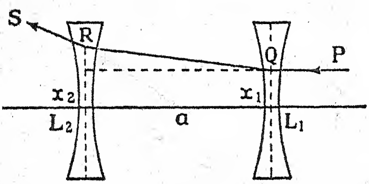


FIG. 177.

*Consider now a system of two lenses placed at a distance, a, apart on a common axis. It is required to determine the focal length of the combination. To simplify matters by having all the quantities involved positive, consider two concave lenses, L_1 and L_2 (Fig. 177), of focal lengths, f_1 and f_2 respectively. Then, the deviation of the ray, PQ, produced by L_1 is equal to x_1/f_1, and the deviation of QR by L_2 is approximately equal to x_2/f_2. Thus, the total deviation is given by

$$\frac{x_1}{f_1} + \frac{x_2}{f_2}.$$

Since the deviation of PQ produced by L_1 is measured by $\frac{x_1}{f_1}$ we have $x_2 = x_1 + \frac{ax_1}{f_1}$,

$$\text{i.e. } x_2 = x_1 + a.\frac{x_1}{f_1} = x_1\left(1 + \frac{a}{f_1}\right).$$

Therefore the total deviation is given by

$$x_1\left(\frac{1}{f_1} + \frac{f_1 + a}{f_1 f_2}\right) = x_1\left(\frac{1}{f_1} + \frac{1}{f_2} + \frac{a}{f_1 f_2}\right).$$

But, if F denote the focal length of the equivalent lens, then

the deviation which would be produced by it is x_1/F. Thus, by definition,

$$x_1 \cdot \frac{1}{F} = x_1 \left(\frac{1}{f_1} + \frac{1}{f_2} + \frac{a}{f_1 f_2} \right);$$

$$\therefore \frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{a}{f_1 f_2}.$$

The focal length of the equivalent lens is thus determined in terms of the focal lengths of the lenses of the combination and the distance between them. If a be zero—that is, if the lenses are in contact—then the relation becomes

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2},$$

which is identical with the result obtained previously (see page 121).

Example.—*Find the focal length of a single lens equivalent to a combination of a convex lens of* 8 *in. focal length and a concave lens of* 12 *in. focal length, placed* 4 *in. apart on a common axis.*

Applying the relation, $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{a}{f_1 f_2}$,

$f_1 = -8$ in., $f_2 = +12$ in., $a = +4$ in.

Hence, $\frac{1}{F} = -\frac{1}{8} + \frac{1}{12} + \frac{4}{-8 \times 12}$,

and $F = -12$ in.

Thus, a *convex* lens of 12 in. focal length is equivalent to the given combination.

The above relations are general, and if due regard be paid to the signs of f_1 and f_2, a being always positive, they may be applied to determine F for a combination of any two lenses. The image of an object produced by the equivalent lens is of the same *size* as that produced by the combination, but its position is not necessarily the same.

The focal length of a single lens which, placed at the position of the second lens, shall produce an image of a distant object in the same *position* as that of the image produced by the combination, may be determined as follows:—

$$\frac{1}{v_1} - \frac{1}{\infty} = \frac{1}{f_1} \text{ or } \frac{1}{v_1} = \frac{1}{f_1}.$$

$$\text{Also, } \frac{1}{v_2} - \frac{1}{a + v_1} = \frac{1}{f_2},$$

$$\text{From which } \frac{1}{v_2} - \frac{1}{a + f_1} = \frac{1}{f_2}; \quad \therefore \frac{1}{v_2} = \frac{1}{a + f_1} + \frac{1}{f_2}.$$

But v_2 is by hypothesis equal to F, the focal length of the single lens, and

$$\frac{1}{F} = \frac{1}{a + f_1} + \frac{1}{f_2}.$$

The difference between this result and the one above should be noticed.

*

11. Object Glasses and Eye-Pieces

When the refraction through a single lens is *centrical*—that is, when the axis of the incident pencil passes through the *centre* of the lens—the defects due to chromatic aberration (see page 155) are by far the most important. When, however, the refraction is *excentrical*—that is, when the axis of the incident pencil does not pass through the centre of the lens—the defects due to spherical aberration (see page 122) also become serious, and produce indistinctness and distortion of the image.

In a telescope, from the nature of its construction (Fig. 171), the refraction through the object glass is centrical, while that through the eye-piece is excentrical. Hence, in order that the final image, A′B′, may be clear and distinct, the object glass must be corrected for chromatic aberration, and to a slight degree for spherical aberration, while the eye-piece must be corrected for both chromatic and spherical aberrations.

(1) THE OBJECT GLASS.—In the case of the object glass it is essential, in order that the refraction may be centrical through the compound lens, that it should be made up of *thin* lenses *in contact*. Hence, it usually consists of two thin lenses, a convex lens of crown glass and a concave lens of flint glass, of such focal lengths that the combination is *achromatic*. The possibility of constructing such a lens has been indicated already (see page 155), and the conditions of achromatism will now be considered more definitely.

The general relation, $\frac{1}{f} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right)$, has been established whichever sign convention is in use (pages 104 and 123*c*). In this relation, f is the focal length for the mean rays of the spectrum—that is, the focal length in the ordinary meaning of the term. Hence, if fv denote the focal length of a lens for violet rays,

$$\frac{1}{f_v} = (\mu_v - 1)\left(\frac{1}{r} - \frac{1}{s}\right) = \frac{\mu_v - 1}{\mu - 1}\cdot\frac{1}{f},$$

where μ_V is the refractive index of the material of the lens for violet rays. Similarly, for red rays,

$$\frac{1}{f_R} = \frac{\mu_R - 1}{\mu - 1} \cdot \frac{1}{f},$$

where μ_R is the refractive index for red rays. Hence

$$\frac{1}{f_V} - \frac{1}{f_R} = \frac{\mu_V - \mu_R}{\mu - 1} \cdot \frac{1}{f} = \frac{\omega}{f},$$

where ω is the dispersive power of the material of the lens.

Now $f_V f_R = f^2$ approximately,

and the preceding expression may be written

$$\frac{f_R - f_V}{f_R f_V} = \frac{\omega f}{\omega f^2};$$

$$\therefore f_R - f_V = \omega f.$$

If F denote the focal length of a combination of two lenses in contact, of focal lengths, f' and f'', then

$$\frac{1}{F} = \frac{1}{f'} + \frac{1}{f''}$$ (see page 121).

$$\text{Hence, } \frac{1}{F_V} - \frac{1}{F_R} = \left(\frac{1}{f'_V} + \frac{1}{f''_V}\right) - \left(\frac{1}{f'_R} + \frac{1}{f''_R}\right)$$

$$= \left(\frac{1}{f'_V} - \frac{1}{f'_R}\right) + \left(\frac{1}{f''_V} - \frac{1}{f''_R}\right);$$

$$\therefore \frac{1}{F_V} - \frac{1}{F_R} = \frac{\omega'}{f'} + \frac{\omega''}{f''},$$

where ω' and ω'' are the dispersive powers of the materials of which the lenses are made.

In order that the lens may be achromatic, however, the condition, $1/F_V - 1/F_R = 0$, must hold—that is:—

$$\frac{\omega'}{f'} + \frac{\omega''}{f''} = 0, \text{ or } \frac{\omega'}{\omega''} = -\frac{f'}{f''}.$$

These are the required conditions of achromatism, and indicate that *the dispersive powers of the materials of the two lenses in contact must have the same numerical ratio as their focal lengths*, and also, since ω' and ω'' are essentially positive, f' and f'' must be of opposite sign—that is, *one lens must be convex and the other concave.*

As already stated, in practice the convex lens is of crown glass and the concave lens of flint glass. To correct the object glass for defects of spherical aberration, *the curvatures of the surfaces of the lenses are modified, so as to reduce these defects as far as possible.*

Example.—*The dispersive power of crown glass is about* 0·03, *and that of flint glass about* 0·05. *Show how to construct a converging achromatic lens of* 60 *cm. focal length.*

From the relation, $\frac{f'}{f''} = \frac{\omega'}{\omega''}$, we have:—

$$\frac{f'}{f''} = -\frac{0{\cdot}03}{0{\cdot}05} = -\frac{3}{5}, \text{ or } f'' = -\tfrac{5}{3}f',$$

where f' and f'' are respectively the focal lengths of the crown glass and flint glass lenses.

Also, from the relation, $\frac{1}{F} = \frac{1}{f'} + \frac{1}{f''}$,

$$-\frac{1}{60} = \frac{1}{f'} + \frac{1}{f''}.\dagger$$

Substituting, $-\frac{1}{60} = \frac{1}{f'} - \frac{3}{5f'} = \frac{2}{5f'}$,

from which $5f' = -120$;

$$\therefore f' = -24 \text{ cm.}$$

Also $f'' = -\tfrac{5}{3}f' = -\tfrac{5}{3} \times -24 = 40$ cm.

Therefore, the compound lens must be made up of a *convex* lens of *crown* glass of focal length 24 cm., and a *concave* lens of *flint* glass of focal length 40 cm.

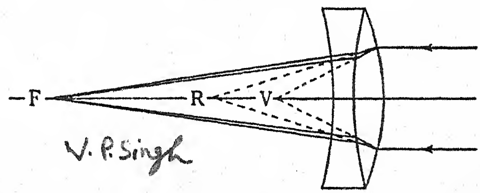
An achromatic lens of this kind is perfectly achromatic only for the two colours chosen—in the above case violet and red. Owing to the irrationality of dispersion (see page 154), the conditions for combining any two rays are not exactly those required for the combination of all the colours, but, by properly choosing the two colours to be combined, the combination may be made approximately achromatic for all colours. The colours chosen for combination are not the red and violet, but two colours from a brighter part of the spectrum, a ray from the orange-yellow being generally combined with a ray from the green-blue.

By using three or more lenses of different dispersive powers, a combination can be obtained which will be achromatic for three or more colours, according to the number of lenses employed. The necessary condition is that $\Sigma\frac{\omega}{f}$ for all the lenses shall be zero.

† Using the Real is Positive convention, we have F = + 60 cm., and we obtain $f' = +24$ cm. and $f'' = -40$ cm., giving the same result.

The action of an achromatic lens composed of a double convex lens of crown glass cemented by Canada balsam to a double concave lens of flint glass is shown diagrammatically (Fig. 178). The parallel rays of light falling on the combination first undergo refraction and dispersion by the convex lens, and, before entering the concave lens, converge to a series of foci lying between V and R. On refraction through the concave lens, however, the dispersion is corrected, and the rays converge to a common focus, F. The spherical aberration of such a lens is diminished by making the front surface of the convex lens of greater curvature than the back surface of the concave lens. This back surface may be either plane or convex outwards depending on the conditions of the case.

(2) THE EYE-PIECE.—(*a*) *Chromatic Aberration.* In the case of an eye-piece, the refraction is excentrical, and it is therefore of no advantage to place the lenses of the combination in contact. Moreover, the errors due to spherical aberration can be corrected more easily and completely in a system made up of separate lenses at fixed distances apart, and it is also possible by this arrangement to obtain achromatism with lenses of the *same* material—that is, of the same dispersive power. For ordinary purposes, two lenses are sufficient, and, in order to deduce the conditions of achromatism for such a system, the general condition determined above must be applied to the relation obtained in Art. 10.



V. P. Singh

FIG. 178.

*The focal length of a lens equivalent to a system of two lenses of focal lengths, f' and f'', arranged on the same axis at a distance, a, apart, is given by

$$\frac{1}{F} = \frac{1}{f'} + \frac{1}{f''} + \frac{a}{f'f''}.$$

$$\text{For achromatism, } \frac{1}{F_V} - \frac{1}{F_R} = 0.$$

$$\text{Now, } \frac{1}{F_V} = \frac{1}{f'_V} + \frac{1}{f''_V} + \frac{a}{f'_V f''_V};$$

$$\therefore \frac{1}{F_V} = \frac{\mu_V - 1}{\mu - 1}.\frac{1}{f'} + \frac{\mu_V - 1}{\mu - 1}.\frac{1}{f''} + \left(\frac{\mu_V - 1}{\mu - 1}\right)^2.\frac{a}{f'f''},$$

$$\text{and } \frac{1}{F_R} = \frac{\mu_R - 1}{\mu - 1}.\frac{1}{f'} + \frac{\mu_R - 1}{\mu - 1}.\frac{1}{f''} + \left(\frac{\mu_R - 1}{\mu - 1}\right)^2.\frac{a}{f'f''};$$

$$\therefore \frac{1}{F_V} - \frac{1}{F_R} = \frac{\mu_V - \mu_R}{\mu - 1}.\frac{1}{f'} + \frac{\mu_V - \mu_R}{\mu - 1}.\frac{1}{f''} + \frac{(\mu_V - 1)^2 - (\mu_R - 1)^2}{(\mu - 1)^2}.\frac{a}{f'f''}.$$

$$\text{Now } \frac{(\mu_V - 1)^2 - (\mu_R - 1)^2}{(\mu - 1)^2} = \frac{(\mu_V + \mu_R - 2)(\mu_V - \mu_R)}{(\mu - 1)^2}$$

$$= \frac{(2\mu - 2)(\mu_V - \mu_R)}{(\mu - 1)^2} \text{ approx.}$$

$$= 2.\frac{\mu_V - \mu_R}{\mu - 1}.$$

$$\text{Therefore } \frac{1}{F_V} - \frac{1}{F_R} = \omega\left(\frac{1}{f'} + \frac{1}{f''} + \frac{2a}{f'f''}\right),$$

where ω is the dispersive power of the material of the lenses. Thus for achromatism,

$$\frac{1}{f'} + \frac{1}{f''} + \frac{2a}{f'f''} = 0,$$

$$\text{From which } f' + f'' + 2a = 0;$$

$$\therefore f' + f'' = -2a,$$

$$\text{and therefore, } a = -\frac{f' + f''}{2}.$$

Hence, in order that a sytem of two lenses of the same material may be approximately achromatic, *the distance between them must be equal to half the sum of their focal lengths*, and, since a is positive, it is evident, from the relation obtained above, that the sum of the focal lengths must be negative—that is, *the combination must be equivalent to a convex lens.*

(*b*) *Spherical Aberration.* It is further necessary to consider the most favourable conditions for correcting the defects due to spherical aberration. It is evident that the greater the refraction a ray undergoes at any surface, the more apparent will the errors of aberration become. Hence, it can be shown that, by dividing

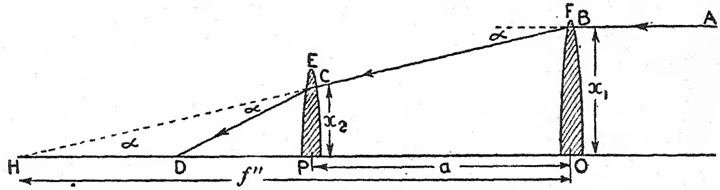


FIG. 179.

the total refraction equally between the two lenses, the errors are reduced to a minimum (see page 92).

Let E and F (Fig. 179) be the two lenses of focal lengths, f' and f'', respectively, and separated by a distance, a. A ray, AB, parallel to the axis, undergoes a deviation, α, at the lens, F, and is refracted towards H, the principal focus of F. At C it strikes the lens, E, and undergoes, by hypothesis, an equal deviation, α, being refracted towards the point, D. Since AB is parallel to OH, the angle, BHO, is equal to α, and therefore DC = DH. In the limit, when AB is very near to OH, DP will be approximately equal to DC, and thus to DH. Therefore a ray converging to a point, H, distant HP from the lens, E, is made to converge to a point, D, only half as far from the lens. Hence:—

$$\frac{1}{\frac{1}{2}\text{PH}} - \frac{1}{\text{PH}} = \frac{1}{f'}, \text{ from which PH} = -f'.$$

$$\text{But OH} = -f'';$$

$$\therefore\ a = f' - f''.$$

Thus, *the distance between the two lenses must be equal to the difference between their focal lengths.* *

When this is the case, the defects of spherical aberration are reduced greatly, for, although the total deviation produced by the two lenses is the same as that which a single lens would be required to produce, yet the aberration at each lens is reduced in a greater ratio than the deviation, and thus the total aberration produced by the two lenses is much less than would be produced by a single lens. The aberration can be corrected to a further slight extent by adaptation of the curvatures of the surfaces of the lenses. The theory of this portion of the subject, however, is difficult and the arrangements adopted are mainly empirical.

(*c*) *Combining the Two Conditions.* Having now deduced the conditions for achromatism and minimum spherical aberration, it becomes necessary to consider whether it is possible to combine

both conditions in one system of lenses. For two lenses of focal lengths, f' and f'', separated by a distance, a, the conditions are†

$$(1)\ a = -\frac{f' + f''}{2}, \text{ for achromatism,}$$

$$(2)\ a = f' - f'', \text{ for minimum spherical aberration.}$$

Hence, to satisfy both conditions,

$$f'' - f' = \frac{f' + f''}{2};$$

$$\therefore f'' = 3f'.$$

Thus, *the focal length of one lens must be three times the focal length of the other, and the distance between them must be equal to the difference between their focal lengths.* The image formed by such a system will be almost free from all defects due to chromatic and spherical aberration.

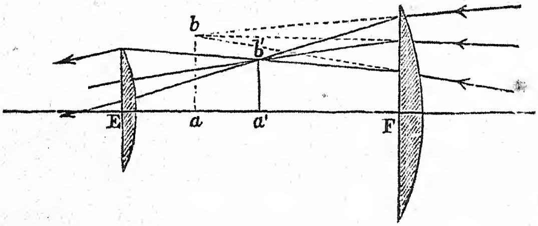


FIG. 180.

12. Special Eye-Pieces

(*a*) HUYGHENS' EYE-PIECE.—An eye-piece satisfying the conditions deduced above (Art. 11) was constructed by Huyghens. It was devised to satisfy the condition of minimum spherical aberration, and it was shown later by Boscovich that it was also achromatic. This eye-piece usually consists of two plano-convex lenses, the plane surfaces being next the eye of the observer. The lens nearest the object glass and farthest from the eye is called the *field-lens*, and the lens nearest the eye the *eye-lens*. The focal length of the former is three times that of the latter. An important function of the field-lens is to concentrate the light on the central portion of the eye-lens, so that the issuing beam has a small cross-section. Thus, pencils emanating from widely separated parts of the object can enter the pupil of the eye at the same time.

† Using the Real is Positive convention, $a = \frac{f' + f''}{2}$, and $a = f'' - f'$, so that $f'' = 3f'$.

* Let E (Fig. 180) represent the eye-lens, and F the field-lens. Rays of light coming from the object glass of the telescope, if uninterrupted, would meet at b, but, falling on the field-lens, they are refracted through b', which should be at the focus of the eye-lens in order that the rays may emerge parallel. To determine the position of ab, in order that $a'b'$ may be at the focus of E,

$$\frac{1}{Fa'} - \frac{1}{Fa} = \frac{1}{3f},$$

where f is the focal length of the eye-lens.

Also, if $a'b'$ is at the principal focus of E, then $Ea' = -f$, and, since $EF = -2f$, then $a'F = -f$, or $Fa' = f$. Hence:—

$$\frac{1}{f} - \frac{1}{Fa} = \frac{1}{3f},$$

$$\text{or } \frac{1}{Fa} = \frac{1}{f} - \frac{1}{3f} = \frac{2}{3f};$$

$$\therefore Fa = \tfrac{3}{2}f. \quad *$$

Thus, the rays coming from the object glass, if uninterrupted, should meet at a point behind the field-lens, or in front of the eye-lens, at a distance from either lens equal to half the focal length of the lens considered. If the object viewed is very distant, then this may be expressed by saying that the focus of the object glass should lie between the lenses of the eye-piece, at a distance from either lens numerically equal to half the focal length of that lens.

It will be seen from what has been said above that the rays from the object glass converge to a point behind the field-lens, and therefore the image, ab, has no *real* existence, the rays being refracted by F to form the image, $a'b'$. For this reason Huyghens' eye-piece has been called a *negative* eye-piece.

The conditions of achromatism of this eye-piece have been deduced by applying the general conditions of achromatism to the relation giving the focal length of a single lens *equivalent* to the system of lenses forming the eye-piece. Now this relation for the equivalent lens has been obtained in accordance with the definition of equivalence given above (Art. 10), and implies only equal deviations by the system and its equivalent. Hence, the rays for which the eye-piece is achromatic will undergo equal deviations, and therefore, on emergence, will be parallel, *but not necessarily coincident.*

However, this implies that images formed by light of different colours will subtend the same *angle* at the eye, so that the red and blue light will ultimately reach the same portion of the retina.

Although it could be proved, by a laboratory experiment, that the red and blue *images* are not coincident, the eye will not be able to appreciate this directly. The action of the eye-piece as an achromatic system is illustrated in Fig. 181. As explained above (page 220), the relation between the focal lengths and the distance implies equal deviation for both red and violet light, and thus the two images subtend equal angles at an eye close to the eye-piece. Strictly speaking, the distance between the lenses can be equal to the mean of the numerical focal lengths for light of one colour only, so that only rays of nearly that colour would be strictly parallel, because the focal lengths change slightly with the colour of the light. In practice, however, this is of minor importance, as the images of various colours subtend very nearly the same angles at the eye, and the system is a great improvement on a single lens.

(*b*) RAMSDEN'S EYE-PIECE.—The eye-piece just described, though perfect theoretically, cannot be used in telescopes where measurements by the aid of cross-wires are made. Such measurements are effected by means of a frame carrying cross-wires of fine platinum wire, silk thread, or spider-threads, in the focal plane of the object glass. The real image of the object formed in this plane is coincident with these threads, and, when viewed through a suitable eye-piece, both image and cross-wires are magnified to the same extent, and any slight distortion produced by refraction through the lenses of the eye-piece is the same for both. Hence, points at which the real image and the cross-wires actually coincide are also the points seen to be coincident on looking through the eye-piece. The actual distance between any two points on the real image may be measured readily by this arrangement. The frame carrying the cross-wires can be moved in its own plane by a micrometer screw fitted with a graduated head, and, by bringing a chosen cross-wire into coincidence first with one point and then with the other, the distance between the two points can be read off in terms of the graduations on the micrometer head. This linear distance is converted readily into angular distance, for, if F be the

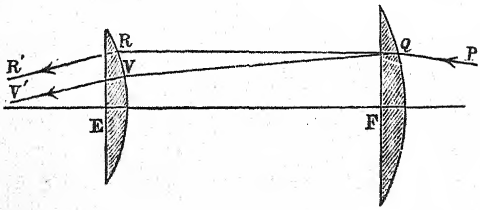


FIG. 181.

focal length of the object glass and d the given distance, then the required angular distance is $\frac{d}{F}$ approximately.

Now it is evidently impossible to adopt the arrangement just described with Huyghens' eye-piece, for the real image of the object by the object glass is not actually formed, and the cross-wires could not be placed at $a'b'$ (Fig. 180), because points on $a'b'$ have not necessarily the same relative positions as the corresponding points of ab would have. For purposes of measurements, therefore, astronomical telescopes are usually fitted with what is known as Ramsden's eye-piece, sometimes called a *positive* eye-piece.

This eye-piece is usually made up of two plano-convex lenses of equal focal length, placed with their convex surfaces facing each other, and separated by a distance equal to two-thirds of the focal length of either.

The conditions of achromatism require that the distance between the lenses should be equal to the focal length of either. Numerically, $a = \frac{f' + f''}{2} = \frac{2f'}{2} = f'$, in this case (see Art. 11). However, if this arrangement were adopted, the field-lens would be at the focus of the eye-lens, and therefore would interfere with distinct vision, especially if the glass were not quite clear, or if dust happened to lie on its surface. For this reason, the distance between the lenses is reduced to two-thirds of the focal length of either, and, although the system thus arranged is not perfectly achromatic, it is very nearly so.

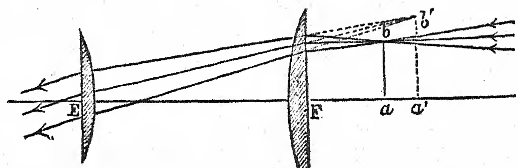


FIG. 182.

*Let E and F (Fig. 182) represent the eye-lens and field-lens respectively of a Ramsden's eye-piece. Rays coming from the object glass converge to a focus at b in front of the field-lens, F, and passing through b, fall on F, where they are refracted. After passing through F the rays diverge from b', which, in order that the rays may emerge parallel from E, should be in the focus of that lens. Thus, Ea' should be equal to $-f$, where f is the focal length of either lens.

$$\text{But EF} = -\tfrac{2}{3}f, \text{ and therefore F}a' = -\tfrac{1}{3}f.$$

Hence, to determine the position of *ab*,

$$\frac{1}{Fa'} - \frac{1}{Fa} = \frac{1}{f}, \text{ and } -\frac{3}{f} - \frac{1}{Fa} = \frac{1}{f},$$

$$\text{from which } \frac{1}{Fa} = -\frac{3}{f} - \frac{1}{f} = -\frac{4}{f};$$

$$\therefore \ Fa = -\frac{f}{4}.$$

Thus, the real image formed by the object glass should fall in front of the field-lens, at a distance from it numerically equal to one-fourth its focal length. This is also the correct position for the frame carrying the cross-wires. *

This eye-piece does not satisfy the conditions of minimum spherical aberration, but the curvatures of the surfaces of the lenses are arranged to remedy this defect as far as possible, and the indistinctness due to this cause is inappreciable.

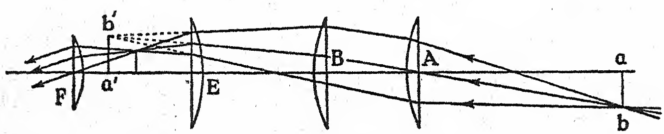


FIG. 183.

(*c*) ERECTING EYE-PIECE.—The only other eye-piece which need be described is the **erecting eye-piece**, referred to above (see Art. 4), which is used to adapt an astronomical telescope for observation of terrestrial objects. It consists of four lenses (Fig. 183). The two nearest the object glass, A and B, are of equal focal length, and placed at any distance from each other. The two nearest the eye, E and F, form a Huyghens' eye-piece.

Rays coming from the object glass meet at *b* in front of the lens, A, and at a distance from it equal to its focal length. Passing through *b*, the rays are refracted through A, and, emerging parallel, fall upon B, which brings them to a focus at *b'*. In this way, an inverted image of *ab*, and thus an erect image of the external object, is formed at *a'b'*, and the system, EF, is adjusted to this image in the way indicated in the diagram, and explained above in (*a*) (see Fig. 180). The four lenses are usually fixed in a tube in their correct relative positions, and the adjustments relative to the image, *ab*, are effected by sliding this tube in and out of the main tube of the telescope.

13. Opera Glasses, Field Glasses, and Prism Binoculars

One of the principal applications of the use of telescopes, as described above, is that employed in the construction of opera glasses, field glasses, and some marine glasses. These forms of the instrument are usually *binocular* (see also Art. 21)—that is, they consist of two telescopes with parallel axes, mounted so that both eyes may be conveniently used in looking at any object.

Opera glasses and *Field glasses* usually consist of two *Galileo's* telescopes, mounted in this way, because of the advantages of this type of telescope, namely that an erect image is obtained directly

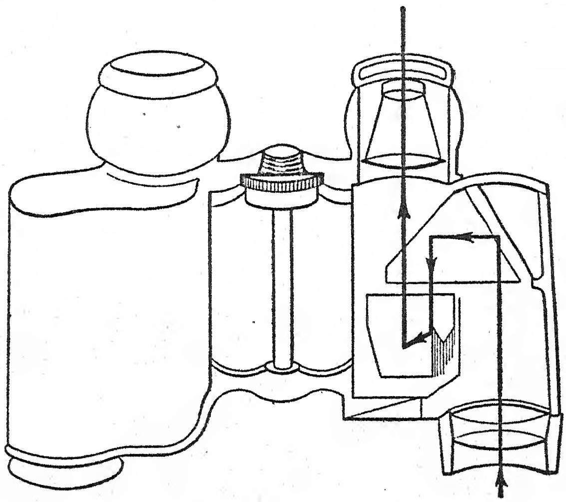


FIG. 184.

without the use of a special eye-piece, and that it is shorter than the corresponding astronomical telescope. In spite of the latter advantage, however, it becomes inconveniently long and cumbersome if a high magnification is required.

Prism binoculars have been designed to overcome this difficulty, and the high magnifying power is obtained by shortening the length of the telescopes, without changing the length of path of the rays, by introducing two right-angled prisms between the object glass and the eye-piece. A modern form of the instrument is illustrated (Fig. 184).

The object glass and eye-piece are both convex, compound systems being used in the more elaborate instruments, but the image is not inverted as in the astronomical telescope, the inversion being completely balanced by arranging the prisms with their axes at right angles. One prism renders the image erect but does not affect the lateral inversion, while the other prism corrects the lateral inversion and so leaves the final image erect and the correct way up.

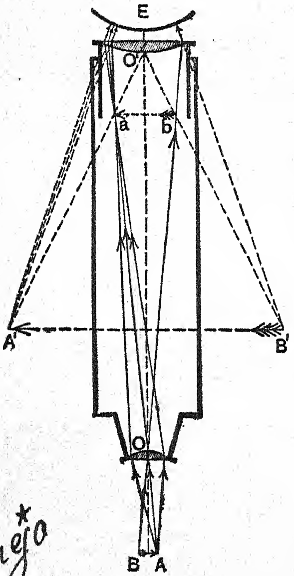


FIG. 185.

14. The Compound Microscope

The compound microscope is exactly similar in principle to the astronomical telescope. The difference between the two instruments results from the adaptation of the object glass, or *objective* as it is called in the case of the microscope, to the purposes of the instrument.

The objective, O (Fig. 185), of a microscope is essentially a convex lens of very short focal length. The small object, AB, to be viewed is placed close to O at a distance slightly greater than the focal length of that lens, and a real image of this object is formed at *ab* in front of the eye-piece, O′. An eye looking through O′ sees a magnified virtual image, A′B′, of this already magnified real image, and, by adjusting the position of O′, this virtual image can be seen at the least distance of distinct vision.

In actual instruments the objective is generally a complicated system of several lenses, constructed and arranged in order to diminish as far as possible the errors of aberration. Owing to the nearness of the object to the objective, the obliquity of the incident pencils of rays is very great, and hence special precautions have to be taken to prevent excessive spherical aberration. The eye-piece is usually an ordinary Huyghens' eye-piece, but when measurements have to be made a Ramsden's eye-piece must be substituted.

Owing to the great magnification produced, the object requires to be illuminated very brilliantly, or the image, A′B′, would be too faint to be of use. Hence, most microscopes are provided with a

reflector, mounted so that it can be placed conveniently so as to concentrate light on the object viewed.

The magnifying power is evidently determined, as in the case of the simple microscope (see page 198), by the ratio, $\frac{A'B'}{AB}$, for both object and image are supposed to be seen at the least distance of distinct vision. Now:—

$$\frac{A'B'}{AB}=\frac{A'B'}{ab}\cdot\frac{ab}{AB}.$$

But, $A'B'/ab$ is the magnification produced by the eye-piece, and is approximately equal to $(1 - D/f)$,† where D is the least distance of distinct vision and f is the focal length of the eye-piece. Also

$$\frac{ab}{AB}=\frac{Oa}{OA}=\frac{v_1}{u_1},$$

where u_1 and v_1 are the *numerical* values of the distances of the object and image ab respectively, from the objective. Hence the magnifying power is given approximately by

$$m=\left(1-\frac{D}{f}\right)\frac{v_1}{u_1}. \dagger$$

In this relation, D, f, and v_1 are constants, for the same adjustment of the same eye-piece. Therefore—

$$m \propto \frac{1}{u_1} \approx \frac{1}{f_0},$$

since v_1 is large compared with u_1.

That is, the magnifying power varies inversely as the focal length, f_0, of the objective.

Experiment. To determine the magnifying power of a microscope.—Focus the microscope on a double scale—one a fine one for observation through the instrument; the other a coarser one for observation with the naked eye. The magnifying power is the true ratio of apparently equal lengths.

Instead of cross-wires, microscopes are often provided with a minutely divided scale in the eye-piece, called a *micrometer scale*, for the purpose of measuring bodies of small dimensions. Before absolute lengths can be obtained, however, the micrometer scale divisions must first be expressed in millimetres. This is done by

† Using the Real is Positive convention the term in brackets should read—

$$\left(1+\frac{D}{f}\right).$$

focusing the microscope on a finely divided scale, called a *stage micrometer*, graduated in millimetres and tenths, and noting how many divisions of the eye-piece scale are apparently equal to a millimetre. The object to be measured is then placed on the stage of the microscope, and its dimensions obtained in terms of the eye-piece scale divisions.

There is no limit to vision. Any particle, however minute, can be seen so long as it can be suitably illuminated. If the dimensions of the particle are much less than half a wave-length of light (see page 316), it is seen only as a whole, and its separate parts cannot be discriminated. Such particles can be seen as bright points if an intense beam of light is focused on them and they are viewed by a microscope with its axis at right angles to the beam of light. This arrangement is called an *ultramicroscope*. In the same way

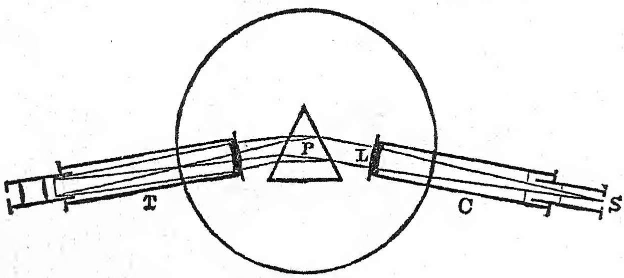


FIG. 186.

dust particles in a beam of light are visible by virtue of the light they scatter towards the eye.

15. The Spectroscope

This is an instrument constructed for the production and careful examination of pure spectra (see page 146).

The instrument consists essentially of three parts—the collimator, the prism, and the telescope. The *collimator*, C (Fig. 186), is a tube like a telescope tube with a *slit*, S, at one end, and a convex lens, L, at the other. The slit is an important part of the instrument. It consists of metal jaws with exactly parallel edges, and its width can be adjusted by means of an attached screw arrangement. The length of the collimator tube is such that, when properly focused, the slit is at the principal focus of the lens. The *prism*, P, is a short prism of glass, or other refracting medium,

of triangular cross-section, similar to those referred to previously (see page 86). The *telescope*, T, is a small astronomical telescope, exactly similar to that described above (Art. 4), the eye-piece being usually fitted with cross-wires.

The arrangement of these three parts is shown in the diagram. The telescope and collimator are attached to a central pillar and table, on which the prism is placed. The source of light whose spectrum is required is placed at S, so that the light from it falls directly on the slit of the collimator. The rays diverging from the slit are refracted through the lens, L, and, emerging parallel, fall upon the prism, P, where they undergo refraction again. Here each ray of the incident beam has the same angle of incidence, and therefore rays of the same refrangibility will be deviated to the same extent. Hence, on emergence from the prism, the rays of each

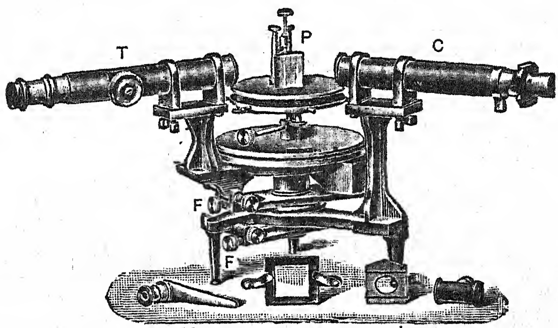


FIG. 187.

constituent of the dispersed beam will be parallel to each other, though not to those of the other constituents. The position of the telescope is adjusted to receive this emergent beam, and, if the eye-piece is focused for parallel rays, an observer looking into the telescope will see a magnified image of the pure spectrum which is formed in the focal plane of the object glass.

16. The Spectrometer

This is a modification of the spectroscope adapted for accurate measurement, one form of the instrument being illustrated (Fig. 187). The collimator, C, is fixed to the base. The telescope, T, is mounted on a rotating piece which carries a circular scale graduated in degrees. The prism, P, is mounted on an adjustable table, which can be levelled by means of three screws, and this in turn

may be raised or lowered and afterwards clamped to the spindle of a circular plate which carries two verniers, 180° apart, sliding alongside the circular scale attached to the telescope. F, F are fine adjustment screws to be used after the prism and telescope are clamped in position. The verniers read to minutes of arc, and for accurate work are read by a small magnifier, M.

ADJUSTMENTS OF THE SPECTROMETER.—Before the spectrometer can be used it must be carefully adjusted. Any time spent on this process will be saved repeatedly by the rapidity and accuracy with which observations can be taken. The adjustments are performed in the following order:—

(1) The eye-piece of the telescope is focused on the cross-wires.

(2) The telescope is focused on a distant object. The eye-piece and cross-wires are in a small separate tube so that this adjustment does not disturb the previous one. The adjustment is correct when, on moving the eye transversely across the eye-piece, the image of the distant object remains at rest relative to the cross-wires.

(3) The collimator and telescope are now brought into a straight line. The slit of the former is illuminated and the adjustable tube of the collimator is moved until the image of the slit, viewed through the telescope, is focused distinctly, the adjustment being tested as in (2).

If a distant object is not available, the following method due to Schuster may be used. Fix a prism with its refracting edge vertical upon the table, and illuminate the slit of the collimator with sodium light. Find the position of the slit at minimum deviation (see page 90), and fix the telescope at about three-quarters of the diameter of the field of view beyond it. Then, on turning the prism round in one direction, the image of the slit moves towards the direction of the incident light, remains still, and then comes back. It may be brought therefore twice to the centre of the field of view. To distinguish between the two positions of the prism when this occurs, call the position when the prism face upon which the light falls is more normal to the incident rays the *normal* position, and the other position the *slanting* position. Place the prism in the slanting position, bring the image to the centre of the field, and focus the telescope until the image is sharp. Rotate the prism to the normal position. Generally the image is out of focus. Adjust the collimator until the image is sharp. Now turn back again to the slanting position and focus the telescope, and then back again to the normal

position and adjust the collimator. After this has been done two or three times, the image will be in focus, without alteration of telescope or collimator, in both positions of the prism, and, when this is the case, the rays leaving the collimator and entering the telescope are parallel.

The proof of this is simple. Since the image of the slit remains in focus, it follows that the virtual image formed by the prism is at the same distance from the telescope in the two positions of the prism; that is to say, the distance between the prism and the virtual image of the slit is not altered by changing the angle of incidence. It can be proved, by calculation or practical geometry, that the distance of the image from the prism varies with the angle of incidence of the rays, except in the one case when the image is at infinity and consequently the incident rays are parallel. The collimator therefore is sending out parallel rays of light, and the telescope is adjusted to focus such rays.

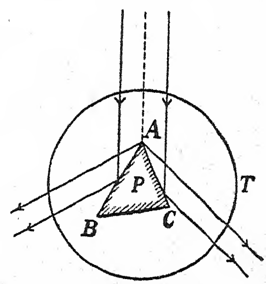


FIG. 188.

17. Experiments with the Spectrometer

The spectrometer is one of the most important instruments used in the study of light, and it is essential that the student should carry out at least the following experiments with the apparatus. Other experiments are described in text-books of Practical Physics (see Bower and Satterly, *Practical Physics*, §§ 150-162).

(1) The refracting angle of a prism is determined easily by means of a spectrometer. Two methods are available, and the agreement of the results obtained by the two methods serves as a test of the accuracy of the instrument and observer.

(*a*) Mount the prism on the table, T (Fig. 188), with the angle, A, which is to be measured pointing towards the collimator, and level T until the refracting edge of the prism is exactly vertical. Parallel light leaving the collimator is split into two parallel beams by reflection at the prism faces, AB, AC. Clamp the table, T, in position. Sight the telescope upon each face in turn, bring the images of the slit to the cross-wires, and read the verniers. It is obvious that the angle through which the telescope has been rotated is equal to twice the angle of the prism (cf. page 24).

(*b*) Mount the prism so that its refracting edge, A (Fig. 189), is just over the centre of the table. Rotate the telescope to a position about 90° from the collimator, and clamp it. Now rotate the prism until the beam of light reflected from the face, AC, enters the telescope. Adjust until the image of the slit is on the cross-wires, and read the verniers attached to the prism table. Next rotate the prism table until the beam reflected from the face, AB, enters the telescope, adjust the image of the slit on the cross-wires, and again read the verniers. The angle through which the prism has been turned is obviously equal to the supplement of the angle, BAC, of the prism.

(2) The refracting angle of a prism being known, the determination of the refractive index of the material of the prism is easily made. The slit is illuminated by monochromatic light, say by a sodium flame. First set the telescope to get a direct reading of the collimator slit. Then place the prism on the table in a suitable position, and locate the refracted image of the slit by the naked eye. Rotate the telescope to view this image, and adjust to the position of minimum deviation as described previously (see page 140). Take the reading of the telescope by means of the verniers. The angle through which the telescope has been rotated from its first position is the angle of minimum deviation, D, in the relation,

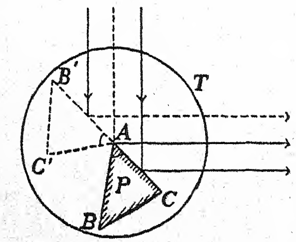


FIG. 189.

$$\mu = \frac{\sin \frac{1}{2}(D + A)}{\sin \frac{1}{2}A},$$

from which μ can be calculated.

If the refractive index of a liquid is required, the liquid may be placed in a hollow glass prism, P′ (Fig. 187), or in a prism whose sides are composed of three parallel-sided glass plates cemented together at the edges. Observations are made as in the case of the glass prism.

(3) By similar experiments, the dispersive power of the material of the prism may be determined. If a very accurate value of the dispersive power is not required, the slit may be illuminated by an ordinary luminous batswing flame. Set the prism in the position of minimum deviation for the mean or brightest rays. The yellow

sodium light will do well for this. Put a little common salt in the flame, and take the reading for this yellow light. Now remove the salt and take the readings for the extreme limits of the red and violet portions of the spectrum. The refractive indices, μ_R, μ, μ_V, may now be calculated, and by means of the relation,

$$\omega = \frac{\mu_V - \mu_R}{\mu - 1},$$

the dispersive power, ω, can be found. If a more accurate result is required, two definite kinds of light must be used. The red light given by a lithium salt in the flame, and the blue light given by a strontium salt are suitable for this purpose.

18. Refractometers

The refractive index of a given liquid at a given temperature and for the same light is a constant. Hence, the refractive index may be employed as a means of identification of a liquid. For a more rapid measurement of the refractive index than is possible with the spectrometer several instruments have been devised, of which the **Pulfrich Refractometer** is probably the best. A short glass tube (Fig. 190), some 15 mm. in diameter and containing about 2 gm. of the liquid, is cemented on one face of a right-angled prism placed so that this face is horizontal. A beam of mono-chromatic light, made to pass at *grazing incidence* along the surface between the liquid and the glass prism, is refracted downwards into the prism at an angle, r, with the normal which cannot be greater than the critical angle from glass to liquid. The light finally emerges into the air at an angle, e, with the normal to the vertical face of the prism. This latter angle is observed by means of a telescope, T, moving over a graduated circular scale shown diagrammatically.

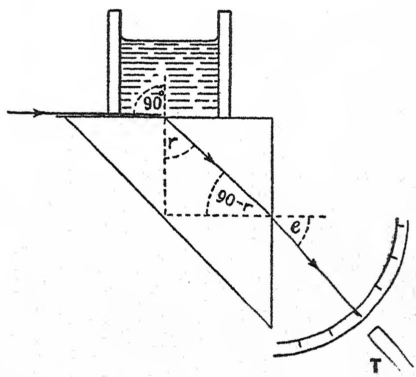


FIG. 190.

If ${}_a\mu_l$, ${}_a\mu_g$ represent the refractive indices of the liquid and the glass of the prism respectively, then for the first refraction,

$$\frac{{}_a\mu_g}{{}_a\mu_l} = \frac{\sin 90^\circ}{\sin r} = \frac{1}{\sin r}.$$

For the second refraction,

$${}_a\mu_g = \frac{\sin e}{\sin (90 - r)} = \frac{\sin e}{\cos r}.$$

$$\text{Hence, } {}_a\mu_l = {}_a\mu_g . \sin r = {}_a\mu_g \sqrt{1 - \frac{\sin^2 e}{{}_a\mu_g{}^2}};$$

$$\therefore\ {}_a\mu_l = \sqrt{{}_a\mu_g{}^2 - \sin^2 e}.$$

19. Direct Vision Spectroscope

This is a convenient form of spectroscope for the qualitative examination of the spectra of flames and incandescent bodies.

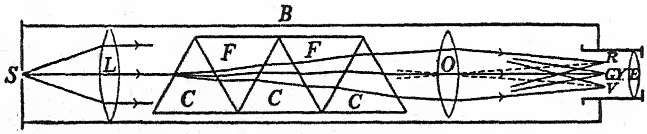


FIG. 191.

As explained above (see page 153), a prism of crown glass and a flint glass prism may be combined to produce dispersion without deviation of the mean ray. By using several prisms, with their edges alternately in opposite directions, a very large dispersion may be obtained. The prisms are enclosed in a brass tube, B (Fig. 191), provided at one end with an adjustable slit, S, placed parallel to the refracting edges of the prisms. Light leaving the slit is rendered parallel by means of the lens, L. It then falls on the prism combination, which may consist of three crown glass prisms combined with two prisms of flint glass. The refracting angles of the prisms and the dispersive powers of the glasses are so chosen that the brightest ray in the spectrum passes through the system without deviation, while the red and violet rays are deviated in opposite directions. The paths of the rays arising from the central incident ray only are shown in the diagram. The spectrum formed may be viewed directly, or examined by means of an objective, O, and an eye-piece, E, arranged as a short telescope, as shown in the figure.

For their size, these spectroscopes can be made very powerful,

and are of great service in chemical, physiological, and other observations.

20. The Photographic Camera

The simplest type of camera is the fixed focus or box camera, which consists of a light-proof box with a lens at one end and a photographic plate or film at the other (see also page 181). The sensitive plate or film is in the focal plane of the lens, so that well-defined images of distant objects may be formed on the plate. In order to take photographs of objects which may be at any distance away, however, a camera of variable length is used, the lens holder being joined to the back of the camera by folding bellows so that the length of the camera can be varied at will.

The quantity of light entering the camera can be regulated by means of an adjustable *diaphragm*, usually arranged with a series of openings or apertures of different diameter. If d is the diameter of the aperture, and f the focal length of the camera lens, the quantity, d/f, is called the *aperture ratio*, and it is usual to specify it by giving it as a fraction, f/n, indicating an aperture whose diameter is $\frac{1}{n}$th of the focal length. The number, n, is called the *f number*, and it is customary to mark the apertures in a series such that the exposure needed is doubled in passing from one to the next. This is because it may be assumed that under given conditions the exposure needed is proportional to the square of the f number (see page 182). This consideration gives the series $\frac{f}{1}, \frac{f}{1\cdot4}, \frac{f}{2}, \frac{f}{2\cdot8}$, etc.

Very special care must be taken in the choice of photographic lenses. If good photographs are to be obtained, the chief defects of lenses, spherical aberration, distortion, and chromatic aberration, must be eliminated. In order to overcome chromatic aberration, all lenses in good cameras are made achromatic over the required range of wave-lengths, to which the ordinary photographic plate is sensitive. Spherical aberration is much reduced if the size of the diaphragm opening is reduced, but distortion can be overcome only by special arrangements of lenses.

Amongst such arrangements of lenses is the so-called RR or *rapid rectilinear* lens. In this arrangement, two achromatic lenses are placed a certain distance apart with the diaphragm between them. Distortion is produced by each lens, but they are arranged

so that the distortions are in opposite directions and thus balance each other. Such lenses give good results, especially in portraiture, provided that the diaphragm opening is small, but for general work they have been largely superseded by the more modern anastigmatic lens.

Anastigmatic lenses are constructed of a special kind of glass, and are so designed that they actually remove the defect of astigmatism and curvature of the image at the same time, even with a large aperture. Such lenses are used generally in pairs, and produce an image in which the outer parts are as clearly defined as the central portions.

The *telephoto lens* is another special arrangement of lenses, designed so that as large an image as possible of a distant object shall be formed without having to use an inconveniently long camera. The system consists of a pair of lenses, both of which are usually achromatic. The negative component, L_2 (Fig. 192),

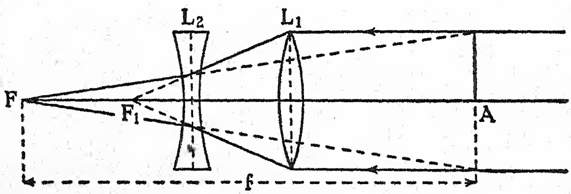


FIG. 192.

occupies the normal lens position in the camera, the positive component, L_1, being some distance in front. The distance between the lenses is less than the focal length of the convex lens, L_1. A parallel beam of light incident on L_1 converges towards F_1, the principal focus of L_1. Before reaching F_1, however, the rays pass through the concave lens, L_2, which reduces the convergency of the beam and brings it to a focus at F. If these rays are produced back to cut the original parallel beam, it will be seen that the telephoto system is equivalent to a single convex lens of focal length, f, placed at A. Thus the telephoto camera has an effective length which can be made much greater than its actual length.

21. The Stereoscope

The field of view of the human eyes, if fixed in their sockets and if the head were fixed, would be very limited. The eyes, however, can be moved about 55° in every direction about their

mean positions, and distances are judged usually by the amount of convergence it is necessary to impress on the optic axes. It is extremely difficult to judge a distance accurately with one eye only, as the reader will find if attempts are made to place quickly the point of a pen upon any small object on a table while one eye is closed.

Now to every point on the retina of one eye there is a corresponding point on the retina of the other, so that, although two images of an object are formed by the eyes, the brain is cognisant only of one. When a relatively near object of three dimensions—that is, having length, breadth, and depth—is looked at, the images formed on the retinas of the two eyes are not exactly alike, as the positions of the eyes are slightly different. This is rendered very apparent if, when looking at a near object, each eye in turn is opened and shut quickly. The brain, however, blends these images into one, and the effort required to do this gives an idea of the solidity of the object. In the case of an ordinary picture, the two images are almost exactly alike, and hence the flatness which is nearly always very apparent. Indeed, artists sometimes attempt to surmount this difficulty by exaggerating the perspective effects.

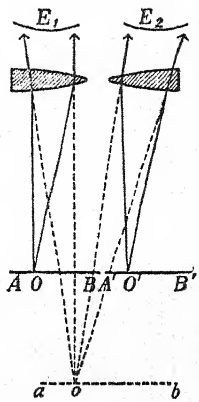


FIG. 193.

The *stereoscope* is an instrument devised by Wheatstone by means of which ordinary photographic pictures may be made to yield to the eye the appearance of depth. Two photographs, AB, A′B′ (Fig. 193), of the same object are taken in two slightly different positions—the positions that a person's two eyes would occupy if he were actually observing it from about the same position as that in which the camera is placed. These are then correctly mounted on one card, and viewed through separate, very acute-angled prisms. These prisms are set with the refracting angles inwards, so that the rays from a point, O, in the right-hand picture, AB, are deviated outwards and enter the right eye, E_1, as if they came from a virtual image much to the left of O. Similarly, the rays from the corresponding point, O′, in the picture, A′B′, enter the left eye, E_2, as if they came from a virtual image to the right of O′. By varying the distance of the pictures, AB and A′B′, from the prisms, it is possible to make the virtual images of O and O′ coincide at a point,

O, say. At the same time, other virtual images will coincide, and thus, instead of two different pictures being seen, only one, *ab*, a virtual image of these two, is perceived. The impression produced on the brain of an observer is the same as if he were looking at the object itself, the front parts of the object appearing to stand out, and the rear parts to sink back.

The surfaces of the prisms are usually curved convex, as indicated in the diagram, so that the images are magnified as well as superposed, thus making the detail much clearer. The best results are obtained usually when the two pictures are mounted so that the distance between corresponding points is nearly equal to the distance between a person's two eyes.

APPENDIX TO CHAPTER XI

2. Thick Lenses

Refraction at a single spherical surface has already been dealt with (see page 93) and the relation

$$\frac{\mu}{v} + \frac{1}{u} = \frac{\mu - 1}{r}$$

was obtained, using the Real is Positive convention. In Fig. 193 (*a*), let *a* be the image of A in the first surface BN. For this surface *u* and *v* are to be measured from N. Now consider refraction of pencil apparently diverging from *a* in the surface CN′, A′ being the image of *a* in this surface. For this refraction the relation is

$$\frac{\frac{1}{\mu}}{v} + \frac{1}{u} = \frac{\frac{1}{\mu} - 1}{r}$$

since the light is now travelling from glass to air. The distances

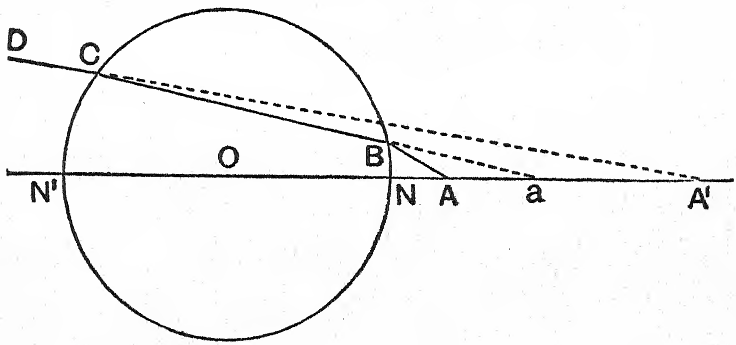


FIG. 193 (*a*).

are now to be measured from N′. Let U be the distance from A to O, and V the distance from A′ to O, the signs being determined by the convention. (The reason for measuring these distances from O will appear later.) Then, for the surface BN, we have if R is the *numerical* value of the radius of the sphere.

$u = \mathrm{AN} = \mathrm{AO} - \mathrm{ON} = \mathrm{U} - \mathrm{R}$, provided $\mathrm{U} > \mathrm{R}$.

$r = \mathrm{R}$, since the surface is convex to the direction of travel of the incident light.

$v = a\mathrm{N} = -\mathrm{W}$ say, since the intermediate image is virtual in the case shown in Fig. 193 (*a*).

Thus $$-\frac{\mu}{W}+\frac{1}{U-R}=\frac{\mu-1}{R} \quad \ldots\ldots\ldots\ldots\ldots \quad (1)$$

For the surface CN′ we have a refractive index of $\frac{1}{\mu}$, since the light is travelling from glass to air. Also:—

$u = aN' = W + 2R$ in the case shown in Fig. 193 (*a*). u must be reckoned positive for the second surface, because a behaves like a real image from the point of view of the second surface.

$v = A'N' = A'O + ON' = V - R$ in the case shown in Fig. 193 (*a*).

$r = -R$, since the second surface is concave to the direction of the incident light. v being negative because the final image is virtual, in the case shown in Fig. 193 (*a*).

Thus $$\frac{\frac{1}{\mu}}{V-R}+\frac{1}{W+2R}=-\frac{\frac{1}{\mu}-1}{R} \quad \ldots\ldots\ldots\ldots \quad (2)$$

Clearing equation (1) of fractions:—

$$-\mu R(U-R)+RW=(\mu-1)W(U-R)$$

or $W[R-(\mu-1)(U-R)]=\mu R(U-R)$.

Treating equation (2) similarly,

$$R(W+2R)+\mu R(V-R)=(\mu-1)(V-R)(W+2R)$$

or $W[R-(\mu-1)(V-R)]$

$$=-2R^2-\mu RV+\mu R^2+2\mu RV-2\mu R^2-2RV+2R^2,$$

i.e. $W[-\mu V+V+\mu R]=\mu RV-\mu R^2-2RV$.

Eliminating W, and dividing through by R, we have:—

$$\mu(U-R)(-\mu V+V+\mu R)=(\mu R-\mu U+U)(\mu V-\mu R-2V),$$

which becomes, after reduction,

$$\mu RV+\mu RU=2(\mu-1)UV.$$

Dividing by μRUV,

$$\frac{1}{U}+\frac{1}{V}=\frac{2(\mu-1)}{\mu R}.$$

Thus, a sphere of glass behaves like a thin convex lens of focal length $\frac{\mu R}{2(\mu-1)}$ situated at its centre (with the usual restriction that the angular width of the pencils of rays is small).

9. The Magnifying Power of a Telescope

Example.—*An astronomical telescope is used to view an object placed at 20 yd. distant, and the eye-piece is adjusted for nearest distant vision of the image. Find the magnification, given that the focal length of the object glass is 2 ft., and that of the eye-piece 4 in. What will be the magnifying power of the instrument when adjusted for normal vision of a very distant object?*

If N (Fig. 171) denote the position of the real image formed by the object-glass,

$\frac{1}{ON} + \frac{1}{60} = \frac{1}{2}$, whence $ON = \frac{60}{29}$ ft., positive since the image is real.

Also, if the least distance of distinct vision be taken as 10 in., then, since the final image is virtual,

$$-\frac{1}{10} + \frac{1}{O'N} = \frac{1}{4}, \text{ from which } O'N = \frac{20}{7} \text{ in.} = \frac{5}{21} \text{ ft.}$$

But the magnifying power is given by the ratio, $-\frac{ON}{O'N}$, so

$$m = -\frac{ON}{O'N} = -\frac{60}{29} \times \frac{21}{5} = -\frac{252}{29} = -8\frac{20}{29}.$$

That is, the final image is inverted and appears about $8\frac{3}{4}$ times as great as the object.

When adjusted for normal vision of a very distant object the magnifying power is given approximately by

$$m = \frac{F_1}{f_1} = \frac{24}{4} = 6 \text{ (numerically).}$$

It should be noticed from this example that the magnifying power of a telescope varies with the distance of the object and with the adjustment of the eye-piece.

10. Equivalent Lens

Consider now a system of two lenses placed at a distance, a, apart on a common axis. It is required to determine the focal length of the equivalent lens, defined as above. In accordance with the Real is Positive sign convention it is natural to give the deviation produced by a lens the same sign as its focal length, in other words, distances such as x_1 and x_2 in Fig. 177 are to be reckoned as positive, whether the lenses be converging or diverging. Put in another way, *deviations towards the principal axis are to be reckoned positive.* Thus, if we have two lenses (Fig. 177), the total deviation is given by

$$\frac{x_1}{f_1} + \frac{x_2}{f_2}.$$

Now, since the deviation produced by L_1 is $\frac{x_1}{f_2}$ we have $x_2 = x_1 - \frac{ax_1}{f_1}$, reckoning the distance a as positive, and remembering that a positive deviation is towards the principal axis.

Therefore the total deviation is given by—

$$x_1\left(\frac{1}{f_1}+\frac{1}{f_2}-\frac{a}{f_1 f_2}\right).$$

But, if F denote the focal length of the equivalent lens, the deviation which would be produced by it is x_1/F by definition.

Thus we find

$$\frac{1}{\mathrm{F}}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{a}{f_1 f_2},$$

f_1 and f_2 being given their proper signs, and a always being reckoned positive. If a is zero, *i.e.* if the lenses are in contact, this relation becomes

$$\frac{1}{\mathrm{F}}=\frac{1}{f_1}+\frac{1}{f_2},$$

which is identical with that obtained previously (see page 123*n*).

Example.—*Find the focal length of a single lens equivalent to a combination of a convex lens of 8 in. focal length and a concave lens of 12 in. focal length placed 4 in. apart on a common axis.*

Applying the relation, $\frac{1}{\mathrm{F}}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{a}{f_1 f_2}$,

$f_1=+8$ in., $f_2=-12$ in., $a=+4$ in.

$$\frac{1}{\mathrm{F}}=\frac{1}{8}-\frac{1}{12}-\frac{4}{8\times-12}, \text{ and } \mathrm{F}=+12 \text{ in.}$$

Thus, a *convex* lens of 12 in. focal length is equivalent to the given combination.

The image of an object produced by the equivalent lens is of the same *size* as that produced by a combination, but its position is not necessarily the same. The focal length of a single lens which, placed at the position of the second lens, shall produce an image of a distant object in the same *position* as that of the image produced by the combination, may be determined as follows:—

$$\frac{1}{v}+\frac{1}{\infty}=\frac{1}{f_1}, \text{ or } v_1=f_1.$$

Also, $\frac{1}{v_2}+\frac{1}{a-f_1}=\frac{1}{f_2}$, because, if f_1 is positive but less than a, the intermediate image is actually formed, and must therefore be reckoned as real for both lenses, while if f_1 is positive but greater than a the intermediate image is not formed because of the presence of the second lens, and is thus real from the point of view of the first lens, but virtual from the point of view of the second. Lastly,

if f_1 is negative, the intermediate image is behind the first lens, and is thus virtual from the point of view of the first lens, but acts like a real object for the second lens. In all these cases, w is to be reckoned positive, and we have:—

$$\frac{1}{v_2} = \frac{1}{f_2} + \frac{1}{f_1 - a}.$$

But v_2 is by hypothesis equal to F, the focal length of the single lens, so

$$\frac{1}{F} = \frac{1}{f_2} + \frac{1}{f_1 - a}.$$

The difference between this result and the one above should be noticed.

11. Object Glasses and Eye-Pieces

(2) THE EYE-PIECE.—(*a*) *Chromatic Aberration.* The focal length of a lens equivalent to a system of two lenses of focal lengths f' and f'', arranged on the same axis at a distance, a, apart, is given by

$$\frac{1}{F} = \frac{1}{f'} + \frac{1}{f''} - \frac{a}{f'f''}.$$

For achromatism $\quad \frac{1}{F_V} - \frac{1}{F_R} = 0.$

Now, $\quad \frac{1}{F_V} = \frac{1}{f'_V} + \frac{1}{f''_V} - \frac{a}{f'_V f''_V};$

$$\therefore \quad \frac{1}{F_V} = \frac{\mu_V - 1}{\mu - 1} \cdot \frac{1}{f'} + \frac{\mu_V - 1}{\mu - 1} \frac{1}{f''} - \left(\frac{\mu_V - 1}{\mu - 1}\right)^2 \frac{a}{f'f''}.$$

Similarly $\quad \frac{1}{F_R} = \frac{\mu_R - 1}{\mu - 1} \cdot \frac{1}{f'} + \frac{\mu_V - 1}{\mu - 1} \frac{1}{f''} - \left(\frac{\mu_R - 1}{\mu - 1}\right)^2 \frac{a}{f'f''};$

$$\therefore \frac{1}{F_V} - \frac{1}{F_R} = \frac{\mu_V - \mu_R}{\mu - 1} \cdot \frac{1}{f'} + \frac{\mu_V - \mu_R}{\mu - 1} \cdot \frac{1}{f''} - \frac{(\mu_V - 1)^2 - (\mu_R - 1)^2}{(\mu - 1)^2} \frac{a}{f'f''}.$$

Now $\quad \frac{(\mu_V - 1)^2 - (\mu_R - 1)^2}{(\mu - 1)^2} = \frac{(\mu_V + \mu_R - 2)(\mu_V - \mu_R)}{(\mu - 1)^2}$

$$= \frac{(2\mu - 2)(\mu_V - \mu_R)}{(\mu - 1)^2} \text{ approx.}$$

Therefore $\quad \frac{1}{F_V} - \frac{1}{F_R} = \omega\left(\frac{1}{f'} + \frac{1}{f''} - \frac{2a}{f'f''}\right).$

Where ω is the dispersive power of the material of the lenses. Thus, for achromatism,

$$\frac{1}{f'}+\frac{1}{f''}-\frac{2a}{f'f''}=0.$$

From which, $$a=\frac{f'+f''}{2}.$$

Hence, in order that a system of two lenses of the same material may be approximately achromatic, *the distance between them must be equal to half the sum of their focal lengths*, and, since a is positive, it is evident from the relation obtained above, that the sum of their focal lengths must be negative, that is, *the combination must be equivalent to a convex lens.*

(*b*) *Spherical Aberration.*—It is further necessary to consider the most favourable conditions for correcting the defects due to spherical aberration. It is evident that the greater the refraction

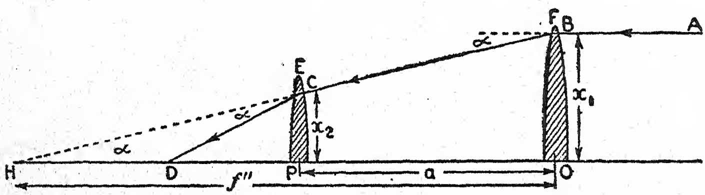


FIG. 193 (*b*).

a ray undergoes at any surface, the more apparent will the errors of aberration become. Hence, it can be shown that, by dividing the total refraction equally between the two lenses, the errors are reduced to a minimum (see page 92). Let E and F [Fig. 193 (*b*)] be the two lenses of focal lengths, f', and f'', respectively, and separated by a distance, a. A ray, AB, parallel to the axis undergoes a deviation, α, at the lens, F, and is refracted towards H, the principal focus of F. At C it strikes the lens, E, and undergoes by hypothesis an equal deviation, α, being refracted towards the point O. Since AB is parallel to OH, the angle BHO is equal to α, and therefore DC = OH. In the limit, when AB is near to OH, DP is approximately equal to DC, and thus to DH. Therefore, a ray converging to a point H *behind* the lens E (virtual object) is made to converge to a point D, half as far behind the lens, and this image is real.

Hence, $$-\frac{1}{PH}+\frac{1}{\frac{1}{2}PH}=\frac{1}{f'}, \text{ from which } PH=f'.$$

$$\text{But } OH=f'';$$

$$\therefore\ a=f''-f'.$$

Thus, *the distance between the lenses must be equal to the difference between their focal lengths.*

12. Special Eye-Pieces

(*a*) HUYGHENS' EYE-PIECE.—Let E [Fig. 193 (*c*)] represent the eye-lens and F the field-lens. Rays of light coming from the object glass of the telescope, if uninterrupted, would meet at *b*, but, falling on the field-lens they are refracted through *b'*, which should be at the focus of the eye-lens in order that the rays may

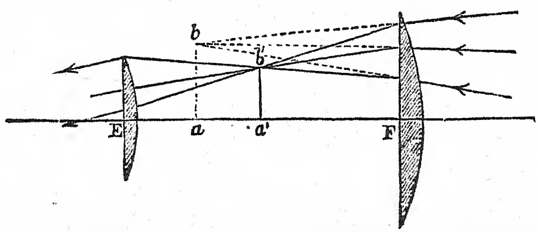


FIG. 193 (*c*).

emerge parallel. To determine the position of *ab*, in order that *a'b'* may be at the focus of E,

$$\frac{1}{Fa'}-\frac{1}{Fa}=\frac{1}{3f},$$

where f is the focal length of the eye-lens. (We have a real image but a virtual object.)

Also, if $a'b'$ is at the principal focus of E, then $Ea'=f$, and since $EF=2f$, then $Fa'=f$. Hence:—

$$\frac{1}{f}-\frac{1}{Fa}=\frac{1}{3f},$$

$$\text{or } \frac{1}{Fa}=\frac{1}{f}-\frac{1}{3f}=\frac{2}{3f};$$

$$\therefore\ Fa=\tfrac{3}{2}f.$$

(*b*) RAMSDEN'S EYE-PIECE.—Let E and F [Fig. 193 (*d*)] represent the eye-lens and field-lens respectively of a Ramsden's eye-piece. Rays coming from the object glass converge to a focus at b in front of the field-lens, F, and passing through b, fall on F, where they are refracted. After passing through F the rays diverge from b', which must be at the focus of lens E in order that the rays may finally emerge parallel. Thus, $Ea' = f$, where f is the focal length of either lens. f is positive since the lens is converging, while Ea' is to be reckoned positive, since $b'a'$ acts like a real object for the eye-lens.

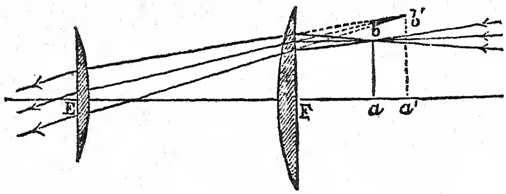


FIG. 193 (*d*).

But $EF = \frac{2}{3}f$, thus $Fa' = -\frac{1}{3}f$, this distance being reckoned negative because $a'b'$ is a virtual image formed by F. Hence, to determine the position of ab,

$$\frac{1}{Fa'} + \frac{1}{Fa} = \frac{1}{f}, \text{ and } -\frac{3}{f} + \frac{1}{Fa} = \frac{1}{f},$$

from which $Fa = \dfrac{f}{4}$, so a should be in front of F. Thus, the real image formed by the object glass should fall in front of the field-lens at a distance equal to a quarter of the focal length. This is also the correct position for the frame carrying the cross-wires.

CHAPTER XII

VELOCITY OF LIGHT

THE velocity of light is one of the most important of the so-called *universal* constants. It is the highest speed attainable, and is the speed with which X-rays, wireless waves, and other similar forms of radiation travel. Until the seventeenth century the transmission of light was thought to be instantaneous. The finite velocity of sound has been accepted as an established fact almost from time immemorial, but the methods adopted for its measurement failed, when applied to light, and it was concluded therefore that light possessed an infinite velocity.

The velocity of light has been determined in three general ways—

(1) From observations of celestial phenomena.

(2) By direct terrestrial experiments.

(3) By indirect electrical methods.

The first and last of these give the approximate velocity *in vacuo*, the second the velocity in air. The first two methods only will be dealt with in this chapter, the third method being essentially a part of the study of electrical phenomena (see Hutchinson, *Advanced Textbook of Electricity and Magnetism*).

The velocity with which light travels through any medium is very great, but varies somewhat with the nature of the medium. The velocity *in vacuo* is taken as *the* velocity of light, and the velocity in any other medium may then be determined from the absolute refractive index of that medium (see Chapter XIV., Art. 5). For, if V denotes the velocity of light *in vacuo*, and V_m its velocity in any given medium, then

$$\frac{V}{V_m} = \mu,$$

where μ denotes the absolute refractive index of the medium.

Hence, if the velocity of light in any medium, such as air, can be determined, its velocity in any other medium, or *in vacuo*, can be calculated. Thus, from the above, if V_a be the velocity in air, and μ_a be the absolute refractive index of air, the velocity, V, *in vacuo* is given by—

$$V = V_a . \mu_a.$$

And, if V_x be the velocity in a medium, X, and μ_x be the absolute refractive index of X, then:—

$$V = V_x . \mu_x, \text{ and } \frac{\mu_a}{\mu_x} = \frac{V}{V_a} \Big/ \frac{V}{V_x} = \frac{V_x}{V_a};$$

$$\therefore V_x = \frac{\mu_a}{\mu_x} \times V_a.$$

It will be remembered, in passing, that the absolute refractive index of air differs very little from unity (see Table I., page 353).

1. Roemer's Method

The first computation of the velocity of light from observations of celestial phenomena was made in 1675, by Roemer, a Danish astronomer. He deduced his result from observations of the

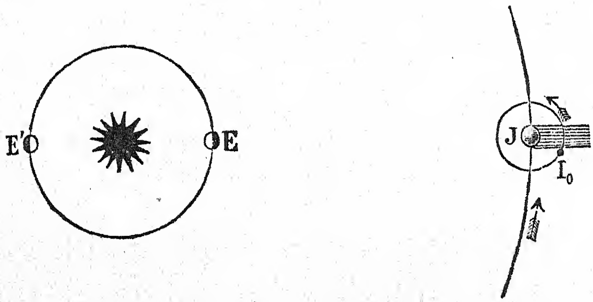


FIG. 194.

eclipses of Jupiter's first satellite, I_0. This satellite is eclipsed once during each revolution when it passes behind the planet into the shadow cast by the sun. This occurs at intervals of about 42 hours.

The instant at which the eclipse should take place can be calculated accurately from dynamical considerations based on the mean of a large number of observations. Roemer timed the eclipse when the earth was in that part of its orbit nearest to Jupiter, and from this time *calculated* the times of the eclipses that would occur throughout the year. During the succeeding months, he set himself to *observe* these eclipses, and found that the observed times were always *later* than the calculated times, and also that the difference between these two times varied with the relative positions of the earth and Jupiter. From a careful analysis of the observations it was found that the difference between the observed and calculated times increased as the earth moved away from Jupiter,

reached a maximum about six-elevenths of a year afterwards, when the distance between the two planets had attained its greatest value, and gradually decreased again to zero in another six-elevenths of a year, when the earth was again in a position nearest to Jupiter.

From this it is evident that the interval between the actual occurrence of the eclipse and the instant of its observation on the earth is equal to the time taken by light in travelling from Jupiter, or more correctly Jupiter's satellite, to the earth, and that the difference between the maximum and minimum intervals is the time taken by light to traverse the diameter of the earth's orbit round the sun.

Let the black star (Fig. 194) represent the sun, EE′ the earth's orbit round the sun, J the position of Jupiter, and I_0 the position of the satellite about to be eclipsed. Suppose the eclipse when the earth is at E′ is t seconds later than that obtained from observation when the earth is at E. Then t seconds is evidently the time taken by light to travel the distance, EE′ miles, the diameter of the earth's orbit. The velocity of light is therefore $\frac{EE'}{t}$ miles per second.

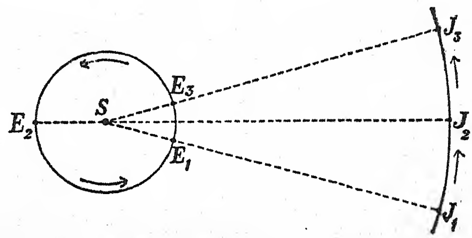


FIG. 195.

To consider this in greater detail, let S (Fig. 195) represent the sun, $E_1E_2E_3$ the orbit of the earth, and $J_1J_2J_3$ that of Jupiter. Both planets move round the sun in the same direction, the times of revolution being one year and twelve years respectively. Starting with the planets in conjunction at E_1, J_1, they will be in opposition at E_2, J_2, six-elevenths of a year later, and again in conjunction at E_3, J_3, twelve-elevenths of a year after the previous conjunction.

It is evident that, as the earth moves to E_2 and Jupiter to J_2, the observed times of the eclipses lag behind the calculated times, the lag being a maximum at E_2, J_2. As the motion still ensues, the lag decreases, and by the time the planets have reached the positions, E_3, J_3, the observed and calculated times agree once more. It is also evident that the maximum difference between the observed and calculated times is equal to the difference in the times taken by light in travelling the distances, J_1E_1 and J_2E_2—that is, equal to the time taken in travelling a distance equal to the diameter of the earth's orbit.

The diameter of the earth's orbit is about 185,600,000 miles. The eclipse at E_2 is always about 16·5 minutes later than the time calculated from observations at E_1. Hence

$$\text{Velocity of light} = \frac{185{,}600{,}000}{16 \cdot 5 \times 60} = 187{,}000 \text{ ml. per sec. (approx.)}$$

2. Bradley's Method

About fifty years after the time of Roemer, **Bradley**, an English astronomer, gave an explanation of the phenomenon of *astronomical aberration*, based on the fact that light travels through space with a definite velocity. This phenomenon is due to the fact that both the earth and light travel through space with definite velocities, and hence the direction in which light from a star reaches the earth will be in the direction of the velocity of the light *relative to that of the earth*.

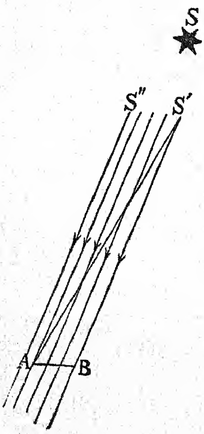


FIG. 196.

Thus, let A (Fig. 196) represent the position of the earth when light from a star, S, starts from S′. Many of the stars are at such great distances from the earth that, neglecting aberration, they are seen apparently in the same direction whatever the position of the earth in its orbit. The distance of the nearest fixed star is greater than 200,000 times the distance of the sun. If the velocities of the earth and of light be such that the earth travels from A to B, while light travels from S′ to B, then the direction in which the star is seen from the earth is parallel to AS′, and not to the true direction, AS″. From the diagram it is evident that—

$$\frac{AB}{BS'} = \frac{\text{Velocity of the earth}}{\text{Velocity of light}},$$

and, when the angle, S″AB, is a right angle—that is, when the true direction of the star is perpendicular to that in which the earth is moving in its orbit, then

$$\frac{AB}{BS'} = \tan BS'A = \tan S'AS'',$$

and the angle, S′AS″, is called the aberration of the star. Thus, if V denote the velocity of light, and v the velocity of the earth in its orbit:—

$$\frac{v}{V} = \tan \theta,$$

where θ denotes the aberration of the star. Of the quantities involved in this relation, v and θ can be determined by astronomical observation, and V can then be calculated.

Aberration is understood more clearly by considering a man running through a shower of rain falling vertically. The drops will strike him in the face, or his face will strike against the drops, and therefore the rain will appear to come from a point not vertically above but somewhat in front. The effect depends entirely on the velocity with which the rain is falling *when it reaches the man,* not on how long it has been falling. So the displacement of a star by aberration is the same for all stars, and is quite independent of their distances.

As the star is never seen in its true position, the angle of aberration cannot be measured directly. The angle measured is the angle between the apparent positions when the earth moves in

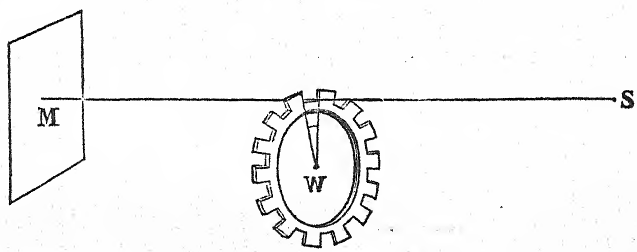


FIG. 197.

opposite directions relative to the star. This angle is double the aberration, and has been found to be 20·44 seconds of arc.

The value of the velocity of light obtained by this method is about 185,000 ml. per sec.

3. Fizeau's Method

Early attempts to measure the velocity of light by terrestrial methods were unsuccessful, and the first successful method was carried out by Fizeau in 1849.

The principle of the method is simple. Let S (Fig. 197) represent a source of light, and M a plane mirror. If a ray of light, SM, be incident normally on the mirror, M, it will be reflected back along MS, and an observer behind S will see an image of S in the mirror. However, if a toothed wheel, having the teeth and spaces of equal width, be interposed at W, as indicated in the diagram, it may be

rotated at such a rate that the light passing through any space will be received, after reflection by M, on the back of the next tooth, and thus no image of S will be seen in the mirror. When this is the case, it is evident that, during the time taken by the wheel to rotate through the angular width of one of the spaces, light travels from W to M and back again. Hence, to determine the velocity of light by this method,

$$V = \frac{2WM}{t},$$

where t denotes the time in which the wheel rotates through the angle subtended at the centre of the wheel by one of the spaces.

It is also evident that, if the wheel be rotated at twice the above rate, the reflected ray will pass through the next space, and the image will become visible again, and if rotated at treble the rate, extinction again takes place, and so on.

In the application of this principle, Fizeau employed somewhat complicated apparatus, the essential parts of which are shown

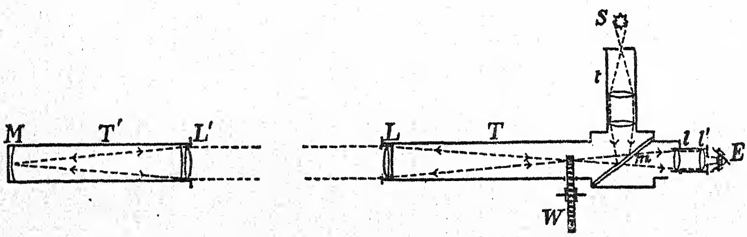


FIG. 198.

diagrammatically (Fig. 198). A source of light, S, was placed so as to send a beam of light through the side tube, t, to the mirror, m, made of unsilvered glass. This mirror is inclined at an angle of 45° to the axis of t, and reflects the light along the main tube, T, to the lens, L. The position of L is so adjusted that the rays emerge parallel, and, after traversing a distance of about $5\frac{1}{2}$ miles, fall on the lens, L′, which causes them to converge through the tube, T′, to the mirror, M, from which they are reflected back along the same path. On reaching the mirror, m, the light is partially reflected to S, but a portion passes through m and reaches the observer's eye at E, after passing through the lenses, l and l', which are adjusted to give distinct vision of the image.

The wheel, placed at W, is driven by clockwork, and, by adjusting

its rate of rotation, the image can be made to disappear and reappear successively several times.

The wheel employed by Fizeau had 720 teeth and 720 spaces, the width of the latter being equal to that of the former. The distance, WM, was about 8,663 metres. The first eclipse of the image took place when the wheel revolved 12·6 times per second. Hence, the time taken by the wheel to rotate through the angle subtended by one of the spaces was $\frac{1}{2 \times 720 \times 12 \cdot 6}$ second. In this time light travels from W to M and back again, a distance of $2 \times 8,663$ m. Hence, the velocity of light is given by

$$V = 2 \times 8,663 \times 2 \times 720 \times 12 \cdot 6 = 314,000,000 \text{ m. per sec.}$$

This result is about 195,000 ml. per sec., and is somewhat in excess of the result obtained by more recent experiments.

Fizeau's method had one great defect, arising from the fact that it is impossible to determine the exact rate of rotation of the wheel at which extinction of the image takes place. The rate can be varied appreciably without allowing the image to become visible, because the quantity of light reaching the eye, when the rate of rotation is approximately equal to that producing exact extinction, is too small to affect the retina.

In 1876 Cornu carried out a careful determination by Fizeau's original method, adopting an electrical device which enabled the rate of rotation of the wheel to be found at any instant, and using a greater distance, about 15 ml., for the light to travel. The results obtained gave a mean value of 300,330,000 m. per sec. in air, corresponding to 300,400,000 m. per sec., or 186,000 ml. per sec. *in vacuo*.

In 1880 Young and Forbes removed the defect of Fizeau's method by arranging the apparatus so that two images, formed by mirrors at different distances, could be seen. The rate of rotation of the wheel was then adjusted until the two images appeared to be of equal intensity. This method was found to be more practicable, and gave more trustworthy results, the mean value obtained being 301,400,000 m. per sec.

4. Foucault's Method

This method is somewhat more complicated, both in theory and in practice, than Fizeau's method. Adopted in 1850, Foucault utilised the principle of the rotating mirror as first employed in 1834 by Wheatstone to determine the duration of the electric spark.

The principle of the method is as follows:—Solar light was transmitted through a narrow rectangular aperture, *s* (Fig. 199), in the middle of which was a fine vertical wire. The light passed through the achromatic lens, L, fell obliquely on a plane mirror, *m*, and then came to a focus at M. At M was placed a concave mirror whose centre of curvature was at *c*, the middle point of *m*. For a certain position of *m*, a pencil of light, *sc*, starting from *s* is reflected from *m* to M, the central ray being incident along the normal, *c*M, and then reflected back along the same path to *s*. For convenience of observation, a thin parallel-sided plate of glass was inserted between L and *s* at an angle of 45° to the central ray, so that the reflected beam is reflected in part and comes to a focus at *a*, which can be observed through an eyepiece, *b*.

If now the mirror, *m*, be made to revolve, it will pass through the position just considered once in each revolution, and therefore

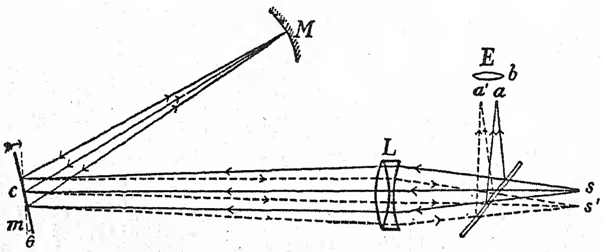


FIG. 199.

an image of *s* will be seen for an instant once in each revolution. When the revolutions become sufficiently rapid, about 30 per second, these quickly succeeding images persist on the retina, and blend into one permanent image, still seen at *a*. When, however, the speed of rotation is increased greatly, the mirror, *m*, turns through an appreciable angle while the light is travelling from *c* to M and back again. Thus, if the mirror, *m*, turns through the angle, θ, while light travels from *c* to M and back to *c*, then the ray, M*c*, will not be reflected along *cs*, but along *cs'*, and the eye at E sees the image of *s* at *a'*. Hence, if the angle, *scs'*, and the distance, *c*M, can be determined, the velocity of light can be calculated.

Now the angle, *scs'*, is equal to 2θ (see page 24), and light travels a distance, 2*c*M, during the time that the mirror revolves through an angle, θ. If the mirror makes *n* revolutions per second, the

angular velocity is $2\pi n$ radians per second, and the time in which the angle, θ, is described is given by

$$t = \frac{\theta}{2\pi n} \text{ sec.}$$

Hence, if cM be denoted by d, the velocity of light is given by

$$V = \frac{2l}{t} = \frac{4\pi n d}{\theta}.$$

Of the quantities involved in this relation, n and d are determined readily, and θ is equal to $\frac{1}{2}scs'$. In practice it would be very difficult to measure scs' with any accuracy, but no difficulty is incurred in an accurate measurement of aa', which is equal to ss', and θ can be evaluated in terms of the several distances involved. If the distances of the lens, L, from the slit, s, and revolving mirror, m, are l and l' respectively, and aa' is denoted by x, it can be shown that

$$x = \frac{2d\theta l}{l' + d}, \text{ from which } \theta = \frac{x\,(l' + d)}{2dl}.$$

Hence, the velocity of light is given by

$$V = \frac{8\pi l n d^2}{x\,(l' + d)}.$$

The distances involved can be measured easily, and thus V can be calculated.

In the actual experiment, the distance, cM, was 20 m., the mirror was a piece of silvered glass, and was rotated by means of an air turbine. The deflection, aa', amounted only to 0·7 mm., but by means of the micrometer eye-piece this could be read to an accuracy of 1 in 150. The result obtained finally by Foucault was 298,000,000 m. per sec.

5. Michelson's Methods

In its original form Foucault's method had several drawbacks. Apart from the time measurement, it requires a measurement of the very small displacement of the image, and this could be increased only by increasing the path of the light, since the speed of rotation of the mirror was limited by mechanical difficulties. In 1880, Michelson introduced great improvements in Foucault's method, the chief being the transference of the lens, L (Fig. 199), to a position between c and M. In this way the distance, cM, could be increased greatly, a distance of 600 m. being attained, without any diminution in the brightness of the image. A deflection of 133

mm. was obtained, the plate of silvered glass and the micrometer eye-piece could thus be discarded, and ss' measured directly. The rotating mirror was driven by an air turbine under perfect control, and its speed was measured by an electrically driven vibrating tuning-fork. The final result obtained was 299,882,000 ± 60,000 m. per sec.

In 1882, another improvement adopted by Michelson and Newcomb was to replace the plane mirror, M, by a prism having four or more reflecting surfaces, so that the brightness of the image was increased fourfold, and the distance, cM, was further increased to 3600 m. The value obtained in this case was 299,810,000 ± 60,000 m. per sec.

In 1926, Michelson carried out a series of measurements at Mount Wilson, U.S.A., using a still further improved form of Foucault's method. The optical arrangement is shown diagram-

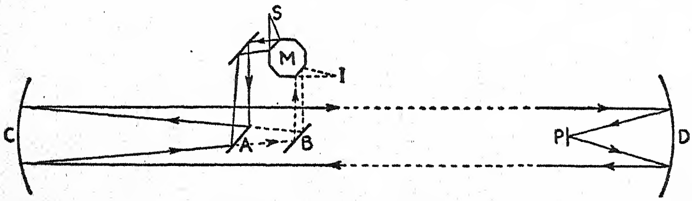


FIG. 200.

matically (Fig. 200). Light from a source, S, is reflected at one of the faces of an octagonal mirror, M, and, by means of a system of mirrors, or right-angled prisms, it reaches a concave mirror, C, which produces a parallel beam. This beam travels to another concave mirror, D, at the principal focus of which is a small plane mirror, P. The beam is thus made to return along a similar path to C, where it is again reflected to pass through A, which is half-silvered, to another small mirror, B. Reflection again occurs, and the beam falls on another face of M before being brought to a focus at I, where it may be observed conveniently by a micrometer eye-piece.

The octagonal mirror is rotated, and its speed gradually increased until the image, I, is in exactly the same position as it occupied when M was at rest. For this to happen, M must turn through one-eighth of a revolution while the light travels to P and back, and thus, instead of the returning beam striking the same face of M, it now strikes the adjacent face. The same effect occurs for

greater speeds of rotation of M, so that while the light travels to P and back, M turns through one-, two-, three-, . . . eighths of a revolution.

Taking the first case, M must have turned through $2\pi/8$ radians. Hence if d is the distance the light travels from S to P, and ω is the angular velocity of M in radians per second, the velocity of light, V, is given by

$$\frac{2\pi}{8\omega}=\frac{2d}{V}; \quad \therefore V=\frac{8d\omega}{2\pi}.$$

The distance, d, was approximately 22 ml., and the required speed of M was about 528 revolutions per sec. Great accuracy was attained in the experiments, and the value obtained for the velocity of light was 299,830,000 m. per sec.

6. Direct Measurements In Vacuo

In the experiments described above, the velocity of light in air was measured. From this, the velocity of light *in vacuo*, V_0, is calculated from the relation, $V_0 = \mu V$, where μ is the absolute refractive index of air. The value of μ depends upon the conditions of temperature and pressure of the air. Now, although Michelson's experiments were regarded as of extreme accuracy, the exact conditions of temperature and pressure of the air were not known, and hence the accuracy of the calculated value of V_0 was limited. For this reason, Michelson devised an experiment to measure the velocity in a vacuum directly. The experiment was commenced in 1929 in collaboration with Pease and Pearson, but was not completed until after Michelson's death in 1931.

The method was similar to that described above (Art. 5), except that the path of the light was inside a long evacuated iron tube. The tube was 3 ft. in diameter and a mile long, its joints being so carefully sealed that it was possible to maintain a pressure of only 0·5 mm. The mean value obtained from a series of about 3000 observations was 299,740,000 m. per sec.

7. Velocity of Light and Refractive Index of Medium

If a long tube containing water, or other transparent medium, be placed between the mirrors, *c* and M (Fig. 199), of Foucault's apparatus, the displacement, *aa'*, of the image will be greater or less, according as the velocity of light in the given medium is less or greater than the velocity in air. Experiment shows that light travels more slowly through a dense medium than through a rare

medium—that is, the greater the refractive index of the medium, the less is the velocity of light through it.

Foucault was the first to carry out experiments on different media in this manner, but he did not succeed in measuring the ratio of the velocities. Michelson, however, using a tube of water 3 ft. long, found the ratio of the velocity of light in air to that in water to be 1·330. Experiments on other transparent media have yielded similar results.

The question has also arisen whether the velocity of light of various colours is the same for all colours in air, or in *vacuo*. Evidence shows that the velocities are identical, for if not a star would appear coloured at an eclipse or at its reappearance after an eclipse, and the image in Foucault's experiment would be dragged out into a spectrum. In other transparent media, however, the velocity is greater for red rays than for violet rays. Thus, Michelson found that the velocity of red light in carbon bisulphide was about 5 per cent. greater than that of blue light.

8. Is the Velocity of Light Changing?

It has often been suggested that the velocity of light *in vacuo* is decreasing slowly with time, and a cursory examination of the values obtained by different observers does lend some support to such a view. There is no valid reason why such a change should not occur, for there are many physical quantities that show a secular variation. Moreover, we know that the Universe is expanding, and a correlation between the mean density of the Universe and the velocity of light cannot be ruled out in the present state of our knowledge.

Recently, a thorough examination of the available experimental material has been made by statistical methods, as a result of which it has been concluded that there is at present no evidence of such a change.

CHAPTER XIII

PHOTOMETRY

LIGHT, like radiant heat, is undoubtedly a form of energy, and, as such, is capable of measurement. The problem of the measurement of light has received a great deal of attention in recent years. The importance of adequate illumination, and the extent to which the welfare of the human being depends upon it, have been fully realised, and to cope with the many problems involved, a new branch of applied physics, *Illuminating Engineering*, has been developed. An important section of this is concerned with the measurement of light.

The quantity of light in any space at any instant is measured by the corresponding amount of energy available for illumination in that space at the instant considered, and the physical intensity of the light is measured by the energy transmitted through that space in unit time. For light of a given colour, the intensity conditions the brightness as perceived by the eye, or by a sensitive plate in a photographic camera. The physical intensity is also strictly proportional to the heating effect produced in a black surface exposed to the light.

There are thus three methods of measuring light—the *photometric*, the *photographic*, and the *calorimetric*. In a photographic camera, the light does work in producing chemical changes in the salts on the sensitive film. In the calorimetric method, a thermopile (see *Textbook of Heat*), or some similar instrument is exposed to the radiation, and the energy received is transformed into heat energy, which produces an electric effect measurable by a delicate galvanometer. The defect of both the photographic and calorimetric methods is that they measure other forms of energy physically similar in every respect, except that of being detected by the eye, to luminous energy. In this chapter the photometric method only will be considered. The eye may be used as the measuring instrument, but other methods of detecting and measuring luminous energy as such are now available, and are often more suitable.

1. Photometric Quantities

The four most important photometric quantities are known as *luminous flux, luminous intensity, illumination, and brightness.*

The meanings of these terms can be made clear by means of simple illustrations.

If a candle in a room is replaced by an electric lamp, the latter sends out a more copious quantity of light than the candle. In other words, the electric lamp produces a much greater *luminous flux* than the candle.

With the electric lamp as the source, the amount of light falling on various objects in the room is greater than with the candle as source. In other words, the *illumination* is increased.

Again, if the electric lamp has a *pearl* bulb, the light will be emitted more or less uniformly in all directions. If, however, a suitably shaped reflector is fitted behind the lamp, it can be arranged that the light is concentrated downwards, say, in a narrow cone or beam. The total flux of light from the lamp has not been changed, but merely redistributed so that now very much more light is emitted in some directions than in others. In other words, the source gives a greater *luminous intensity* in some directions than in others.

If two lamps of the *same power*, one having a pearl bulb and the other a clear glass bulb, are used, these give approximately the same amount of light. On looking at them, however, it is seen that the glowing filament of the latter has a much greater *brightness* than the translucent surface of the other. This means that the same amount of light is emitted from a much larger surface in the case of the pearl bulb, and it is evident that the brightness depends upon the relation between the intensity of the light emission and the area of emission.

2. Luminous Flux

The definition of *luminous flux* adopted by international convention is *the rate of passage of radiant energy evaluated with reference to its visual effect.* Since a beam of light is a stream of energy, it seems perfectly natural to refer to luminous flux as the rate of emission of energy. It must be remembered, however, that a source of light emits a good deal of radiation, such as the infra-red and ultra-violet, to which the eye is not at all sensitive (see page 158), and also that the sensitivity of the eye varies widely for different parts of the visible spectrum (see page 156). In photometry, the visual effect of the radiation only is to be considered, and therefore, it would be insufficient to define luminous flux merely as the rate of passage of energy by the source. Thus the statement must be qualified as in the definition given above.

3. Luminous Intensity

Before considering the definition of this quantity, it is necessary that the student should understand what is meant by a *solid angle.* The definition of a solid angle is obtained by a simple generalisation of the method used to define a plane angle in circular measure.

With O as centre (Fig. 201) describe a sphere, S_1, of unit radius. The lines joining O to every point of the edge of a figure, S, will describe on S_1 a similar figure. The area, ω, of this figure is a measure of the solid angle subtended by S at the point, O. Since the total surface area of S_1 is given by 4π, it is evident that the solid angle corresponding to the whole space surrounding O will be 4π.

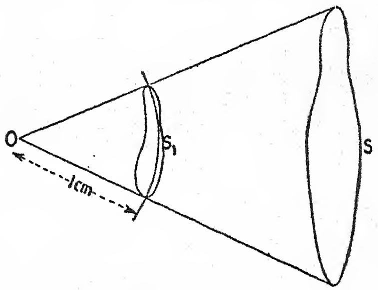


FIG. 201.

The small solid angle subtended at a point by a small area may be found by using this definition. Thus, let S (Fig. 202) be a small plane area at a distance, r, from the point, O, having its normal, PN, inclined at an angle, θ, to the line, OP. Describe a spherical surface with O as centre and radius, r. The cone subtended at O by S will cut off from this spherical surface a small surface of area, S cos θ. The area cut off from a spherical surface, S_1, of unit radius

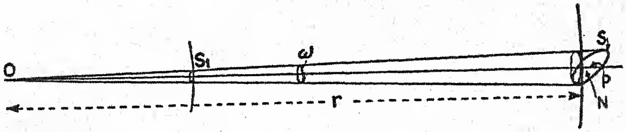


FIG. 202.

about O as centre, will be $\frac{S \cos \theta}{r^2}$, so that the small solid angle is given by

$$\omega = \frac{S \cos \theta}{r^2}.$$

Now, if all luminous sources emitted light equally in all directions, the power of any source could be specified by stating the total luminous flux of the source. This is not generally the case, however, and it is thus necessary to have some means of specifying

the rate at which a source is emitting light in any given direction. It is for this purpose that the quantity *luminous intensity* is introduced.

Let O (Fig. 203) be a small source of light, and let a cone of small solid angle, ω, be constructed about OP as axis, the luminous flux of O within this cone being denoted by F. If this cone is sufficiently small, it may be assumed that the luminous flux is distributed uniformly within it, and the ratio, $\frac{F}{\omega}$, measures the luminous power of O in this particular direction. This ratio is known as the luminous intensity of O in the direction, OP—that is, *the luminous intensity*, I, *of a source in any specified direction is the luminous flux per unit solid angle emitted in the given direction.*

If a given source has the same luminous intensity for all directions within a solid angle, Ω, the total luminous flux emitted is given by

$$F = I\Omega.$$

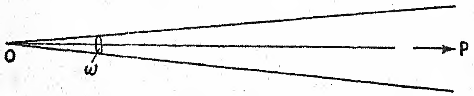


FIG. 203.

If the source emits with the same luminous intensity in all directions, it is said to be a *uniform* source, and the total luminous flux, F_0, is given by

$$F_0 = 4\pi I.$$

4. Units of Luminous Intensity and Flux

In dealing with the definitions in the preceding section, intensity was explained in terms of flux. In deciding upon suitable standards of measurement, however, it is more convenient to begin with luminous intensity. This is done by specifying in detail a source of light, and taking the luminous intensity of this source in a particular direction as the *unit* of intensity.

In the early stages of photometry it was agreed to take as the standard source of light a standard *sperm* candle, weighing six to the lb., and burning at the rate of 120 grains per hour. This is too inaccurate, however, for present-day work, different candles often varying by as much as 20 per cent., and other standards have therefore been devised. The most accurate of these is the Vernon-Harcourt Pentane lamp, which consists of a flame of pentane vapour, mixed with a certain definite proportion of air and burnt at a ring burner made of steatite. Under defined conditions it provides a very constant source of light. The flame is made fairly large so that it

gives an intensity in a horizontal direction of approximately that of 10 standard candles. From this lamp the *International Standard Candle* is defined as *the intensity of a source having $\frac{1}{10}$th the intensity in a horizontal direction of such a pentane lamp burning under specified conditions.*

At the present time, for the sake of convenience, the standard is usually maintained in terms of an electric filament lamp, with a specified constant potential difference between its terminals and carrying a specified constant current.

The luminous intensity of any source of light measured in terms of this standard is called the *International Standard Candle-Power*, or briefly the *Candle-Power*, of the source (see Table IV., page 354).

Having fixed the unit of luminous intensity, the unit of luminous flux can be defined in terms of it. Thus, let O (Fig. 204) be a point source of light of uniform intensity equal to 1 candle-power. The luminous flux emitted by this source within a cone of unit solid angle is taken as the unit of luminous flux, and is called the *lumen*—that is, *the lumen is the luminous flux, or quantity of light emitted per second by a uniform source of one standard candle-power within a cone of unit solid angle.*

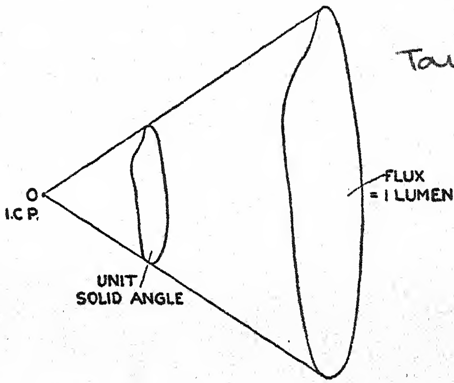


FIG. 204.

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Thus, the total luminous flux emitted by this source in all directions is 4π lumens, and the total luminous flux, F_0, emitted by a uniform source of luminous intensity, I, will be given by

$$F_0 = 4\pi I \text{ lumens.}$$

5. Illumination

The third quantity, ***illumination***, is defined as the total luminous flux received or intercepted by a surface per unit area. If the surface is so illuminated that the same quantity of light falls per second on every small element of the surface, it is said to be *uniformly illuminated*; and the value of the illumination, E, is equal to the

total luminous flux, F, falling upon the surface divided by the area, A. Thus, E = F/A.

In this definition the total luminous flux falling on the surface is considered, quite independently of the direction in which it reaches the surface. Also, the luminous flux which the surface receives only is considered, and not what happens to the light when it reaches the surface. It may all be reflected regularly, or diffused, or it may be completely or partly absorbed or transmitted. These differences, however, may affect the appearance or the *brightness* of the surface. Thus, a piece of black velvet lying on snow may have the same illumination as the snow, but the brightness is very different. This is an important distinction between illumination and brightness which should be noted carefully.

6. Law of Inverse Squares

The *illumination of a surface* due to a source of light diminishes as the surface recedes from the source, and increases as it approaches. With light, as with sound and with radiant heat and other influences which spread from a centre, the intensity at any point in free space depends on the distance of that point in a manner which is expressed by the *Law of Inverse Squares.* The law states that *the illumination at any point of a surface is inversely proportional to the square of the distance from the source,* provided that the medium through which the light passes is perfectly transparent.

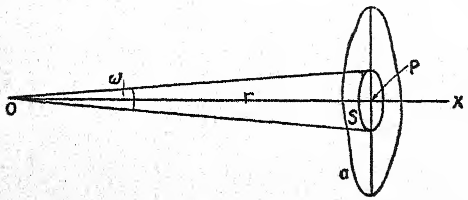


FIG. 205.

The calculation of the illumination produced at any surface by a given arrangement of lamps, or other sources of light, constitutes the main problem in photometry, and the basis of the calculation is the expression for the illumination at any point of a surface produced by a point source of light.

Let O (Fig. 205) be a point source of light having a luminous intensity, I, in the direction, Ox, and let a be a surface the plane of which is perpendicular to Ox, so that the light from O falls normally on the surface at P. Consider a small area, S, with P as its centre. This will subtend at O a small solid angle, ω, which is given by

$\omega = \dfrac{S}{OP^2} = \dfrac{S}{r^2}$. The luminous flux, F, emitted by O within this cone is given by $F = I\omega$. Hence, the illumination of the area, S, at P will be given by

$$E = \frac{F}{S} = \frac{I\omega}{\omega r^2} = \frac{I}{r^2}.$$

It follows from this relation that if E_1 denote the illumination of a surface at unit distance from a source of light and perpendicular to the rays, then the illumination, E_0, of a similarly placed surface at a distance, r, is given by

$$E_0 = \frac{E_1}{r^2},$$

$$\text{for } \frac{E_0}{E_1} = \frac{1}{r^2}, \text{ and hence } E_0 = \frac{E_1}{r^2}.$$

The illumination also varies with the angle of incidence of the light on the surface, the angle of incidence being the angle made by the axis of the incident beam of light with the normal to the surface. Thus, if OP (Fig. 206) is the axis of the incident beam, and θ is the angle between OP and the normal to the surface, S, at the point, P, the solid angle subtended at O by the small area, s, around P is given by

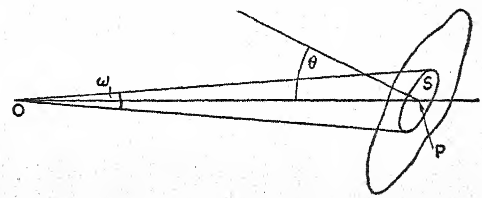


FIG. 206.

$$\omega = S.\frac{\cos\theta}{r^2}.$$

The luminous flux, F, emitted by the source at O within this solid angle is given by $F = I\omega$, and hence the illumination at P is given by

$$E = \frac{F}{S} = \frac{I\cos\theta}{r^2},$$

i.e. *the illumination varies directly as the cosine of the angle of incidence.*

This explains partly why the strength of the sun's heat and light is greater on fields and hillsides which lie directly facing the sun,

and less upon places which slope away from the sun, thus lying obliquely to the rays. Now since:

$$E_0 = \frac{E_1}{r^2}, \text{ and } E = E_0 \cos \theta,$$

$$E = \frac{E_1 \cos \theta}{r^2},$$

which gives the illumination of a surface placed at a distance, r, from a source of light, the angle of incidence being θ, and the source being of such luminous intensity that the illumination of a surface placed at unit distance from the source, and perpendicular to the rays, is E_1.

7. Units of Illumination

From the definition of illumination as luminous flux per unit area incident on a surface, it is evident that the unit of illumination may be defined as that which is obtained when unit luminous flux, one lumen, falls on unit area, and it follows that there will be several units according to the chosen unit of area.

The most common units are the *lumen per square metre*, sometimes called the *lux*, and the *lumen per square foot*. These two units are often termed the *metre candle* and the *foot candle* respectively.

The derivation of these terms may be explained by supposing that a uniform point source of *one* standard candle-power is situated at the centre of a sphere of radius, r, drawn about the source as centre. The total luminous flux emitted by this source is 4π lumens, and this luminous flux is distributed uniformly over the surface of the sphere, producing a uniform illumination, E, which is given by

$$E = \frac{4\pi}{4\pi r^2} = \frac{1}{r^2}.$$

Thus the unit of illumination is given by the illumination produced at the surface of a sphere of unit radius by a uniform point source of one standard candle-power situated at the centre. If the radius of the sphere is one metre, then the illumination at the surface is the *lux*, or *metre candle*—that is, it is the illumination produced by a source of one candle-power at a surface one metre away, the light being incident normally on the surface. Similarly, if the radius of the sphere is one foot, the illumination produced by a source of one candle-power at a surface one foot away, when the light is incident normally, is the *lumen per square foot*, or *foot candle*.

It should be noted that the term, lumen per square foot, expresses the true definition of illumination as luminous flux per unit area, and is therefore the preferable term to use.

8. Brightness

Brightness is concerned with the light *emitted* by a surface, in contrast to the term, illumination, which is concerned with the light *received* by a surface, as explained above (Art. 5).

Suppose a point, P (Fig. 207), on an illuminated surface, or a self-luminous surface, is observed by the eye, along a line, PN, normal to the surface. A small area, S, of the surface around P may be considered as a point source of light. If the luminous intensity in the direction, PN, be denoted by I_0, then the ratio, $\frac{I_0}{S}$, —that is, the candle-power per unit area, is called the brightness, B, of the surface at the point P in the direction PN.

The unit of brightness is thus *candle-power per unit area* (see Table V., page 354).

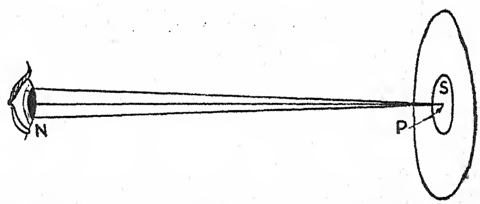


FIG. 207.

9. Comparison of Luminous Intensities

The luminous intensity of any source of light is proportional to the illumination which it produces. This quantity is measured by the illumination of unit area of a surface placed at unit distance from the given source, the light being incident normally on this surface.

If I_1 denote the luminous intensity of a given source of light, A, the illumination produced by this source on a surface at a distance, d_1, when the angle of incidence of the light is θ_1, is given by

$$E = \frac{I_1 \cos \theta_1}{d_1^2} \text{ (see Art. 6).}$$

Similarly, if another source of light, B, be so placed as to produce the *same* illumination, E, for an angle of incidence of the light, θ_2, then

$$E = \frac{I_2 \cos \theta_2}{d_2^2},$$

where I_2 denotes the luminous intensity of B, and d_2 its distance

from the illuminated surface. Equating these two expressions for E gives

$$\frac{I_1 \cos \theta_1}{d_1^2} = \frac{I_2 \cos \theta_2}{d_2^2},$$

which is the general relationship. Usually, however, in the comparison of sources of light, it is arranged that $\theta_1 = \theta_2$. In this case,

$$\frac{I_1}{d_1^2} = \frac{I_2}{d_2^2}, \text{ or } \frac{I_1}{I_2} = \frac{d_1^2}{d_2^2},$$

the distances of A and B being altered until they produce the same illumination of the surface.

This shows that *the luminous intensities of different sources of light are* directly *proportional to the squares of the distances they must be placed from a given surface, in order to produce on it the same illumination.* This statement must be distinguished carefully from that made above (Art. 6), and it should be noticed that it is true only when the angle of incidence of the light is the same in each case. Hence, it is evident that, in an experimental comparison of luminous intensities, care must be taken to satisfy this condition.

Any arrangement suitable for carrying out a comparison of luminous intensities, or candle-powers, of different sources of light by means of this relationship is known as a *photometer.* Many different forms of apparatus have been devised for this purpose, and some of these will be described below (Art. 10). It is important to realise that the eye is unable to estimate the ratio of the illuminations due to different sources of light, but that it is a better judge of the equality of the illumination of two adjacent surfaces, provided that they appear the same, or nearly the same, colour. This power of the eye forms the basis of visual *photometry*, and nearly all methods of photometry depend on the equalisation of the illuminations of two adjacent white screens, and the details of the construction of photometers are devised to facilitate this adjustment.

Examples.—(1) *The illumination of a screen placed* 6 *ft. from a given source of light is denoted by* E. *Find the illumination when the distance of the screen is increased to* 9 *ft.*

Let E′ denote the required illumination. Then (Art. 6),

$$\frac{E'}{E} = \frac{6^2}{9^2} = \frac{36}{81} = \frac{4}{9},$$

and
$$E' = \tfrac{4}{9}E.$$

(2) *A small screen is held* 6 *ft. from a source of light, in such a position that the light is incident normally. It is then removed to a distance of* 10 *ft. and*

rotated, so that the light is incident on its surface at an angle of 60°. *Compare the illuminations of the screen in the two cases.*

Let E and E′ denote the illuminations for the first and second cases respectively.

Then, since the illumination varies *inversely* as the squares of the distances, and *directly* as the cosines of the angles of incidence (Art. 6),

$$\frac{E}{E'} = \frac{10^2}{6^2} \cdot \frac{\cos 0°}{\cos 60°} = \frac{10^2}{6^2} \cdot \frac{1}{\frac{1}{2}};$$

$$\therefore \frac{E}{E'} = \frac{50}{9}.$$

(3) *Two sources of light,* A *and* B, *when placed* 8 *and* 10 *ft. respectively from a screen, produce the same illumination of its surface. Compare the luminous intensities of* A *and* B.

Let I and I′ denote the luminous intensities of A and B respectively.

$$\text{Then, from above,} = \frac{I}{I'} = \frac{8^2}{10^2} = \frac{16}{25}.$$

(4) *The luminous intensities of two sources of light,* A *and* B, *which are placed* 10 *ft. apart, are in the ratio* 4 : 9. *Find at what points on the line joining them the illumination is the same.*

Let x ft. denote the distance of either of the required points from A. Then:—

$$\frac{x^2}{(10 - x)^2} = \frac{4}{9} = \left(\pm\frac{2}{3}\right)^2;$$

$$\therefore \frac{x}{10 - x} = \pm\frac{2}{3}.$$

That is, $3x = 20 - 2x$, or $-3x = 20 - 2x$;

$\therefore 5x = 20$, or $-x = 20$.

Thus, $x = 4$ ft. or $x = -20$ ft.

This means that there is equality of illumination at a point between A and B, 4 ft. from A and 6 ft. from B, and also at a point 20 ft. from A on the side remote from B—that is, the line, AB, is divided internally and externally in the ratio, 2 : 3.

10. Photometers

RUMFORD'S PHOTOMETER.—In this form of photometer, the luminous intensities of two sources of light are compared by adjusting to equality the intensities of the two shadows of a vertical rod cast side by side on a screen by the two given sources. The two sources of light, L_1, L_2, the rod, R, and screen, SS, are arranged as shown (Fig. 208), so that the shadows, S_1x, S_2x, whose edges should be well-defined, appear close together and of equal intensity.

In this way, since each shadow is illuminated by the source to which the other is due, equality of intensity of the shadows cast

by L_1 and L_2 means equality of illumination due to L_2 and L_1. Hence, if the distances, L_1x and L_2x, are measured, the luminous intensities, I_1 and I_2, of the sources are compared by the relation,

$$\frac{I_1}{I_2} = \frac{(L_1x)^2}{(L_2x)^2}.$$

Hence, if the candle-power of one source is known, that of the other can be calculated.

FOUCAULT'S PHOTOMETER.—This form of photometer, sometimes known as the *photoped*, consists of a semi-transparent screen, AB

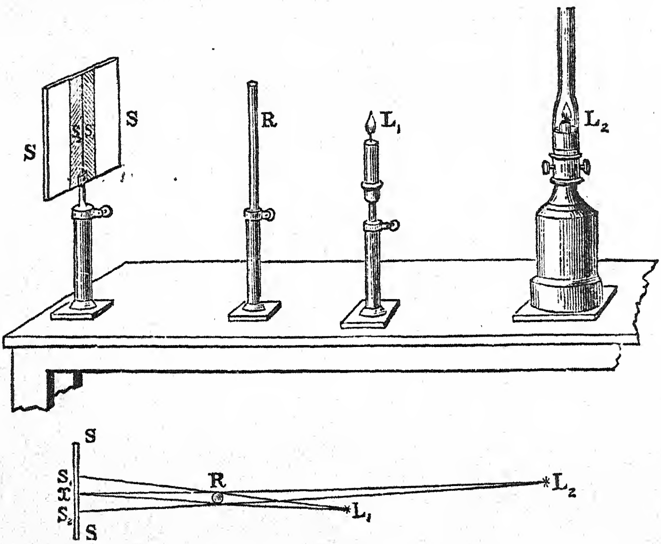


FIG. 208.

(Fig. 209), of thin paper, ground glass, or thin white porcelain, fixed vertically in front of, and at right angles to, a partition, CD, which is movable by means of a screw in the direction of its length. The two sources of light, L_1 and L_2, are placed on opposite sides of this partition in such positions that the angle, L_1EL_2, is bisected by CD. By this arrangement, one portion of the screen is illuminated by one source and the other portion by the other source, and, by adjusting the position of CD until these separately illuminated portions become contiguous, their illuminations may be compared more accurately. The distances of L_1 and L_2 are adjusted until both parts of AB are equally bright.

When both portions appear equally bright, the comparison is complete. If L_1 and L_2 represent the final positions of the sources of light, and if I_1 and I_2 represent their luminous intensities, then, as before

$$\frac{I_1}{I_2} = \frac{(EL_1)^2}{(EL_2)^2}.$$

BUNSEN'S GREASE-SPOT PHOTOMETER.—This was the earliest form of photometer capable of any precision. It consists essentially of a sheet of white paper in the middle of which a part is made translucent by means of paraffin-wax. This part may be of any shape, but its edges should be well defined. If such a sheet of paper is held up to the light, it will be seen that the grease-spot is semi-transparent, and looks brighter than the rest of the paper when viewed from the side remote from the light, but darker when seen from the other side. The reason for this is that more light passes through the region of the grease-spot than through the rest of the paper, and hence, when seen from the side remote from the light, it looks brighter than the rest of the screen, through which little or no light passes. When looked at from the other side, however, the spot looks comparatively dark, because a large proportion of the light incident upon it passes through, and is not used therefore in illuminating its surface.

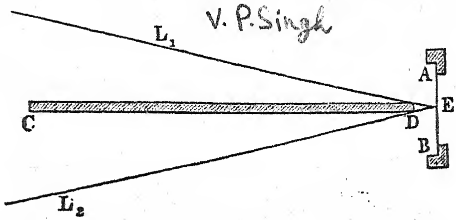


FIG. 209.

It will now be understood that if a suitable paper screen, having a grease-spot at its centre, be placed between two sources of light, A and B, and its position is adjusted until the spot cannot be seen on either side, except by close inspection, then the screen must be illuminated equally on both sides. For, if the spot is not readily distinguishable from the adjacent surface of the screen, the amount of light coming from unit area of both must be the same. Let F denote the quantity of light incident from A on unit area of the surface of the screen, and f the quantity which passes through, per unit area, in the region of the spot. Then, $(F - f)$ denotes the quantity of light spent in illuminating the surface of unit area on the grease-spot, whereas the whole quantity, F, is spent in illuminating the surface of unit area of the screen in the neighbourhood of the spot. It is assumed that the screen is opaque, except at the

grease-spot, and that its surface is such that there is no regular reflection. Hence, on the side of A, the spot will appear dark unless a quantity of light, *f*, is transmitted through it from B to make up for the quantity that has passed through from A.

Thus it appears that a necessary condition for the disappearance of the spot is that the quantities of light passing through it in opposite directions must be equal. If the surface of the spot is the same on both sides, the quantity of light which passes through will be, for each side, the same fraction of the light incident on that side, and consequently, if the quantities of light passing through unit area of the grease-spot are equal, then the quantities of light incident on unit area of opposite sides of the screen are also equal —that is, the two sides of the screen are illuminated equally.

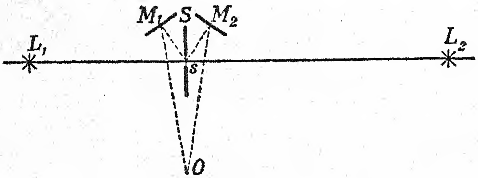


FIG. 210.

If L_1, L_2, and S (Fig. 210) represent the relative positions of the sources of light and the screen when finally adjusted, then, as before,

$$\frac{I_1}{I_2} = \frac{(L_1 s)^2}{(L_2 s)^2},$$

where I_1 and I_2 are the luminous intensities of the sources.

In carrying out the necessary measurements experimentally, at least four different adjustments should be made, as follows:—

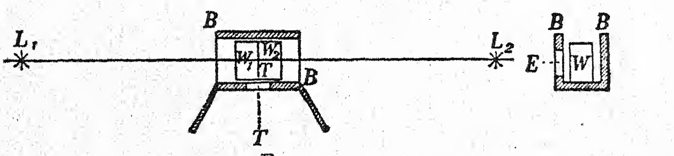


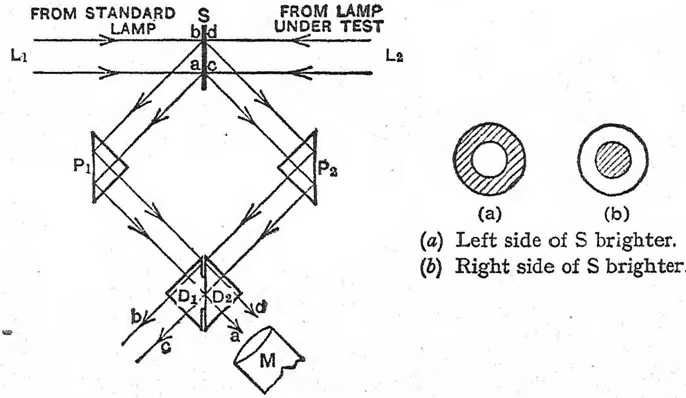
FIG. 211.

(1) Adjust for disappearance of the spot when seen from the side of the screen facing L_1. (2) Turn the screen through 180°, and again adjust for disappearance of the spot from the same surface, now facing L_2. (3) Repeat (1) and (2) with the other surface of the screen. The screen is usually mounted in a light frame, so that it can be turned round easily in its stand, or as a whole.

An alternative method sometimes used is to adjust the position until the two sides show an equal contrast between the grease-spot

and the surrounding screen. For this purpose the screen is mounted inside a suitable box, blackened on the inside, and two plane mirrors, M_1, M_2 (Fig. 210), arranged so that an observer at O can view both sides of the screen at the same time approximately under the same conditions.

JOLY'S PHOTOMETER.—This form of photometer consists of two equal blocks, W_1, W_2 (Fig. 211), of paraffin-wax or opal glass, about a quarter of an inch thick, separated by a smooth sheet of tin-foil, T, and mounted on an open wooden slider, BB. The light coming from either source enters the blocks and is scattered both before and after reflection from the surface of the tin. The tin is opaque, so that W_1 is illuminated solely by L_1, and W_2 by L_2.



(*a*) Left side of S brighter.
(*b*) Right side of S brighter.

FIG. 212.

In taking measurements, the compound block is moved up and down the line joining the luminous sources until a position is found in which the edges of W_1 and W_2 appear equally bright to an observer looking at them through a hole in BB. The position of T is observed and, as before,

$$\frac{I_1}{I_2} = \frac{(L_1T)^2}{(L_2T)^2}.$$

LUMMER-BRODHUN PHOTOMETER.—This photometer is a commercial application of the Bunsen grease-spot photometer used in precision measurements.

Light from the sources, L_1 and L_2 (Fig. 212), falls on opposite sides of the screen, S, which is made of some white diffusing

substance, such as magnesium carbonate or plaster of Paris, and placed at right angles to the line joining the sources. The light scattered from the surfaces is reflected by the plane mirrors, P_1 and P_2, into a double prism as shown. In some forms of the apparatus, the mirrors are replaced by two reflecting prisms (see page 178). This double prism consists of two right-angled prisms, D_1 and D_2, whose hypotenuse faces are optically plane and polished. The outer part of this face of D_1 is cut away, so that when the two prisms are placed together as shown contact is made over a sharply defined circular central area only. By this means, of the light reaching D_1 from the source L_2, the central part only is transmitted into D_2, while the outer parts are totally reflected. Similarly, of the light reaching D_2 from the source L_1, the central part is transmitted into D_1, and the outer parts are totally reflected. A microscope, M, is focused on the hypotenuse face of D_1, and the centre of the field of view is illuminated only by light from L_2, while the outer parts are illuminated by light only from L_1. One of the sources is moved towards or away from S until the whole field of view is uniformly bright—that is, until no dividing line is visible between the centre portion and the surroundings. The two sides of S must then be equally bright, and hence, if the surfaces have the same reflecting properties, they must be illuminated equally.

In order to eliminate any effects due to differences in the surfaces of the screen, the sources are interchanged and a further reading taken.

THE FLICKER PHOTOMETER.—The great disadvantage of all the photometers considered so far is the lack of accuracy which occurs when the sources of light are of different colour. With the Flicker photometer this difficulty does not arise. The principle of the method is to expose to the eye alternately in rapid succession the two illuminated surfaces to be compared. It is found that a frequency of alternation can be obtained for which the colour difference disappears, the colours merging into some intermediate hue, while the flicker due to differences of brightness still remains. The sources of light are adjusted until this brightness flicker disappears also, and the ordinary law is then applied as with other forms of photometer.

A number of different forms of photometer have been developed on this principle. In the simple form illustrated (Fig. 213), F is a fixed white surface illuminated by the source, L_1. A second white disc, M, shaped as shown at N, can be rotated by means of a motor

on the axle, R, and is illuminated by the source, L_2. The two surfaces can be viewed through the tube at E, and, as M rotates, the observer can see alternately an arm of M illuminated by L_2, and F illuminated by L_1. If the surfaces are not illuminated equally there is *flickering*. The distances of L_1 and L_2 are adjusted until the sensation of flickering disappears entirely. The luminous intensities of the sources are then proportional to the squares of the distances of the sources from the parts of the screens seen through the observation tube.

11. Experiments in Photometry

A number of useful experiments, in addition to the comparison or the determination of luminous intensities, may be carried out by using any of the types of photometer described above. Some of these experiments will now be described.

(1) *To prove the inverse square law.*—Set up a Bunsen or a Joly photometer, and compare the luminous intensity of a single candle with that of four similar candles arranged together as one source of light. It will be found that, on adjusting for equality of illumination, the distance of the four-candle source from the screen is twice that of the single candle. Repeat for different distances and different numbers of candles. Show that, for each observation,

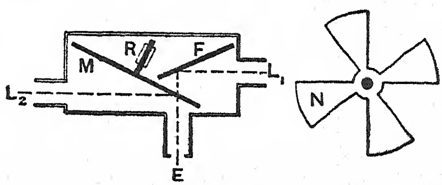


FIG. 213.

$$\frac{I_1}{I_2} = \frac{R_1^2}{R_2^2},$$

where I_1 and I_2 are the luminous intensities of the sources, and R_1 and R_2 are the distances of the sources from the screen, from which it follows that for the same source and a screen at different distances,

$$E \propto \frac{I}{R^2}, \text{ where E is the illumination.}$$

The position of the screen is determined more quickly and accurately if it be made to oscillate between two positions, in one of which one side is too bright and in the other too dark. While making it oscillate, the extent of the oscillation should be reduced

gradually to zero. In this way the position of equality of illumination will be found with great exactness.

(2) *To find the percentage of light transmitted by a glass plate.*—The photometer is set up as usual between any sources of light, and adjusted until the illuminations are equal. The plate is then placed between one source and the photometer. It is then necessary to move this source nearer to the photometer to regain the equality of illumination.

If the initial distance of the source, L_2 (Fig. 214), from the photometer, P, is denoted by x, and the final distance when the plate, S, is interposed is denoted by y, then the initial illumination is given by $\frac{I}{x^2}$, where I is the luminous intensity of L_2, while the final illumination is given by $\frac{pI}{y^2}$, where p is the fraction of the light transmitted by the glass plate. Since the other source, L_1, remains at a fixed distance from P, these illuminations are equal—that is,

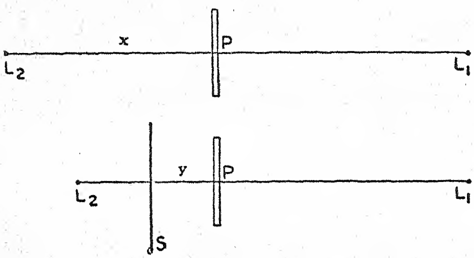


FIG. 214.

$$\frac{I}{x^2} = \frac{pI}{y^2}, \text{ from which } p = \frac{y^2}{x^2}.$$

Hence the percentage of light transmitted is given by

$$100p = 100\frac{y^2}{x^2}.$$

The fraction of the light transmitted is sometimes called the *transmission factor* of the plate.

(3) *To find the reflection factor of a plane mirror.*—The photometer is set up between two sources of light, as in the previous experiment, and adjusted for equality of illumination as usual. Then, the mirror, M (Fig. 215), is placed at an angle of 45° to the axis of the photometer, and the source, L_2, is moved into the position, B, so that its image formed by reflection in the mirror is at B′, say, the distance being adjusted to give equality of illumination again. If the initial distance, AP, of the source, L_2, from the photometer is

denoted by x, and the final distance, B′P = BN + NP, is denoted by y, then

$$\frac{I}{x^2} = \frac{rI}{y^2},$$

where I is the luminous intensity of the source and r is the required reflection factor.

Hence,

$$r = \frac{y^2}{x^2}.$$

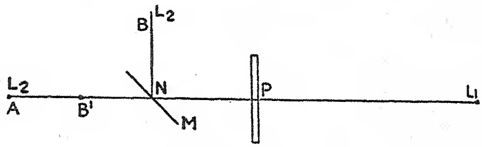


FIG. 215.

12. Measurement of Luminous Flux

The measurement of the luminous intensity, or candle-power, of a source of light by means of the photometer is of fundamental importance. It has become customary, however, to specify sources of light by stating their total luminous flux, or output of light. This quantity could be calculated if the luminous intensity were to be measured in a large number of different directions, but this would be a laborious process, and in order to avoid this, methods of measuring the total luminous flux directly have been devised. Photometers for this purpose are known as **integrating photometers.**

If a source of light is placed inside a hollow sphere, the inner surface of which is perfectly diffusing, it can be shown that the illumination of every part of the surface due to light scattered from the remainder of the surface is the same and is proportional to the total luminous flux emitted by the source. This statement expresses the principle on which integrating photometers are based.

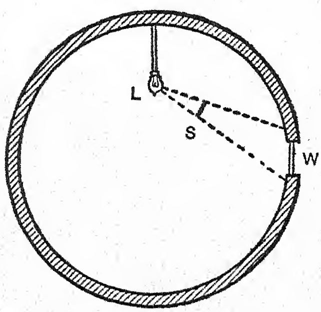


FIG. 216.

The method of use is to introduce the source of light, L (Fig. 216), such as an electric lamp, into a sphere, the inner walls of which are coated with a suitable paint. In the walls is fitted a window, W, of opal glass, and an opaque screen, S, prevents light from L reaching W directly. According to the principle stated above, the luminous flux from L produces the same illumination on the window, W, irrespective of the direction in which it is emitted by the lamp—that is, the illumination of W is proportional

to the total luminous flux of L. The brightness of W, with L in the sphere, is then compared with the illumination from a standard lamp, the total luminous flux of which has been determined by measuring the luminous intensity in all directions.

13. Measurement of Illumination

In planning a lighting scheme for a room or large hall, it is important to know how to produce sufficient illumination where it is actually necessary, and in dealing with the problem it is often more important to measure the illumination at a given place than to know the luminous intensity and the luminous flux of the source of light used. For this purpose, portable instruments known as *illumination photometers* have been devised. Many forms of such apparatus are available, and the principle and use of the Macbeth Illuminometer will be described.

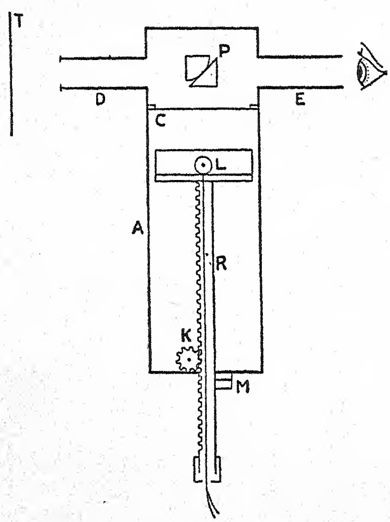


FIG. 217.

A white surface, T (Fig. 217), is placed in the position at which the illumination is to be measured, and its brightness is compared with that of another similar surface, C, fixed in the instrument, the illumination of which can be varied. The instrument consists of a tube, A, fitted with short side tubes, D and E, an eye-piece being fixed in E. P is a double prism similar to that used in the Lummer-Brodhun photometer (see page 267). The comparison surface, C, of opal glass is illuminated by the small lamp, L, and the illumination varied by moving L. To effect this, L is attached to the rod, R, which can be moved in or out by the milled wheel, K, its position being shown by the indicator, M, on a scale fixed along R.

In making a test, the side tube, D, is pointed towards the illuminated surface, T, and the observer looks through E, at the prism, P. The outer part of the field of the eye-piece is illuminated

by C, and the centre of the field by T. The position of L is adjusted until equality of illumination is established, and the scale reading on R is taken. The scale is calibrated previously, by noting the position of the lamp, L, for equality of illumination, when T is illuminated by known values of illumination, the calibration being carried out as a separate experiment.

14. Sources of Light

Nearly all light is produced either by burning or by incandescence. The earliest domestic sources of light consisted either of wicks immersed in oil, *lamps*, or of wicks immersed in solid fats, *candles*. Both these sources are still in use, and have been greatly improved in recent years. Coal-gas was first produced on a commercial scale by Murdoch in 1792, and was then burnt only at naked burners. Argand in 1884 improved the light of both oil and gas lamps by putting a glass chimney around the flame to produce a draught of air which caused better combustion. Bunsen increased the temperature of the flame by mixing air with the gas before combustion, but this made the flame non-luminous. If a body, however, especially a refractory oxide, is placed in a flame, it is heated to incandescence, and gives out a brilliant light. The outcome of this discovery was the Welsbach mantle, 1893. The mantle, a gauze-like structure, is made up of 99 per cent. thorium oxide and 1 per cent. cerium oxide, the latter possessing great light-emitting properties. Acetylene gas, owing to its strongly luminous flame and ease of preparation, is also used largely for lighting.

With the advent of electricity, the electric arc and the electric incandescent lamp were introduced. The former was invented by Davy in 1801, and was used for street lighting and optical lantern work, its light being very white. One form of the arc lamp used carbon rods, impregnated with metallic salts, the arc itself being the principal source of light while the flame is made intensely luminous by the vaporised salts. A modern development of the arc provided the Pointolite lamp. In this an arc is maintained between a small sphere of tungsten and a short length of tungsten wire, the sphere forming almost a point source of light which is often of great use in the laboratory. The ordinary incandescent electric lamp was provided with carbon filaments in a vacuum, the first practical form being produced in 1879 by Edison in America, and Swan in England. In the modern lamp, the filament is made of tantalum or tungsten—mainly the latter, and the filament is in an atmosphere of some inert gas, such as nitrogen or argon.

The development of the electric lamp has been directed towards the increase of its luminous efficiency, which is usually expressed by quoting the total luminous flux in lumens per watt of electrical power supplied. With the early carbon filament lamp, efficiencies of the order of 3·5 lumens per watt was the best that could be obtained. The tungsten filament vacuum type lamp of 1911 had an efficiency of 10 lumens per watt, while the modern coiled-coil lamp has an efficiency of 15 lumens per watt. Experiment shows that if 1 watt of electrical power could be converted completely into luminous energy of the wave-length to which the eye is most sensitive, it would produce a luminous flux of 620 lumens, and even if the light were distributed uniformly over the whole range of the visible spectrum, it would still produce about 250 lumens. The electric filament lamp is very far short of the ideal in efficiency because the greater part of the energy used in heating the filament is lost, partly owing to thermal conduction and convection, and partly because the greater part of the radiated energy falls in the invisible infra-red region of the spectrum. The light given out by the sun carries 35 per cent. of the total output of energy, while that emitted by the glow-worm and firefly carries 99 per cent., so that, in comparison with Nature, man's appliances would appear to be very feeble.

In recent years the development of the so-called gas-discharge lamp has produced a much more efficient source of light, and such lamps, in addition to their familiar use as advertisement signs, are gaining in popularity as sources of illumination, particularly in offices and factories and for street lighting. The principle involved is the passage of an electric discharge through a gas or vapour at low pressure contained in a suitable tube. The first practical application of this was made by Moore in 1895. By using different gases and coloured glass tubes, a large variety of colours can be obtained. Thus, neon gives a red colour in ordinary glass, argon gives blue, carbon dioxide gives white, mercury vapour in brown glass gives green, while a mixture of neon and mercury vapour in ordinary glass gives blue. The earliest gas-discharge lamp required about 10,000 volts for its operation, and its efficiency was low. The most recent lamps, however, work at ordinary supply voltages, and the efficiency is relatively high. Thus the familiar sodium vapour and mercury vapour lamps have efficiencies of from 35 to 40 lumens per watt. By coating the walls of the tube of a mercury vapour lamp with suitable fluorescent material, an efficiency of 60 lumens per watt has been obtained, and in addition a much whiter light, instead of brilliant green, is produced.

15. Photo-electric Methods of Photometry

The methods of photometry, described in this chapter so far, are purely optical and make use of the eye as the instrument of comparison. It is possible to use instead for this purpose various *electrical* effects of light. Among effects that can be used may be mentioned:

(*a*) *Photo-conductivity.* The electrical resistance of certain substances such as selenium varies with the intensity of illumination.

(*b*) *The photo-electric effect.* Certain metals emit electrons when light falls on them (see page 318). The only metals that do this in visible light are those of the alkali group. The photo-electric cell consists of an electrode of potassium or caesium arranged in an evacuated bulb with a collecting electrode. The emission current is proportional to the intensity of light (provided that a sufficient potential is applied to the collecting electrode). This current is very small and requires a valve-amplifier for its detection.

(*c*) *The photo-voltaic effect.* Certain types of contact between a metal and its oxide show peculiar effects, such as electric rectification. In particular, if a plate of copper is covered by a thin layer of the red oxide (Cu_2O), and the whole is illuminated, a difference of potential between the body of the plate and the oxide layer is set up. The mechanism of this effect is complicated and not perfectly understood. A photo-voltaic cell is made by lightly oxidising a copper disk, contact with the oxide layer being made by clamping a metal ring to the surface of the disk near its edge. If the cell is now connected to a microammeter we have an instrument that can measure intensity of illumination *directly*, because the deflection of the microammeter will be a measure of the rate at which radiant energy is falling on the oxide layer. In commercial instruments the microammeter is calibrated directly in foot-candles.

The photo-voltaic cell is much superior to other methods of photo-electric measurement because it needs no battery. A calibration once made persists unchanged for a long time. Such an instrument can easily be made small enough for the pocket, and a survey of the illumination of (say) a schoolroom or workshop can be made in a few minutes, whereas by purely optical methods it would take much longer. The instrument measures intensity of illumination, but it can obviously be adapted to the measurement of candle-power and distribution of radiant energy round a given source of light simply by placing it in various positions round the

source and at known distances from it. In a typical instrument the sensitive surface is about 2 inches in diameter, so the variation of illumination over this surface is not likely to be serious. A smaller version is used as an **exposure-meter** in photography.

Photo-voltaic cells can be made of other materials besides copper and copper oxide. One recipe employs selenium, which can show a photo-voltaic effect in addition to its well known property of photo-conductivity. The two effects should not be confused.

In illuminating engineering, it is desirable that the ratios of the responses of the measuring instrument to light of different colours should approximate to those of the average eye. It is possible to make both photo-electric and photo-voltaic cells that satisfy this requirement very well, even without the use of colour-filters.

CHAPTER XIV

THE THEORY OF LIGHT

MANY of the effects of light and many crude optical instruments were known to the ancients, but of the theory of light they were wholly ignorant. **Pythagoras**, B.C. 540-510, and **Plato**, B.C. 430, maintained that vision was a threefold phenomenon. Their idea was that the human eye sent out a stream of potency or divine fire which combined first with the light of the sun and then with the emanation from the third body, the second combination completing the act of vision. **Aristotle**, B.C. 350, struck the right chord by maintaining that light was not a material emission from a source, but a mere quality or potentiality of a medium existing between the eye and the body seen.

Although the study of science, including optics, was prevalent during the centuries that followed, and great progress was made in many branches of science, the work in optics was mainly experimental. This experimental work was concerned with the principal phenomena of light and the production of optical instruments, the fore-runners of present-day instruments, but little or no thought was given to the theoretical explanation of optical phenomena until the time of **Newton**, 1642-1727, who adopted the *corpuscular* theory of light. About the same time, **Huyghens**, 1678, adopted the *wave* theory of light, although it may be said that this theory really began with Aristotle. The fundamental ideas of these two theories will be explained in this chapter.

Although a tremendous amount of work has been carried out since the time of Newton and Huyghens, in an endeavour to establish which of these two theories will explain completely the nature of light and optical phenomena generally, the position now is that two theories still hold, each explaining part of the facts which are inexplicable by the other. Light seems to show a twofold character, and its behaviour can be explained only by considering it sometimes as waves and at other times as particles or corpuscles. At the end of the nineteenth century it was thought that the wave theory had become a firmly established doctrine. Since then, however, new discoveries as to the behaviour of light, concerned mainly with its emission and absorption by matter, have been made which could not be explained by the wave theory. This led to the evolution of

a new form of corpuscular theory by **Einstein,** which, though very different in detail from Newton's theory, gave a simple interpretation of these new discoveries.

1. The Corpuscular Theory

As stated above, Sir Isaac Newton upheld the **corpuscular** or **emission** theory of light. According to this theory, light was supposed to be a swarm of corpuscles or particles ejected from a luminous body at great speed, and these corpuscles on entering the eye excited the sensation of vision.

It is easy to see that on this theory the rectilinear propagation of light (see page 7) is at once explained, for it is necessary only to make the assumption that the light particles, while travelling in a uniform medium, are not subjected to any force, and that, therefore, in accordance with Newton's first law of motion, they travel with a uniform constant velocity in a straight line.

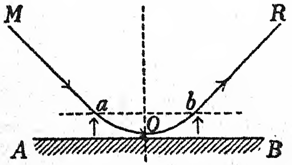


FIG. 218.

Other optical phenomena, such as reflection and refraction, can also be shown to follow from the theory by making very simple assumptions regarding the behaviour of the particles at the boundary surface between two media.

2. Reflection and Refraction on the Corpuscular Theory

Although the corpuscular theory of light, as enunciated by Newton, cannot now be accepted, it is necessary to study it a little in order that the student may realise how decisive is the evidence against it provided by Foucault's experiment (see Art. 8). Briefly, the suggestion was that the corpuscle approaching a reflecting medium was subjected to a *repulsive* field extending for a short distance above the surface and always directed normally to it (Fig. 218). The law of equality of the angles of incidence and refraction was then a simple consequence of the laws of conservation of energy and momentum. For, if the force is normal to the surface, the tangential component of velocity is unaltered, while the normal component is exactly reversed as the particle proceeds from a to b, reaching zero at O (Fig. 218). Similarly, a transparent medium was supposed to exert a normal *attractive* force (Fig. 219). Again the tangential component of velocity is unaffected, and if V_a and V_b are the total velocities in the two media (outside the small region near the surface where the forces are supposed to be

appreciable), this implies that $V_a \sin i = V_b \sin r$, which can be made to agree with Snell's Law, *provided that we assume that light travels* faster *in the medium with* greater *refractive index.* It will be pointed out below (Art. 8) that this is in contradiction with experiment. In addition, since reflection implies a repulsive force on the corpuscle, and transmission into an optically denser medium implies an attraction on the corpuscle, it is clearly not possible to account in any simple way for the experimental fact that a beam of light incident on a transparent medium is *partly reflected and partly transmitted.*

3. The Wave Theory. (*a*) The "Elastic Solid" Theory

The wave theory of light, first advanced by Huyghens in 1678, postulated the existence throughout all space of a medium, the ether, vibrations of which constitute radiation. The exact details of the method of propagation were left unspecified, but it will be shown below (Arts. 5, 6 and 7) that all the ordinary properties of light can nevertheless be accounted for on the wave hypothesis without any detailed assumptions. There is a further set of phenomena known as interference effects (Chapter XV.) which can be accounted for by the wave theory but not by the emission theory of Newton. The phenomena of polarisation (Chapter XVI.) give clear evidence that the vibrations constituting light are transverse to the direction of propagation of the wave.

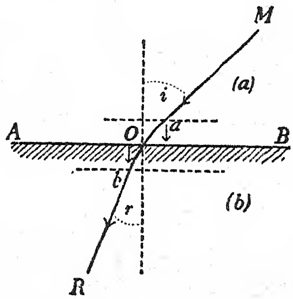


FIG. 219.

So far the postulate is that cosmic rays, gamma rays, X-rays, ultra-violet, visible and infra-red light and wireless waves are all vibrations of the luminiferous ether and differ only in wave-length and frequency. In order to deal quantitatively with such effects as the partial reflection and transmission of light at the boundary of two transparent media it is necessary to make some assumptions about the *mechanism* of propagation. The first attempt to do this was due mainly to Fresnel. He endowed the ether with mechanical properties like those of an *elastic solid*. Since we know that light waves are transverse, it is not possible to think of them as simple compressions and rarefactions like sound waves, but an *elastic* solid is also capable of propagating transverse vibrations or waves

of *shear*, and Fresnel's idea was that these were light waves. This theory accounts quantitatively for very many of the facts, including the distribution of energy between the reflected and refracted beams at the boundary of two media. The reasons for rejecting it are complicated, but the main one will be understood when we consider the electromagnetic theory. In addition, it is not easy to think of celestial bodies moving rapidly, without loss of energy, through a medium of great mechanical rigidity.

4. The Wave Theory. (*b*) The Electromagnetic Theory

The electromagnetic theory of light cannot be understood without consideration of the relationships between electricity and magnetism, for which an advanced textbook on electricity must be consulted (see *Advanced Textbook of Electricity and Magnetism*, Hutchinson). It may be stated here that it was shown by Maxwell that the relationships between electricity and magnetism already established experimentally by Faraday and Ampère had as a logical consequence the possibility of the propagation through space of a self-maintaining disturbance alternately of electric and magnetic nature. The velocity of this propagation could be calculated and was found to be very near the observed velocity of light. Again, the behaviour of electric and magnetic fields at the boundary between two media is known from fairly elementary considerations. For example, it can be shown from the Principle of Conservation of Energy that the component of the electric field lying parallel to the boundary must be the same in both media. It was then found that these very same boundary conditions enabled deductions to be made (without introducing any new assumptions) about the ratio between the intensities of the reflected and transmitted components of light at the boundary between two transparent media that are in agreement with experiment. In particular, Brewster's Law (Chapter XVI., Art. 4) can be accounted for.

Thus the electromagnetic theory is an almost direct deduction from experimental laws and will explain everything that the "elastic solid" theory will, without the necessity of introducing arbitrary assumptions about the properties of the ether. To be sure, the ether, or, if one prefers it, space, must have *some* properties to enable action at a distance to occur at all, but the electromagnetic theory has the great merit of accounting for the facts without introducing any more assumptions about space or the ether than are already necessary to account for the experimental facts of electricity and magnetism, while the elastic solid theory

necessitates the assumption of hypothetical properties of the ether *ad hoc*.

A more serious difficulty that would now confront any attempt to revive the elastic solid theory is provided by the now famous Michelson-Morley experiment, which together with other experiments, was devised to try to detect the motion of the earth through the ether. Briefly, the conclusion from these experiments is that two observers will always obtain the *same* experimental value for the velocity of light even if their beams of light are differently oriented with respect to the earth's orbit. A logical extension of electro-magnetic theory (the so-called restricted theory of relativity), explains these experiments in a natural way, but they are quite unintelligible on the elastic solid theory.

For most purposes it is therefore unnecessary to consider the properties of the ether. In the remainder of this chapter the "wave theory" is to be taken merely as the assumption that light phenomena consist of transverse oscillations of very short wave-length and high frequency travelling in a vacuum with a velocity of 186,000 miles per second.

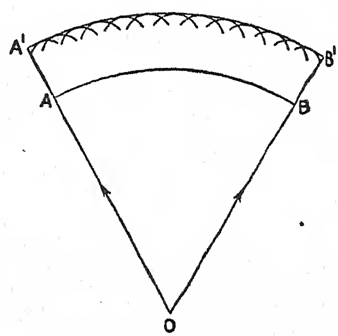


FIG. 220.

5. Rectilinear Propagation on Wave Theory

The explanation of the rectilinear propagation of light was a strong point with the supporters of the corpuscular theory, for, granted that the corpuscles moved in straight lines until acted on by the surfaces of media, the theorem that light travels in straight lines is self-evident. On the wave theory, however, it proved at first a great stumbling block.

In order to overcome the initial difficulty of explaining rectilinear propagation on the wave theory, Huyghens made the assumption that each point on a wave front could be treated as a source of secondary waves.

Thus, suppose that O (Fig. 220) is the centre of a spherical disturbance which at the time considered has reached AB. It must be remembered that the disturbance is actually spread over the whole surface of a sphere of which O is the centre so that AB merely

represents a section of a circle in the plane of the diagram. AB is therefore a portion of the wave front due to a source at O. Each point on AB is the centre of a new disturbance known as a secondary wave, according to Huyghens. At a time, t, after the disturbance has reached AB, each of these secondary disturbances will have travelled a distance, Vt, where V is the velocity of light in the medium. So that, if $AA' = BB' = Vt$, the wavelets will all touch a curve, A′B′. Consequently A′B′ becomes the new wave front.

When the distance of the wave front from the source of disturbance is very great, as in the case of light coming from a star, the wave front becomes plane if considered in small portions—that is, portions of a sphere of infinite radius.

So far, Newton fully appreciated Huyghens' theory, but he abandoned it in favour of the corpuscular theory because he could not reconcile it with the fact that all waves known to him, such as sound waves and water waves, could bend round corners whilst light was propagated in straight lines. Very simple experiments with ripples on water and with sound waves are sufficient to show clearly that the extent to which waves bend round the edge of an obstacle depends directly upon the wave-length of the disturbance and its relation with the linear size of the obstacle. Now the wave-length of the sound waves caused by a tuning-fork of frequency 512 c.p.s. is about 70 cm., while the wave-length of the yellow or mean light of the spectrum is only 0·000059 cm., and hence it would be expected that the bending in the case of light would be extremely small. In other words, the bending is so small that it is justified to regard the propagation of light as approximately rectilinear. Fresnel, by using Huyghens' theory, together with the idea of interference introduced by Young (see page 287), gave a satisfactory explanation of this, and showed that light does in fact bend round corners, or obstacles, but that, owing to the extreme shortness of the wave-lengths of light, this bending is very small, and hence it appears to travel in straight lines.

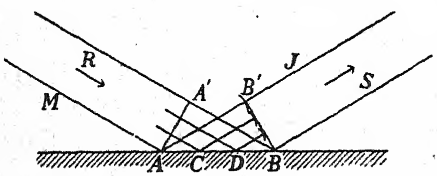


FIG. 221.

6. Reflection on Wave Theory

(*a*) PLANE WAVES.—Suppose that AA′ (Fig. 221) is a wave front of a plane wave incident on the plane reflecting surface, AB.

According to Huyghens, when the wave front reaches AB, secondary disturbances immediately form at the points touched. Thus points, A, C, D become centres of disturbance in turn, and a series of spherical waves spreads outwards from each point, being reflected back into the medium. During the time that it takes for A′ to travel to B, a time, *t* say, a secondary wave from A travels a distance, AB′, equal to V*t*, where V is the velocity of the wave in the medium.

To determine the new wave front, draw a circle with centre, A, and radius, V*t*. Draw the tangent, B′B, to this circle, passing through B. Then B′B is the new wave front; for a wave leaving A′ will reach B at the same time as the secondary wave leaving A reaches B′, and similarly for waves between A′ and A. Thus, the wave front proceeds in a direction perpendicular to B′B. Now

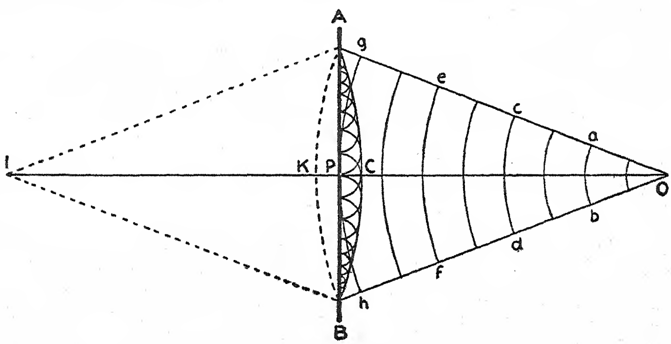


FIG. 222.

since A′B = AB′, it is evident that the angle, A′AB, is equal to the angle, ABB′, from which it follows that the angle of incidence is equal to the angle of reflection. Also, it is evident from the figure that the incident beam and the reflected beam lie in the same plane, that of the figure. Thus, the results are in agreement with the laws of reflection at a plane surface (see page 17).

(*b*) SPHERICAL WAVES.—Suppose that AB (Fig. 222) represents a plane reflecting surface, and that a source situated at O emits spherical waves, the wave fronts at different distances from O being represented by *ab*, *cd*, etc. If OP is perpendicular to AB, the wave front, *gh*, will touch AB at one point only, P. This gives rise to a secondary disturbance from P. If the reflecting surface was absent, the wave front, *gh*, would have reached the position,

AKB, in a time, t say, and thus the secondary wave has a radius equal to PK or PC. Each point in the wave front, *gh*, will in turn reach AB, and set up similar disturbances, and it is evident that all these secondary waves will touch the curve, ACB, where the radius of ACB is equal to that of AKB, but opposite in direction. Thus, the reflected wave front is ACB, which is such that it appears to diverge from a point, I, the image of P, the distance, IP, being equal to the distance, OP. This is in accordance with the formation of the image of an object by reflection at a plane surface (see page 19).

(*c*) SPHERICAL WAVES AND SPHERICAL SURFACE.—Suppose that APB (Fig. 223) represents a concave spherical reflecting surface, the centre of curvature being C, and the radius of curvature, PC, and that a source situated at O emits spherical waves. Let the wave front be ADB when the waves first meet the reflecting surface. If it is assumed that the radius of curvature, PC, is large compared with the aperture of the surface, AB, which is usually the case in practice, then while the disturbance at D travels to P, secondary disturbances will be set up at A and B which will travel to E and F respectively, where AE = PD = BF. The reflected wave front is thus EPF, which is assumed to be spherical and which will converge to a point, I, on the axis of the reflecting surface. Draw perpendiculars, AG = EH, from A, E, to PC.

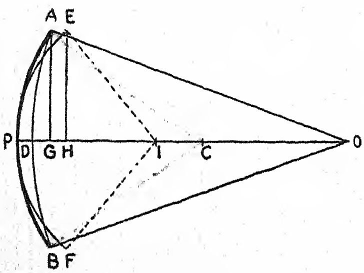


FIG. 223.

By geometry, $AG^2 = 2OD.DG$, from which $DG = \dfrac{AG^2}{2OD}$. Similarly, $PH = \dfrac{EH^2}{2ID}$, and $PG = \dfrac{AG^2}{2PC}$, approximately.

$$\text{But, } PH = PG + GH = PG + PD = 2\,PG - DG,$$

$$\text{or } PH + DG = 2\,PG;$$

$$\therefore \frac{EH^2}{2ID} + \frac{AG^2}{2OD} = 2\frac{AG^2}{2PC}.$$

Since $AG = EH$, and putting $OD = u$, $ID = v$, and $PC = r$, according to the usual notation and either sign convention,

$$\frac{AG^2}{2v} + \frac{AG^2}{2u} = 2 \cdot \frac{AG^2}{2r},$$

$$\text{from which } \frac{1}{v} + \frac{1}{u} = \frac{2}{r},$$

the well-known general relation for reflection at a spherical surface.

7. Refraction on Wave Theory

(*a*) PLANE WAVES.—Suppose that AB (Fig. 224) represents the plane surface of separation of two transparent media, (*a*) and (*b*), and that AA′ represents the wave front of a disturbance incident in the direction shown on the surface of separation. As before, each point of AA′ becomes in turn a source of secondary disturbances as it reaches the surface at A, C, D, etc., and a series of spherical waves spreads out into the medium, (*b*). Assuming that the velocity of the waves is not the same in each medium, the new wave front after some time, *t* say, will be in a position such as B′B. If the velocities in the two media are represented by V_a and V_b, and if the wave front travels from A′ to B in the time, *t*, then $A'B = V_a t$. The new wave front will be such that the secondary waves from A travel to B′ in the same time, *t*, so that $AB' = V_b t$.

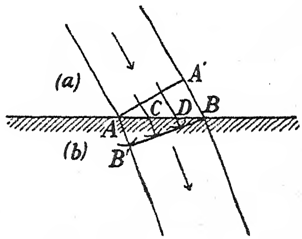


FIG. 224.

Thus to determine the new wave front, draw a circle with centre, A, and radius, $V_b t$. Draw the tangent, B′B, to this circle through B. Then B′B is the new wave front, and this wave front will travel in a direction perpendicular to B′B in the medium (*b*).

Thus, if $V_a > V_b$, as in the figure, $A'B > AB'$, and the beam is deviated towards the normal in the second medium.

$$\text{Also, } \frac{\sin i}{\sin r} = \frac{\sin A'AB}{\sin ABB'} = \frac{A'B/AB}{AB'/AB} = \frac{A'B}{AB'}.$$

$$\text{But, } A'B = V_a t, \text{ and } AB' = V_b t;$$

$$\therefore \frac{\sin i}{\sin r} = \frac{V_a}{V_b},$$

a constant for the same two media. Thus, the results are in

accordance with the laws of refraction at a plane surface (see page 59). It should be noted particularly that, on the wave theory, the ratio, $\frac{\sin i}{\sin r}$—that is, the refractive index *from medium* (*a*) *to medium* (*b*)—is equal to *the velocity in* (*a*) *divided by the velocity in* (*b*).

*(*b*) SPHERICAL WAVES AND LENS.—Suppose that CDE (Fig. 225) represents the wave front of a system of spherical waves emitted from an object, O, on the principal axis of a lens, AB. In this case, the central portion of the wave front strikes the lens first at D. Assuming that the material of the lens is more dense than the surrounding medium, and that the secondary disturbances set up when the wave front meets the surface of the lens travel more slowly in the lens, the disturbances at C and E travel to F and H respectively, while that at D will travel to G. Thus the new transmitted wave front, which is assumed to be spherical, is FGH with its centre at I to which it converges.

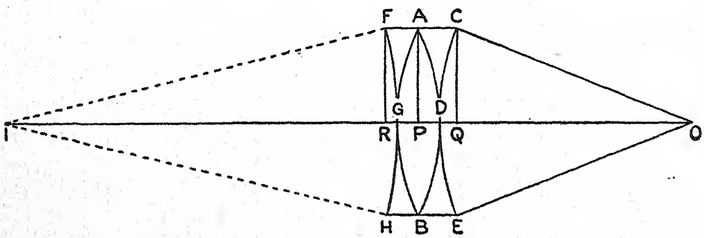


FIG. 225.

If the refractive index of the material of the lens is denoted by μ, then, since the optical paths are equal, CF $= \mu$.DG (see (*a*) above).

Draw perpendiculars from A, C, and F, to the axis of the lens, and let AP = CQ = FR = h. Adopting the usual notation, so that OP is denoted by u, IP by v, and the radii of curvature of the surfaces of the lens are denoted by r_1 and r_2 respectively, and also adopting the usual sign convention, then by geometry

$$QD = \frac{h^2}{2u},\quad DP = \frac{h^2}{-2r_1},\quad PG = \frac{h^2}{2r_2},\ \text{and}\ GR = \frac{h^2}{-2v}\ \text{approx.}$$

$$\text{Now } CF = \mu.DG,$$

$$\text{or, } QD + DP + PG + GR = \mu\,(DP + PG),$$

$$\text{or, } QD + GR = (\mu - 1)\,(DP + PG).$$

$$\text{Hence } \frac{h^2}{2u} - \frac{h^2}{2v} = (\mu - 1)\left(-\frac{h^2}{2r_1} + \frac{h^2}{2r_2}\right),$$

$$\text{or } \frac{1}{v} - \frac{1}{u} = (\mu - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right),$$

which is the general relation for the position of the image of an object, formed by refraction through a lens (see page 105).*

8. Crucial Test between the Corpuscular and Wave Theories

If the refractive index between two transparent media, *a* and *b*, be denoted by μ, and the velocities of light in these media be denoted by V_a and V_b respectively, then the relation between these three quantities, according to the Emission theory, is given by

$$\mu = \frac{V_b}{V_a} \text{ (see Art. 3)},$$

and, according to the Wave theory, is given by

$$\mu = \frac{V_a}{V_b} \text{ (see Art. 7)}.$$

Thus, in a medium such as water, for which $\mu > 1$, the corpuscular theory states that the velocity of light is greater than the velocity in air, whereas the wave theory postulates the reverse. Foucault decided this point (see page 251) by experiment in favour of the wave theory, and although this does not prove the wave theory to be correct, it certainly proves that the emission theory, as enunciated above, is wrong. These results were assumed in the work on refractive indices in a previous chapter (see page 65).

Other phenomena met with in the study of light, such as interference, diffraction, polarisation, which will be dealt with in following chapters, are quite in harmony with the corresponding phenomena met with in the study of transverse wave motion in general, and support the wave theory of light. Recent work on infra-red rays or waves, and electric waves, including those used in *wireless*, is greatly in favour of the electromagnetic theory. Infra-red waves have been traced to a wave-length greater than that of the shortest known electric waves, so that there is now a distinct region of overlap which can be produced and detected by two quite distinct techniques (see page 279). Also, the properties of the infra-red waves approximate, as the wave-length increases, to the properties of the electric waves, tending to show that the difference between electric waves and light waves is essentially one of wave-length only—that light waves are in fact electromagnetic waves just the same as wireless waves, but of very much shorter wave-length.

APPENDIX TO CHAPTER XIV

ALTERNATIVE PASSAGE IN TERMS OF THE REAL IS POSITIVE SIGN CONVENTION

7. Refraction on Wave Theory

(*b*) Spherical Waves and Lens.—Suppose that CDE [Fig. 255 (*a*)] represents the wave-front of a system of spherical waves emitted from an object, O, on the principal axis of a lens, AB. In this case the central position of the wave front strikes the lens first at D. Assuming that the material of the lens is more dense than the surrounding medium, and that the secondary disturbances set up when the wave front meets the surface of the lens travel more slowly in the lens the disturbances at C and E travel to F and H respectively, while that at D will travel to G. Thus, the new transmitted wave front, which is assumed to be spherical, is FGH with its centre at I to which it converges.

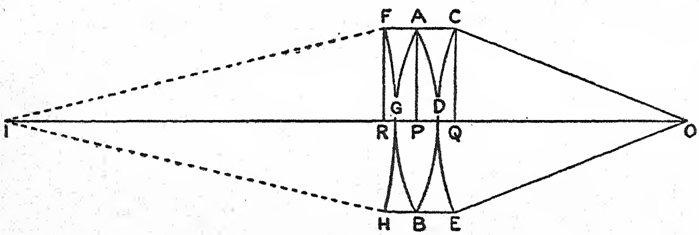


Fig. 225 (*a*).

If the refractive index of the material of the lens is denoted by μ, then, since the optical paths are equal, F $= \mu$.DG [see p. 283, Art. 7 (*a*)]. Draw perpendiculars from A, C, and F, to the axis of the lens and let AP $=$ CQ $=$ FR $= h$. Then, as the figure is drawn O is a real object and I a real image, so that OP $= u$, IP $= v$ which are both positive. Again, ADB is a surface convex to the incident wave front, and its radius of curvature must be reckoned positive and equal to r, while AGB is concave to the incident wave front and its radius of curvature r_2 must be reckoned negative, because, although it is a converging surface, the refraction is from glass into air.

By geometry

$$QD = \frac{h^2}{2u}, \quad DP = \frac{h^2}{2r}, \quad PG = \frac{R^2}{-2r_2}, \quad GR = \frac{h^2}{2v} \text{ approx.}$$

$$\text{Now } CF = \mu DG$$

$$\text{or } QD + DP + PG + GR = \mu\,(DP + PG),$$

$$\text{or } QD + GR = (\mu - 1)\,(DP + PG).$$

$$\text{Hence } \frac{h^2}{2u} + \frac{h^2}{2v} = (\mu - 1)\left(\frac{h^2}{2r_1} - \frac{h^2}{2r_2}\right)$$

$$\text{or } \frac{1}{u} + \frac{1}{v} = (\mu - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

which is the general relation for the position of the image of an object formed by a lens (see page 123d).

CHAPTER XV

INTERFERENCE AND DIFFRACTION

IT is fully explained in textbooks on Sound that when wave motion from more than one source is travelling through any medium, the disturbance at any point in the medium is the *resultant* of the individual component disturbances reaching the point at that instant from the several sources.

Thus, consider two sources sending out transverse waves of equal wave-length and amplitude, in the same direction, and consider any fixed point in the medium. If the individual wave systems at this point are in the same phase—that is, if they arrive, as it were, *crest to crest* and *trough to trough*—they will reinforce each other, and the amplitude of the disturbance at the point will be doubled. If, however, one system is half a wave-length behind the other—that is, if they arrive, as it were, *crest to trough*—they will cancel each other, and the disturbance will be nil. This is the principle of **interference** or superposition as applied to wave motion.

Young, 1801, was the first to state clearly the principle on which the phenomenon of interference depends, and in 1807 published a paper containing experimental details of the interference of light. The interpretation of the results obtained were criticised, however, until **Fresnel,** a few years later, completely removed the doubts and showed by means of his *bi-prism* (see Art. 4) that interference was an established fact. These were very important advances in the development of the Wave Theory of light.

It has been stated already that **Newton's** objection to the wave theory lay in the fact that light travels in straight lines and does not, as in the case of sound, bend round corners. **Fresnel,** however, proved conclusively that light does bend round corners, and the comparison with sound is merely a question of degree. **Grimaldi** discovered **diffraction,** a particular case of this.

1. The Conditions for Interference

If two sources of light, X, Y (Fig. 226), very near to one another, could be made to vibrate exactly in phase, then the waves would arrive in phase at all points on the plane bisecting XY at right angles, but at a point such as P a phase difference would be set up. As XP − YP increases, the phase difference becomes larger,

and when this distance is equal to half a wave-length, the effect of the two sources taken together exactly cancels at all times, because there will be a constant phase difference of π. If we increase this distance still further, the cancellation will not be complete, and when XP — YP becomes equal to a whole wave-length the effect of the two sources will reinforce. A little consideration of the geometry of the arrangement will show (Fig. 226) that as P moves along the screen, the distance XP — YP will be alternately an even and an odd number of half-wave-lengths so that one would expect a pattern of bright and dark bands, shading continuously into one another, to be formed on the screen. The apparent paradox of two lights producing local darkness does not mean that there is any conflict with the principle of conservation of energy. Dark bands cannot be produced without bright bands, and it can easily be shown that the energy missing from the dark bands reappears in the bright bands.

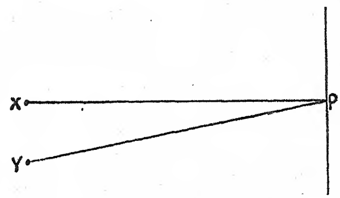


FIG. 226.

In practice, it is not possible to produce interference from two separate sources of light. All interference experiments involve the use of light from *one* source which is split up into two or more beams travelling along different paths. The reason for this can be understood if one refers to the quantum theory of spectra (Chapter XVII.). Atoms and molecules do not emit light continuously, but in very short bursts or quanta of the order of 10^{-8} second in duration, so that any real source of light is subject to rapid and random phase changes. Thus, it is only when the interfering beams come from one source, in which case any phase change will affect equally all the component beams that an interference pattern lasting long enough to be observed can be expected. The simplest method of demonstrating interference is by means of two slits illuminated by one source.

2. Young's Experiment

Young admitted a beam of sunlight through a slit, X (Fig. 227), into a darkened room. The beam was then allowed to fall on a screen containing two pinholes, S and S′. The effect produced on the screen, PY, was the result of the interference of waves from the two sources, S and S′, vibrating in the same phase, bright and dark bands or fringes being produced.

If P is the central point on the screen, and PA bisects SS′ at right angles, then if PY is parallel to SS′, there will be brightness or darkness at Y according as S′Y − SY is equal to an even or an odd multiple of $\frac{\lambda}{2}$, where λ is the wave-length of the light.

$$\text{Let } SS' = 2d, \ PA = D, \text{ and } PY = y.$$

$$\text{Then, } S'Y^2 = D^2 + (y + d)^2,$$

$$\text{and } SY^2 = D^2 + (y - d)^2.$$

$$\text{Hence, } S'Y^2 - SY^2 = 4yd,$$

$$\text{or } S'Y - SY = \frac{4yd}{S'Y + SY}.$$

But the interference bands produced are all close to P, and SS′ is small, so that both y and d are very small compared with D. Hence:—

$$S'Y - SY = \frac{4yd}{2D} = \frac{2yd}{D} \text{ approximately.}$$

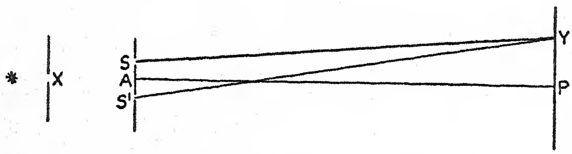


FIG. 227.

Thus the path difference is $\frac{2yd}{D}$, so that for *bright* bands,

$$\frac{2yd}{D} = 2n\frac{\lambda}{2}, \text{ giving } y = \frac{n\lambda D}{2d},$$

and for *dark* bands,

$$\frac{2yd}{D} = (2n + 1)\frac{\lambda}{2}, \text{ giving } y = \frac{(2n + 1)\lambda D}{4d},$$

where n is a whole number.

Thus the distance between consecutive bright or dark bands is $\frac{\lambda D}{2d}$.

When monochromatic light is used, alternate bright and dark bands are produced. In the case of white light, the central band only remains white, the others being coloured and ill-defined owing to overlapping of various wave-lengths.

Instead of forming the bands on a screen, a travelling microscope, or a Ramsden eye-piece may be used, and the bands observed directly.

3. Fresnel's Mirrors and Fresnel's Bi-prism

These methods make use respectively of reflection and refraction to produce two close images of a single slit. The arrangement of Fresnel's mirrors is shown in plan in Fig. 228, A being a point on the slit, and S and S′ its images in the two mirrors.

Since O is on both mirrors, we have OA = OS = OS′;

$$\therefore \text{ Angle SOS}' = 2 \text{ Angle SAS}' = 2\phi,$$

where $\pi - \phi$ is the obtuse angle between the mirrors. For ϕ small, we have $SS' = 2OA \times \phi$ and the rest of the theory is exactly the same as for Young's experiment.

The arrangement of Fresnel's bi-prism is given in Fig. 229,

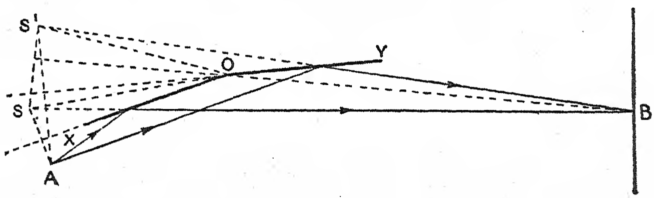


FIG. 228.

the bi-prism consisting of two prisms of very small refracting angle placed base to base. Thus two images of a single slit placed symmetrically behind the bi-prism are formed, one by refraction through each prism. If the distance between the images is known the theory of Young's experiment (Art. 2) can again be applied. If the refractive angles of the prisms are known, this distance can at once be calculated by the theory of Chapter VI. (Art. 2), but, in practice, this distance can be ascertained very rapidly by an indirect method, which avoids the troublesome process of measuring the refractive angles.

Experiment. To find the wave-length of sodium yellow light.—A bi-prism is set up on an optical bench with its edges vertical and at the same height as a vertical slit illuminated by a sodium flame. A travelling microscope is arranged in line with the bi-prism so that, sighting along the microscope, two images of the slit are

seen, one produced by each half of the prism. The slit is then rotated slowly until the interference bands are seen most clearly in the eye-piece, thus fixing the position at which the slit is parallel to the axis of the bi-prism. The width of the slit and the position of the microscope are then adjusted until the bands are clearest. (It is a good plan to make these three adjustments, in turn, several times over, until any alteration makes the bands less clear.)

The distances between the bands are measured either by traversing the microscope to set the cross-wires on each in turn or directly by means of a scale in the eye-piece. In the latter case the scale must be calibrated (Chapter XI., Art. 14) *without altering the distance between eye-piece and objective of the microscope.* The distance from the slit to the plane for which the microscope is in focus must be measured, and it now only remains to determine the distance S_1S_2. A convex lens is placed between the bi-prism and the microscope, and adjusted in position, without altering the distance between bi-prism and microscope, or the focusing of the latter, until two images of the slit are seen in focus through the eye-piece. If the lens is of sufficient power [its focal length must obviously be less than $\frac{SP}{4}$ (Fig. 229), XY being the plane on which the microscope is focused], there will be two positions for which this is possible and (see page 133) the magnification in one position will be the inverse of that in the other. The distances between the two images are measured in both positions (by traversing the microscope or by the scale in the eye-piece), and S_1S_2 is the geometric mean of the two values so obtained. The wave-length can now be calculated by the theory given above (Art. 2).

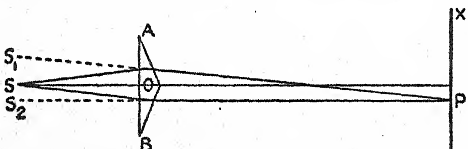


FIG. 229.

4. Lloyd's Single Mirror

By arranging that light from a slit, S (Fig. 230), should fall on a plane mirror, MM, at almost grazing incidence, interference bands are produced on a screen at A. The interference occurs between the direct rays from S, and the reflected rays which appear to come from S_1, the image of S formed by the mirror. The slit is

arranged to be parallel to the mirror and perpendicular to the line, MP. Since S is slightly above the level of the mirror, no reflected rays reach the screen at P. This means that no central band can be observed. To produce such a band, a thin sheet of mica is placed in the path of the direct rays to the screen so that the lengths of the paths, SA and SOA, may be optically equal.

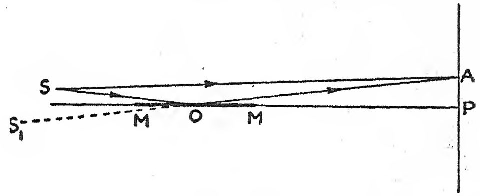


FIG. 230.

K. K. Singh.

Then a point such as A is the centre of the system of bands.

It is found by experiment that the central band is dark with bright bands on each side, so that for no path difference the two interfering wave trains are out of phase. This indicates that the reflected light must have its phase changed by 180° when reflection occurs. This phase change occurs only when the light is reflected at the surface of a medium such as glass which is optically denser than the medium through which the light is travelling.

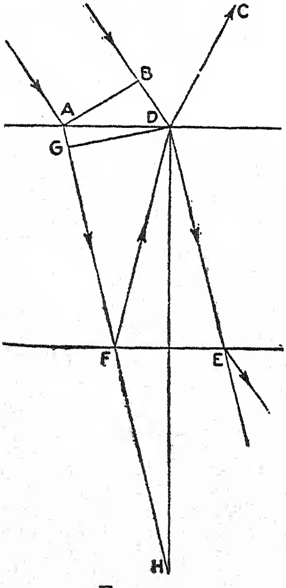


FIG. 231.

5. Interference in Thin Films

The colours produced by thin layers of transparent substances are due to interference. When a piece of orange peel is pressed into a puddle of water by a child, the oil from the peel forms a thin film on the water, and this produces colours by the interference of light from the upper and lower surfaces of the film. Soap bubbles, very thin glass, and oil films formed when oil drips from a car, are other examples of interference phenomena by thin films. The interference may occur by reflected light, or by transmitted light.

Consider a plane parallel plate of a transparent medium of refractive index, μ, and let AB (Fig. 231) be a plane wave front incident in air on one surface of the plate. Let DC be a ray reflected

at the front surface and FD a ray reflected internally at F. Interference between these two rays will occur at D. Similar effects will occur at all other points on the surface where the incident wave front falls. When the wave front from B reaches D, that from A will reach G, where DG is perpendicular to AF. Produce AF to H, making FH = FA, and join HD. Then, if t is the thickness of the plate, HD = $2t$, and the angle of refraction, θ, is equal to the angle, GHD.

The path difference between the rays under consideration is AF + FD in the plate minus BD in air. Thus, for the path difference we have:—

$$\text{Path difference} = \text{GF} + \text{FD in the medium} = \text{GH} = \text{DH} \cos \text{GHD}$$
$$= 2t \cos \theta.$$

This path difference is equivalent to a path difference μ times as great in air. Hence, for interference to occur:—

$$2\mu t \cos \theta = (2n \pm 1)\frac{\lambda}{2},$$

where n is an integer, and λ is the wave-length of the light.

In this case, however, the light reflected at D undergoes a phase change equal to half a period, while that reflected at F undergoes no change, so that the condition for *darkness* becomes:—

$$2\mu t \cos \theta = n\lambda.$$

Interference between the transmitted light may also occur. A ray which is reflected twice internally before emergence must travel a further optical path equal to $2\mu t \cos \theta$ than a ray which is transmitted directly. Since the reflections occur at surfaces of less dense media, at F and D, there will be no change of phase, and hence the condition for *brightness* is

$$2\mu t \cos \theta = n\lambda.$$

This means that the interference bands seen by reflected light are complementary to those seen by transmitted light.

The two cases are illustrated by an experiment carried out by **Young.** A convex lens is placed on a glass plate and the air film immediately surrounding the point of contact is illuminated by monochromatic light. A system of concentric rings (see Art. 7) results when the reflected light is viewed, the central spot being *dark* owing to the phase change at reflection. Young used a lens of crown glass and a plate of flint glass. Then a layer of oil of *sassafras*, which has a refractive index less than that of flint glass but greater than that of crown glass, was placed between

the two. In this case the central spot was *bright* because no net phase change occurs at reflection.

6. Newton's Rings

Although named after Newton, this example of the effects produced by interference in thin films was first studied by Hooke, 1665, and thus constitutes one of the earliest observations of the phenomenon. Newton studied this very carefully without arriving at an explanation of the origin, and the results were explained by Young.

The method adopted usually is to place a plano-convex lens of known large radius of curvature on a plane sheet of glass, the curved surface of the lens being downwards, P (Fig. 232). At the point of contact the air film is infinitesimally thin, but gradually increases in thickness from the centre outwards. Light from a sodium flame, S, passes through a convex lens, L, and is reflected by a glass plate, T, fixed at an angle of 45° to the horizontal, on to the central portion of the air film. Interference occurs, and a large number of bright rings surrounding a central dark spot become visible. The lens, L, is adjusted to give a parallel beam. A travelling microscope, M, is used to examine and measure the diameters of the rings.

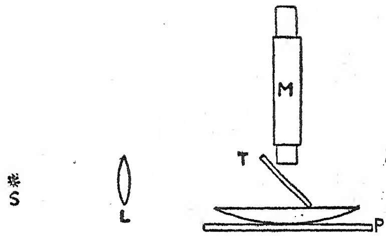


FIG. 232.

By measuring the diameters of a number of rings, the wavelength, λ, of the light can be determined. Since the air film may be considered as a parallel plate of gradually increasing thickness, the condition for darkness (Art. 6) is

$$n\lambda = 2\mu t \cos \theta.$$

In this case, however, the interference takes place in air for which $\mu = 1$, and the light is at normal incidence so that $\cos \theta = 1$, and the condition for interference becomes

$$n\lambda = 2t.$$

If R is the radius of curvature of the surface of the lens, and d the diameter of any dark ring under consideration, then by geometry (Fig. 233),

$$R^2 = (R - t)^2 + \frac{d^2}{4},$$

$$\text{or } 2Rt = \frac{d^2}{4}, \text{ neglecting } t^2, \text{ which is very small.}$$

The condition for interference may thus be written

$$n\lambda = \frac{d^2}{4R},$$

which means that d^2 is proportional to n, or the square of the diameter of any ring is proportional to the number of that ring.

Usually the central spot is not well defined, and it is found better to measure the diameters of a number of rings by the microscope, M, traversing the field from one side until the corresponding ring on the other side is reached. Then, if the values of d^2 are plotted against the values of n, a straight line is obtained.

If dn is the diameter of the nth bright ring, and d_{n+m} that of the $(n + m)$ *th*, then:—

$$d_n^2 = 4Rn\lambda, \text{ and } d^2_{n+m} = 4R(n + m)\lambda,$$

$$\text{from which } d^2_{n+m} - d^2_n = 4Rm\lambda,$$

$$\text{and } \lambda = \frac{d^2_{n+m} - d^2_n}{4Rm}.$$

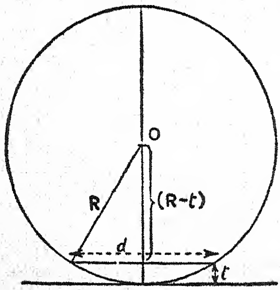


FIG. 233.

Thus, the actual order of the ring is not required, but the difference, m. R may be found by means of a spherometer, or optical bench, and thus λ may be calculated.

7. Diffraction

If light from a very small source is intercepted by an obstacle, the edges of the shadow are never perfectly sharp, and a number of bright and dark bands can be seen. This was discovered by Grimaldi in 1665 and the phenomenon is called **diffraction,** though there is no real distinction between diffraction and interference. Diffraction effects can easily be observed experimentally.

Experiment. To observe diffraction effects.—If an adjustable slit to form a source is not available a fair substitute can be contrived by smoking a glass plate and scoring a line along it with a razor-blade. The slit is illuminated as strongly as possible and set up on an optical bench. Parallel to it set up a fine wire and observe the shadow by an eye-piece or travelling microscope. Try wires of different thicknesses and note that the finer the wire, the less the appearance seen corresponds to a geometrical shadow. Replace the wire by a wide obstacle such as a metal ruler. The effects are now confined to a few bands near the edge of the shadow. Now

try an adjustable slit. As the slit is made narrower the illumination is reduced, but the illuminated area tends to become *wider* instead of narrower as one would expect on geometrical optics. (This result is of importance in connexion with the resolving power of optical instruments.)

The theory of diffraction due to extended regions such as slits is distinctly more difficult than that of interference due to point-sources. Indeed, only a few of the simplest cases have been worked out rigorously by the electromagnetic theory of light, but it is possible to solve many problems approximately by an application of Huyghens' construction. We shall consider the case of a single slit (illuminated by parallel light), as an example.

In Fig. 233 (*a*), let AB be a slit of length $2l$, illuminated by parallel light, and let XY be a screen. Let O be the mid-point of AB, and let ON be perpendicular to XY and of length p. It is required to find the illumination at a typical point P on the screen. Let $NP = x$. Let Q be a typical point on the slit, and let $OQ = y$. We consider only the case where l is small compared with p. We have:

$$OP^2 - PQ^2 = [p^2 + x^2] - [p^2 + (x - y)^2] = 2xy - y^2.$$

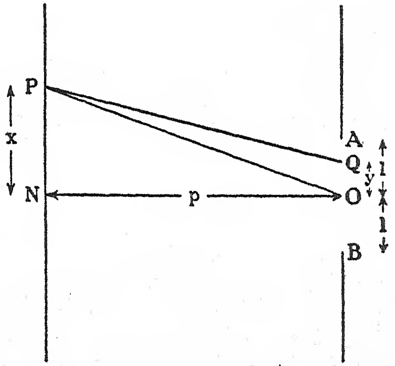


FIG. 233 (*a*).

For y small compared with d, OP and PQ are nearly equal, so that approximately

$$OP + PQ = 2\,(p^2 + x^2)^{\frac{1}{2}};$$

$$\therefore\ OP - PQ \simeq \frac{2xy - y^2}{2(p^2 + x^2)^{\frac{1}{2}}},$$

which determines the phase difference between a small strip of width dy of the slit at O, and a similar strip at P. To determine the total amplitude of oscillation at P we must sum over the whole slit, taking account of phase differences. (There is no phase difference between O and Q because the slit is illuminated by parallel light.)

The contribution to the amplitude at P from a strip of width dy at Q is thus proportional to $\dfrac{dy}{(p^2 + x^2)^{\frac{1}{2}}} \cos\left[v\left(t - \dfrac{2xy + y^2}{c(p^2 + x^2)^{\frac{1}{2}}}\right)\right]$

where V is the frequency of the light and c its velocity. The total amplitude can be obtained in various ways, either graphically by dividing the slit up into a number of portions and taking for each portion the phase appropriate to its mid-point, or analytically as follows. The total amplitude is proportional to the expression:

$$\frac{1}{(p^2+x^2)^{\frac{1}{2}}}\int_{-l}^{+l} dy \cos\left[v\left(t-\frac{(2xy+y^2)}{c(p^2+x^2)^{\frac{1}{2}}}\right)\right],$$

an integral which cannot be evaluated by elementary methods

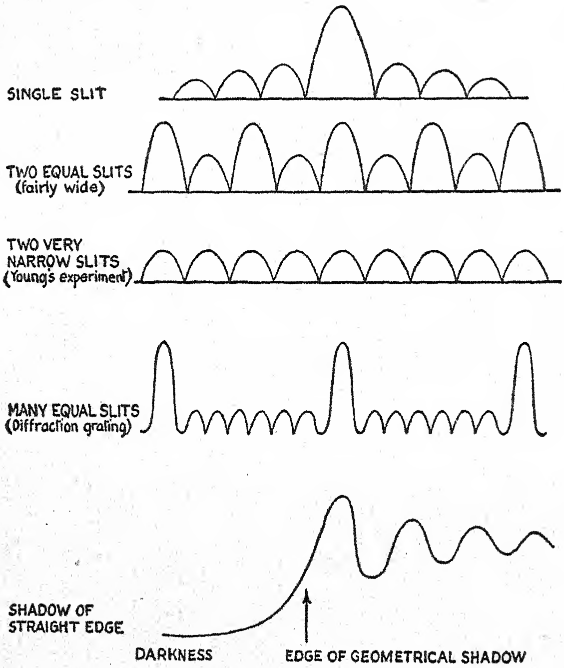


FIG. 234 (*a*).

but may be regarded as known because it can be expressed in terms of tabulated functions. It is evident that calculations can be made for many different cases, such as the shadow of a straight edge or wire, or single slit or two slits close together, or many equal slits (the diffraction grating) simply by taking different limits of integration. For two or more slits one can simply add up the amplitudes of oscillation due to single slits. Some of the more important cases are plotted (not to scale) in Fig. 234 (*a*).

The quantity plotted is intensity of illumination (obtained by squaring the amplitude).

It is found that the behaviour of the integral is mainly determined by the quantity $\frac{xl^2}{p\lambda}$ (λ being the wave-length) so that the smaller l is, the *larger* x has to be before we come to the first minimum of illumination. The situation is, in fact, very similar to that of Young's experiment (Art. 2). (If we make the distance between the two slits less, the interference fringes become wider.)

8. The Resolving Power of Optical Instruments

In Chapters X. and XI. we considered optical instruments on the basis of geometrical optics, and we found that the defects of chromatic and spherical aberration prevented a point-image of a point-source from being formed. Even if these defects could be perfectly corrected, one would still get a finite image of a point-object for a reason quite unconnected with geometrical optics, which we shall now examine. The reason is that every telescope or microscope must have a finite *aperture*. The case of a round aperture is qualitatively similar to that of a slit (due allowance being made for the fact that the problem is now three-dimensional), and if such an aperture is illuminated by a point-source or parallel light, the illumination pattern along any line parallel to the aperture will be qualitatively the same as that for a single slit illustrated in Fig. 234 (*a*). Now suppose we introduce a second point-source, independent of the first one. Since there is no correlation between the phases of two *independent* sources, each will produce its own diffraction pattern just as if the other were not there, and if the sources are close together these patterns will overlap. If the overlapping is not too great the eye will be able to distinguish two images.

A certain amount of arbitrariness exists in defining resolving power, but, in practice, two point-sources are said to be *resolved* by an optical instrument, if, when the observer examines the final images of the two sources, he sees the central maximum due to one source coinciding with the first minimum due to the other source. In practice, this does seem to represent the limit for good observers fairly closely. It will thus be clear that, if a telescope is being used to examine a double star, it will not be possible to see the two components separately unless the angular separation between them is greater than a certain value depending inversely on the aperture

of the telescope. The effect of increasing the aperture is two-fold. More light is collected from a point-source, yet this greater amount of light is concentrated into an image of smaller dimensions. A similar consideration applies to microscopes. There is thus a practical limit for a given size of aperture beyond which it is not economic to refine the corrections of the lenses for chromatic and spherical aberration. Conversely, it is useless to increase the aperture without improving these corrections, which become more and more difficult to make as the aperture increases.

It will be clear from the above discussion that the resolving power can be improved if the wave-length is shortened. A slight improvement can be made by using ultra-violet light detected photographically, and, of recent years, much work has been done on the project of short-wave-length microscopy. The electron-microscope exploits the fact that a stream of electrons has been found to behave, in some respects, like a wave-train with a very short wave-length.

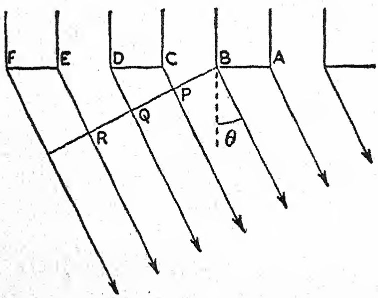


FIG. 234 (*b*).

9. The Diffraction Grating

A diffraction grating consists of a large number of lines ruled parallel and very close to one another on a sheet of glass or metal. A usual figure is 10-20,000 lines to the inch. The ruling is done by a dividing engine which operates on the same principle as a screw-gauge. The ruling process takes some time, and the temperature must be maintained constant to within a fraction of a degree during the process to avoid uneven spacing of the lines. An original grating is therefore expensive, and, for ordinary laboratory work, photographic reproductions or impressions taken in celluloid are used.

It is usual to consider a grating as a series of narrow parallel slits with obstacles in between, so that a plane wave falling on the grating sets up secondary sources at each of the slits. The diffraction grating is covered by the general theory of diffraction by systems of slits given above (Art. 7), but it is possible to give a much more elementary theory for the case in which the number of slits, N, is very large. In Fig. 234 (*b*), let ABCDEF be part of the grating, and suppose we have a telescope adjusted to receive light

leaving the grating at an angle θ. The difference in phase between a point on the slit BC and a corresponding point on the slit DE is represented by the distance DQ. If this distance is a whole number of wave-lengths the wavelets from corresponding points will reinforce, and the amplitude due to the whole grating will be N times that due to a single slit. If this distance is *not* a whole number of wave-lengths, take any point X on the slit BC. If the number of slits is very large, we can find a corresponding point Y on one of the other slits such that the phase-difference between X and Y is exactly an odd number of half-wave-lengths so that the total contribution of X and Y to the amplitude is zero. This matching-up can always be done point by point, except in the special case in which DQ is a whole number of wave-lengths, when it is not possible, because now every slit is, so to speak, optically equivalent to every other slit.

One must conclude, then, that practically all the light is concentrated in certain definite directions with darkness in other directions [Fig. 234 (*b*)]. The condition for illumination is

$$n\lambda = \mathrm{DQ} = \mathrm{BO}\sin\theta = g\sin\theta,$$

where g represents the total width of a slit and an obstacle and is usually called the *grating element*. If white light is used θ will vary with wave-length, so that the angle at which the telescope is set will be different for each colour. If the telescope is replaced by a screen a set of spectra will be formed. $n = 1$ gives the first order spectrum, $n = 2$ the second order spectrum, and so on. Notice that, in contrast with a prism, violet light is deviated less than red light. For θ small, up to 20° say, $\sin\theta$ is nearly proportional to θ, and the wave-length is directly proportional to the deviation.

Experiment. To determine the wave-length of light by a diffraction grating.—For this purpose a spectrometer (see page 231) is used. The telescope and collimator of the spectrometer are first focused for parallel light in the usual way. The grating is then mounted on the table of the instrument, and adjustments are made so that the plane of the grating is vertical and also the ruled lines on the grating are vertical.

To effect the first adjustment, the slit of the collimator is illuminated, and the reading of the vernier of the telescope is taken when the telescope and collimator are directly in line so that an image of the slit is formed on the cross-wires. The grating is placed on the table of the spectrometer with its plane perpendicular

to the line joining two of the levelling screws (see Fig. 187). The telescope is rotated through 90°, and the table is rotated until light from the collimator is reflected from the unruled surface of the grating to form an image of the slit on the cross-wires. One of the levelling screws is then adjusted until this image is in the centre of the field of view. The plane of the grating is then vertical.

To carry out the second adjustment, the slit is illuminated by monochromatic light, such as that from a sodium flame, and the grating is rotated through 45° so that its plane is perpendicular to the axis of the collimator, the ruled surface being towards the telescope. The diffracted images are located by the eye directly. Commencing in the position in which the undeviated light can be seen, the eye is moved slowly in either direction until the first diffracted image is seen. This image is then viewed through the telescope, and the third levelling screw of the table is adjusted until this image is in the centre of the field of view.

Having made these adjustments, the two positions of the telescope in which the first order image can be seen are obtained, and the vernier of the telescope is read. Half the difference between these readings will give the angle of deviation, θ_1, for the first order image. The wave-length of the light is then given by the relation,

$$\lambda = g \sin \theta_1,$$

where g is the grating element.

Similar observations are made with the second order images, and the angle of deviation, θ_2, obtained. In this case:—

$$2\lambda = g \sin \theta_2.$$

The third order images may also be observed, but usually images beyond the second order are so faint that their positions are difficult to locate.

The value of g may be found by finding the number of lines per centimetre with the aid of a microscope, or calculated from the number of lines if given. Alternatively, light of known wave-length may be used, and the value of g calculated from observations of the images produced.

10. The Fabry-Perot Interferometer and the Lummer-Gehrcke Plate

The theory of the diffraction grating illustrates an important difference between the effect of a small number of sources and that of a large number. With two sources the effective width of dark

and bright bands is practically the same [Fig. 234 (*a*)] whereas with a great many sources the light is concentrated in certain preferred directions, and monochromatic light forms very narrow bright bands separated by wide intervals of darkness. Such a result is just what is wanted for the *resolution of spectral lines*. If one has a spectral line consisting of two components very close to one another and one analyses them by means of a spectrometer the **resolving-power** is defined generally as $\frac{\lambda}{\Delta\lambda}$, where λ is the wave-length of the light and $\Delta\lambda$ is the smallest wave-length separation between two lines which enables them to be recognised as a doublet. High resolving power can be obtained in two distinct ways.

(*a*) By increasing the effective number of sources contributing to the diffraction pattern, thus sharpening up the maxima. The detailed theory of this is difficult and will not be given here. It will be seen that the simple theory of the grating given above (Art. 9), predicting infinitely narrow bright bands, only applies when the number of slits is infinite. It can be shown that the resolving power is proportional to the effective number of sources, that is to the total number of lines in the grating.

(*b*) By observing spectra of very high order, corresponding to big path differences. (The effect of this will be understood more clearly when we consider the Michelson interferometer, which relies on this effect alone, and does not use multiple sources.) Let us assume that we could observe high-order spectra from a diffraction-grating. Then we have, according to the simple theory,

$$n\lambda = g \sin \theta.$$

For a slightly different wave-length we have

$$n(\lambda + \Delta\lambda) = g \sin (\theta + \Delta\theta) \sim g \sin \theta - g \cos \theta . \Delta\theta \text{ (for } \Delta\theta \text{ small)},$$

whence $\frac{\lambda}{\Delta\lambda} \sim \frac{\tan \theta}{\Delta\theta}$. Thus, for a given angular separation $\Delta\theta$ the resolving power increases as θ approaches 90°, that is, as the order of the spectrum increases. In practice, it is not possible to observe spectra of high order with a diffraction grating, both because the high-order spectra are extremely faint, and also because the various orders overlap very much in the region of $\theta = 90°$.

The **Fabry-Perot** and **Lummer-Gehrcke** interferometers are practical instruments, combining big path differences with a large number of effective sources. Their resolving power is thus extremely high. The principle of the Fabry-Perot instrument is illustrated

by Fig. 234 (*c*). It consists of two half-silvered plates mounted parallel to one another, the silver film being thick enough to reflect the light back and forth many times, and yet thin enough to allow an appreciable transmission at each reflection. Much time and skill are required to determine the optimum thickness of silvering (which varies somewhat with the wave-length of the light to be examined). As usually used, the instrument is supplied with light converging towards a focus, and the repeated reflections produce a great many images spaced out equally. The theory is thus very similar to that of the diffraction-grating, except that the spacing between the effective sources is very much larger (the distance between the plates can be made as much as a centimetre), and also, because of the axial symmetry, the fringes are *circular* and not straight. It is possible to observe fringes from the transmitted or the reflected light just as with the thin film (see Art. 5 above).

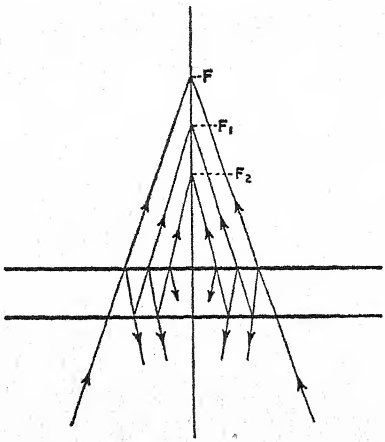


FIG. 234 (*c*).

The Lummer-Gehrcke plate also combines large path-differences with a large effective number of sources. It consists simply of an optically flat plate to which is cemented a small prism. Fig. 234 (*d*) illustrates the path of a typical ray of light. At B, C, D, etc., the angle of incidence is slightly less than the critical angle for total reflection. The result is that the emergent rays are comparatively feeble; the light is beaten backwards and forwards between the surfaces of the plate a great many times, so one obtains a large number of emergent rays, appearing to come from a corresponding set of equally spaced sources *of approximately equal strength.* Again the path-differences can be made large by using a thick plate. The prism is used to avoid the excessive loss of light by reflection which would occur if the neighbourhood of A were illuminated directly, by enabling the angle of incidence to be chosen so that the transmitted beam is of maximum amplitude. (Normal incidence is best for this.)

11. The Michelson Interferometer

This instrument is an extremely versatile one and only a few typical uses can be described. We have already investigated the effect of multiple sources. The use of very long path-differences lends to results of equal importance. The interferometer is illustrated in Fig. 234 (*e*).

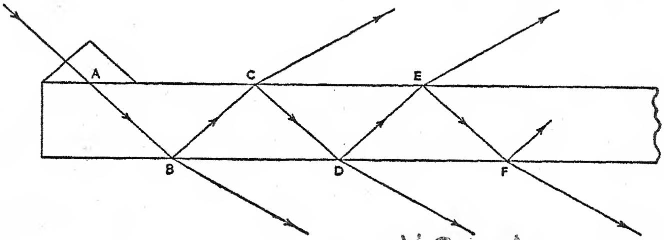


FIG. 234 (*d*). V. P. Singh

The mirror, A, is half-silvered so that the reflected and transmitted beams are of approximately equal amplitude. The reflected beam from A reaches the mirror B, is reflected back along its path, and part of it is transmitted through A and reaches the screen. Similarly the beam transmitted by A is reflected back by C, is

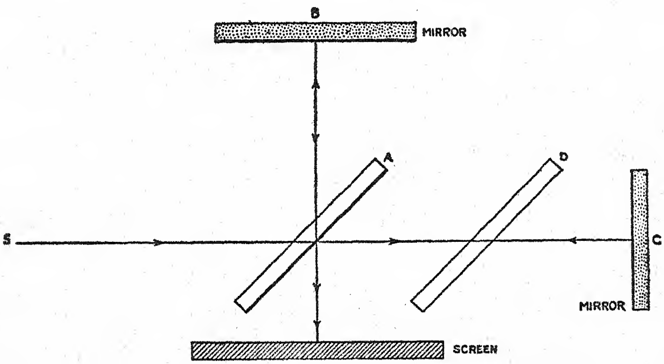


FIG. 234 (*e*).

partially reflected by A and also arrives at the screen. (D is a plain piece of glass, of the same thickness as A, inserted to compensate for the path-difference introduced by the fact that a ray originally reflected by A traverses its thickness three times before reaching the screen, while a ray originally transmitted by A traverses the thickness of A only once.) As the angle of incidence

of the original light alters so does the difference in the length of the two possible paths and a system of fringes appears on the screen. The "zero" fringe, corresponding to zero path-difference, will appear in the centre of the field of view if B and C are placed at exactly equal distances from the centre of A. By moving one of these mirrors backwards the "zero" fringe can be displaced to any desired extent and the fringe at the centre of the screen can be made to correspond to any desired path-difference.

Suppose now that our source emits light of two distinct wave-lengths very close to one another. Each wave-length will give its own interference pattern, the spacing between corresponding fringes being proportional to the wave-length. Near the "zero" fringe the patterns will be practically superposed, but, as we increase the path-difference the two patterns will become more and more out of step. Finally, when we have introduced a path-difference d given by the relation,

$$\frac{d}{\lambda_1} - \frac{d}{\lambda_2} = \tfrac{1}{2},$$

which for $\lambda_1 - \lambda_2$ small may be written approximately

$$\frac{d\,(\lambda_2 - \lambda_1)}{\lambda^2} = \tfrac{1}{2},$$

we shall, at the centre of the field, have a dark fringe due to light of wave-length λ_1 superimposed on a bright fringe of wave-length λ_2. If the components of the spectral line are of equal intensity the fringe pattern will disappear, if they are not of equal intensity the fringe pattern will become successively stronger and weaker as d is altered. Thus, if we have reason to think that a certain line is a doublet, this simple method enables us to estimate both the wave-length separation and the relative intensities. On the other hand, we are unable to decide whether the component of longer wave-length is the stronger or the weaker one, so that we cannot determine the structure of a spectral line uniquely by the Michelson inferometer alone. It has, however, greater resolving power than the Fabry-Perot or Lummer-Gehrcke interferometers, although the latter have the advantage of giving actual pictures of the spectral line, as it would appear if intensity were plotted against wave-length.

12. The Resolving Power of Interferometers

The maximum resolving power of the Michelson interferometer can be calculated as follows. For interference to be possible at

all, the path-difference must be short enough for light emitted by one atom and thus belonging to one quantum to reach the screen *simultaneously* along the two alternative paths. On the quantum theory of light this emission is discontinuous so that each quantum has a finite length. The life of excited atoms emitting visible light has been determined in various ways to be of the order of 10^{-8} sec. [One method uses the Kerr cell (see page 311) as a very rapid shutter.] Since the velocity of light is 3×10^{10} cm. per sec., this means that the effective length of a quantum is of the order of 300 cm., which is a measure of the maximum permissible path-difference. The corresponding resolving power is easily calculated. We have, for separation of two components of wave-lengths λ_1 and λ_2

$$\frac{d(\lambda_1 - \lambda_2)}{\lambda^2} = \tfrac{1}{2};$$

$$\therefore \text{ R.P.} = \frac{\lambda}{\lambda_1 - \lambda_2} = \frac{2d}{\lambda} = \frac{600}{5 \times 10^{-5}} = 12 \text{ million for visible light.}$$

The calculation of the resolving powers of instruments like the grating or Fabry-Perot interferometer which involve multiple sources, is beyond the scope of this book. As we should expect, the separation of two wave-lengths is proportional to the order of the spectrum observed, and it can be shown that the effective breadth of the image is inversely proportional to the effective number of sources. (We have proved above that it would be zero for infinitely many sources.) The resolving power of such instruments is, in fact, just equal to the product of the number of sources and the order of the spectrum observed. Thus, for a grating of 100,000 lines a resolving power of 300,000 could be attained using the third order spectrum. For a Fabry-Perot instrument with the plates separated 1 cm. the order of the spectrum will be $\frac{2}{5 \times 10^{-5}}$ or 40,000 for visible light, while the number of effective sources might be of the order of 20, giving an overall resolving power of 800,000. It will be seen that these figures are distinctly inferior to that for the Michelson interferometer at its best.

13. The Determination of the Metre in Terms of the Wave-length of Light

Suppose that we have a measuring rod whose length is known accurately in terms of the standard metre. We may now determine

this length in wave-lengths of light of any desired colour by moving one mirror of a Michelson interferometer through a distance equal to the length of the rod, and counting the number of fringes that go past. We can estimate to a fraction of a fringe by eye, which already gives a very accurate measurement. Still further accuracy is obtainable in the estimate of the fraction if we use two wave-lengths whose ratio is known, the two fringe systems of slightly different spacing thus being used as a *vernier*. It is thus possible to determine the wave-length of light to eight significant figures, such measurements being among the most accurate in physics. (It is only in certain astronomical observations that this accuracy is surpassed.) It has been proposed to use the red cadmium line, which the Michelson interferometer has shown to be extremely homogeneous, as the primary standard of length, as it can easily be reproduced anywhere in the world. The suggestion has not yet been generally adopted.

14. Applications of Interferometry to Physical Problems

The possibilities of interferometry as a tool in physical investigations are by no means exhausted, and many new uses are being discovered. The physicist naturally turns to it whenever he has to measure a very small change of position or length.

Suppose that one wants the coefficient of expansion of a crystal of which only a small piece is available. The crystal is set up instead of one of the mirrors of a Michelson interferometer and heated through a known range of temperature. The movement of the front face of the crystal can then be followed by the shifting of the fringes, and, not only can one make a very accurate determination, but by observing when the individual fringes cross the centre of the field one can determine whether the expansion is linear with temperature or not. (A "blank" experiment is of course necessary to correct for the expansion of the rest of the apparatus.)

Another application is to the determination of refractive indices which differ only slightly from unity, such as those of gases. A special type of interferometer, the **Jamin Interferometer**, has been developed for this purpose [Fig. 234 (*f*)].

It consists simply of two optically flat plates of glass supported nearly parallel to one another. For a given angle of incidence there are two possible paths of nearly equal length, such as ABCDE and AFGDE. If the plates are not quite parallel the difference between the lengths of the two paths will change slightly for

different angles of incidence, so that a system of fringes will be obtained if a screen is placed at S. To measure the refractive index of a gas, a long glass tube, with flat ends, of known length, is placed in the path of *one* of the beams, say between C and D. The number of fringes that go past a fixed point when the tube is evacuated is counted, which gives a measure of the change in optical path introduced by the presence of a known thickness of gas, and hence of its refractive index.

15. The Correction of Mirrors and Lenses

From what has been said, it is clear that interferometry is a very sensitive method of determining small distances. It can, in particular, be used to detect errors of the order of a wave-length of light in the figuring of lenses or mirrors. A simple example will suffice. In the Newton's rings experiment (Fig. 233), suppose

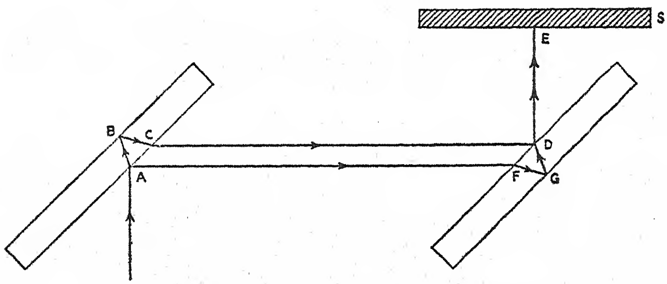


FIG. 234(*f*).

that there is a local departure of the lens from the spherical shape. Near this point the thickness of the air layer t will not attain its proper value, and consequently the rings, which represent *contours* through points corresponding to equal values of t will not be circular. Since the refractive index of glass is 1·5, a bulge on the glass 1 wave-length thick will behave like $1\frac{1}{2}$ wave-lengths of air, and the path-difference will be altered by $2(1\frac{1}{2} - 1) = 1$ wave-length, corresponding to a shift of a whole ring, and a departure from the spherical shape of 1 wave-length will cause a quite noticeable distortion of the ring pattern. Thus, we can not only locate the defect, but the method also tells us its approximate size and extent and, even more important, whether glass has to be locally *removed* to remedy it, or whether there is a local *hollow*. A great many very ingenious tests for flat plates, mirrors and lenses have been devised, but we cannot enter into details here.

CHAPTER XVI

POLARISATION AND DOUBLE REFRACTION

IT has been stated in previous chapters that light consists of a wave motion in space, and it has been shown how it is possible to explain the phenomena of interference and diffraction by means of the wave theory. It was assumed that the wave-motion is transverse in character. A wave motion, however, may in general be either (1) *longitudinal*—that is, one in which the vibrations are in the direction in which the waves are travelling, as is the case with sound waves in air—or (2) *transverse*—that is, one in which the constituent vibrations are perpendicular to the direction of propagation, as is the case with waves travelling along a stretched string. In the latter case the vibrations are not necessarily confined to one direction, but might take place in any direction in a plane perpendicular to the line of advance of the waves (Fig. 235). In longitudinal waves, however, it is clear that the vibrations are necessarily restricted to one direction. It is possible, therefore, that one train of transverse waves may differ from another, otherwise similar, in that the directions of vibration in the two trains are different. Such a distinction between two trains of longitudinal waves would be impossible.

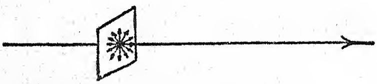


FIG. 235.

1. Polarised Waves

Transverse waves in which the vibrations are confined to one direction would not be symmetrical, and might be expected to exhibit properties having some relation to direction. This characteristic of sidedness, the dependence of certain properties on direction, is called *polarity*, and such a train of waves is said to be polarised. If, as in this case, the vibrations are executed in one direction only, the wave motion is **plane-polarised**.

Some of the phenomena of polarised waves may be reproduced by the aid of simple apparatus: a length of rubber cord, or rubber tubing filled with sand in order to reduce the velocity of waves along it, and two pieces of wood or other material, each having a parallel-sided slit a few inches long, the slit being of such width as to

allow the cord to just pass freely through. Fix one end of the cord to a support, pass it through the two slits which must be pushed at first up to the fixed end, and hold the other end in the hand. By moving this end to and fro through a small distance and continually changing the direction of the movement, while keeping it always in a plane perpendicular to the cord, a series of transverse waves will be sent along the cord, and the vibrations will be in various directions indifferently, on the whole as often in one direction as another. This is an unpolarised wave motion.

Now move one of the slits along the cord, and arrange that it is held rigidly in a stand, and that the cord passes freely through it. Send a train of waves as before, when it will be seen that the waves which get through the slit have their vibrations confined to a direction parallel to the slit. The waves are plane polarised.

Move up the second slit midway between the end of the cord and the first slit, and support it so that its length is perpendicular to that of the first (Fig. 236). Repeat the experiment, and it will be seen that scarcely any wave motion gets through the second slit. In this position the slits may be said to be *crossed*.

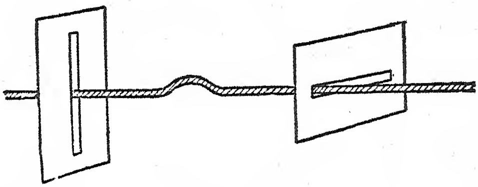


FIG. 236.

Rotate the second slit gradually until it is parallel with the first, from time to time sending waves as before. It will be found that the amplitude of the wave motion transmitted through the second slit gradually increases until, when the slits are parallel, the plane-polarised waves coming through the first slit pass on unaltered through the second.

From these simple experiments it is seen that a train of plane-polarised waves in a cord differs from a train of unpolarised waves in that it will not pass through a slit held in one certain direction, while it passes unaltered when the slit is turned through a right angle.

2. Polarisation of Light. Double Refraction

It was observed by **Bartholinus** in 1669 that under certain conditions it was possible to obtain a beam of light which manifested similar properties of *sidedness* or **polarisation** to those indicated above (Art. 1). This occurred when light passed through a crystal of Iceland spar, a natural form of calcium carbonate.

This substance occurs in rhombohedral crystals (Fig. 237), the six faces of the rhomb being parallelograms having angles of 78° and 102° approximately. Two, and two only, of the opposite corners, O, O′, are contained by three obtuse angles. A line, OA, O′A′, drawn through either of these corners, so as to make equal angles with the three sides meeting at the corner, is called the *optic axis* of the crystal. The crystal, so far as its optical properties are concerned, may be taken to be symmetrical about such an axis. It is clear that this axis is a *direction*, and not any fixed line in the crystal.

If now a crystal be cut so as to have two plane and parallel faces parallel to this axis, or even if a crystal as it ordinarily cleaves is taken, and an illuminated pinhole is viewed normally through the crystal, two images of the pinhole are seen. One of them is normally above the object, and is found to occur according to the ordinary laws of refraction, while the other is to one side of the normal, and is not formed in accordance with these laws. This phenomenon is called **double refraction.** When the crystal is rotated above the object, the *ordinary* image remains still, but the *extraordinary* image, as it is termed, rotates with the crystal round the ordinary image.

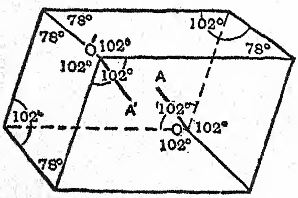


FIG. 237.

If a similar crystal be placed above the first one, by its help the two beams of light diverging from these two images can be studied. It is found that, when the two crystals are placed with their optic axes parallel, the two images are separated more widely —that is, the effect is that of a thicker crystal. As the upper crystal is rotated slowly, each of the images gives rise to two—an ordinary and an extraordinary from each of the original images. When the optic axes of the crystals are at 45° these four images are of equal brightness. As their inclination to one another approaches 90°, one of each pair of images gradually becomes weaker and disappears when the axes are perpendicular. There are then left two images, an *extraordinary* one of the first *ordinary* image, and an *ordinary* one of the first *extraordinary* image.

This shows that the two beams of light separated by the first crystal have properties depending upon direction, since they give rise to different types of image according as they pass through a second crystal placed similarly to the first, or placed in a direction

at right angles to the first, whereas the original beam of light gives two images however the first crystal is turned. They are, in fact, plane polarised, the one in a plane perpendicular to that of the other. The crystal is so built up that it transmits, in a direction at right angles to its optic axis, wave trains whose vibrations are in one direction with a different velocity from those whose vibrations are in a direction at right angles. When the unpolarised light falls on the crystal, the vibrations may be regarded as being resolved in these two directions, and the two components travel on differently, leading to different laws of refraction, and to the formation of two distinct images. Similar, or more complicated double refraction occurs in the majority of cases when light travels through crystalline media.

Various pieces of apparatus which will allow light, whose vibrations are in one plane only, to pass through, will be described below. If a beam of ordinary light passes through such an arrangement, only the components of the vibrations in one definite plane are transmitted, and thus a beam of plane polarised light is obtained. This is, of course, of weaker intensity than the original beam, containing only half its energy.

The apparatus thus used is called a **Polariser.** However, exactly the same arrangement may be used to examine any beam of light to discover whether it is polarised, for if the apparatus be rotated slowly, no change will be observed in the intensity if the light is unpolarised, while if it is plane polarised, it will be blocked out completely for one position of the apparatus, and transmitted completely for the position at right angles. If the light is partially plane polarised, it will be changed in intensity as the apparatus is rotated. Used in this way, the Polariser is called an **Analyser.** It should be mentioned that, except in the case of plane polarised light, the information given by the use of an Analyser alone is capable of more than one interpretation.

If ordinary light passing through such a Polariser, A (Fig. 238), such as a tourmaline crystal (see Art. 3), then falls on a second similar arrangement, B, placed exactly similarly, almost all the light transmitted by A passes through B, and of course remains polarised in the same direction as before. This corresponds to the case of parallel slits (Art. 1). If, however, B is rotated relatively to A, then B gradually quenches the light which has passed through A, and when B has been turned through 90°, no light that has passed through A can get through B. The polariser and analyser are then said to be *crossed*—the case of crossed slits (Art. 1).

If an analyser is employed to examine the light from the two images formed by Double Refraction in Iceland spar, or in any other case, it is found that, for one position of the analyser, one image disappears, and when the analyser is turned through 90°, the other image disappears. Thus the light from each is plane polarised and in perpendicular directions.

Huyghens showed that double refraction could be explained in terms of the wave theory of light, but was not able to suggest a reason why the two beams should present these characteristics of *sidedness*, since he supposed the waves constituting light to be *longitudinal*. It was not until much later that it was realised, due to Young and Fresnel, that these phenomena were to be brought into line with the wave theory, and this theory itself to gain in precision, by interpreting them to indicate that light waves are *transverse* and not longitudinal.

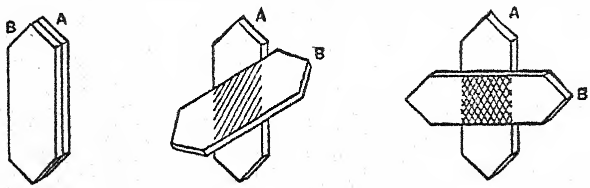


FIG. 238.

3. Polarisation by Refraction

(*a*) A crystal of Iceland spar, as stated above, splits up a beam of light into two beams plane polarised in directions at right angles to each other. With an ordinary sized crystal, however, the two beams are very close to one another, and in order to separate them sufficiently a very large crystal would be necessary. Suitable crystals are rare, and this method, though very efficient otherwise, is not practically serviceable.

(*b*) A plate of *tourmaline* is frequently used. Tourmaline is a doubly refracting crystal of a slightly greenish tinge, and has the property of separating a beam of ordinary light into two beams of polarised light, ordinary and extraordinary. The ordinary beam, however, is absorbed if the thickness exceeds 1 mm., while the extraordinary beam is almost fully transmitted. Thus a comparatively thin plate of material produces from ordinary light a beam of plane polarised light.

(*c*) *The Nicol Prism.* This prism, invented in 1828, is a device for getting rid of one of the polarised beams produced in a crystal of Iceland spar.

C and G (Fig. 239) are the opposite corners of a rhomb of Iceland spar, which are contained by three obtuse angles, and C*a*, G*a'* are in the direction of the optic axis. The plane, ACEG, containing this axis is called the *principal section* of the crystal. To form a Nicol prism, a section, CKGL, of the crystal is made at right angles to the principal section. The two halves are then cemented together with Canada Balsam, which forms a thin parallel transparent film between them.

If now a beam, MN, of unpolarised light falls on the face, ABCD, it will, on entering the crystal, since it is travelling in a direction inclined to the optic axis, be divided into an ordinary beam, NO, and an extraordinary beam, NQ, plane polarised in planes perpendicular to one another. Now the refractive index of Canada Balsam is less than that of Iceland spar for the ordinary beam, and greater than that of Iceland spar for the extraordinary. Hence, if the inclination of NO to CKGL exceeds a certain value, total reflection of the ordinary beam will occur, while the extraordinary beam will pass on through the crystal and emerge at Q, thus yielding a pure beam of plane polarised light. The dimensions of the crystal are chosen so as to make the angle of incidence of NO on CKGL greater than the critical angle. The crystal is mounted in a tube blackened on the inside, so that the reflected beam is absorbed.

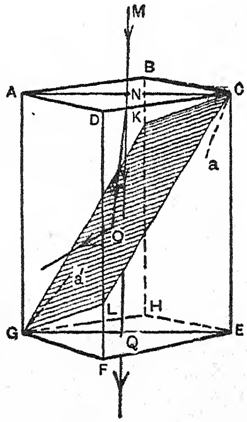


FIG. 239.

The Nicol prism is one of the most valuable sources of polarised light.

4. Polarisation by Reflection

In 1810 Malus noticed that light reflected from a glass window showed signs of polarisation when examined through a doubly refracting crystal, whereas the direct light showed no such sign. Thus it became clear that a change in the character of the light had occurred in the process of reflection—the reflected light was partially plane polarised.

If ordinary light reflected from a polished glass surface be examined with an analyser (Art. 2), the degree of polarisation is found to vary with the angle of incidence, and to be complete when the angle is about $57\frac{1}{2}°$. This is known as the *polarising angle* for glass.

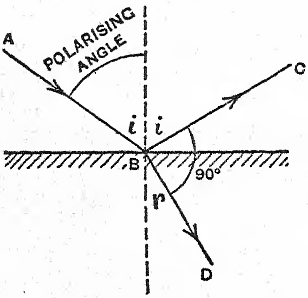


FIG. 240.

This angle varies for different substances, being related to the refractive index of the reflecting substance by the simple relation, $\tan i = \mu$, where i is the polarising angle. This relation is known as *Brewster's* law. The polarising angle, therefore, evidently varies slightly with the colour of the light. Also, it follows from the law that at the polarising angle the reflected and refracted rays are at right angles, for (Fig. 240) by the law of refraction,

$$\sin i = \mu \sin r,$$

$$\text{and by Brewster's law, } \tan i = \mu;$$

$$\therefore \cos i = \sin r,$$

so that the angles, i and r, are complementary, and the angle, CBD, is 90°.

Let AB (Fig. 241) be a beam of unpolarised light incident upon the surface, S, at the polarising angle. In the wave front, L, the vibrations are occurring successively in all directions. At B all vibrations which are not parallel to the surface are transmitted, together with a certain proportion of the vibrations which are parallel to the surface. Thus the refracted beam is only partly plane polarised. The reflected beam, BC, consists entirely of vibrations parallel to the surface and therefore is completely plane polarised. Those vibrations in the incident wave front, L, which

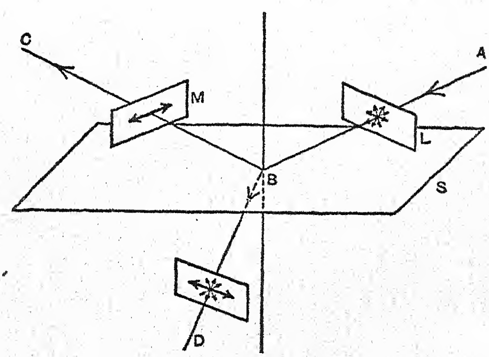


FIG. 241.

are inclined to the plane of incidence may be regarded as being resolved into vibrations respectively parallel and perpendicular to this plane, when the components undergo reflection or refraction as above.

Light polarised by reflection is defined as *polarised in the plane of incidence*, but the vibrations constituting it are perpendicular to this plane—that is, the plane perpendicular to the direction of vibration is called the *plane of polarisation*.

If the reflected beam, BC (Fig. 241), is received on a second surface of the same kind with its normal in a plane perpendicular to the plane of incidence on the first surface —that is, so that the planes of reflection are perpendicular, vibrations which were perpendicular to the first plane of incidence will be in the plane of incidence for the second surface. In consequence, if the inclination of this latter surface is adjusted so that the beam strikes it at the polarising angle, no light is reflected. The two surfaces now act like polariser and analyser crossed.

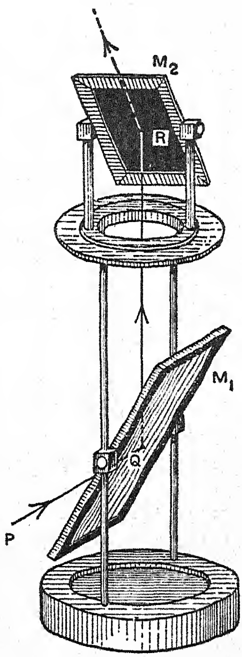


FIG. 242.

These facts may be verified with the apparatus shown (Fig. 242). M_1 is a plane glass plate backed with black paint, and can be rotated about a horizontal axis. M_2 is a similar plate which can be turned about a horizontal and also a vertical axis. In each case the amount of rotation can be measured on a scale of degrees, the scales not being shown in the diagram. M_1 and M_2 are arranged so as to be equally inclined to the horizontal, and a beam, PQ, of parallel light is directed on to M_1 at the polarising angle, so that the plane of incidence is normal to the surface, and so that the reflected beam, QR, is vertical and falls on M_2, which it must then strike necessarily at the same angle. As M_2 is rotated about a vertical axis, it is found that the intensity of the beam reflected by it varies from a maximum, when the normals to M_1 and M_2 are parallel, to zero, when the planes of incidence are at right angles. This apparatus is called a *Polariscope*. It is clear that by its use the polarising angle can be determined experimentally without the aid of any other form of polariser or analyser.

It has been pointed out already that the refracted beam, BD (Fig. 241), is by no means completely polarised, although it contains a great proportion of light whose vibrations are in the plane of incidence. If now this light is allowed to fall on another plate parallel to the first, still more of the light whose vibrations are perpendicular to the plane of incidence will be filtered out by reflection. It is found that if about 24 such parallel plates are used, about 90 per cent. of the light transmitted is constituted of vibrations in the plane of incidence. Such a *pile of plates*, as it is called, thus yields an easily made piece of apparatus for producing nearly plane polarised light. Carefully cleaned microscope cover glasses are recommended. From 12, giving light rather more than 80 per cent. polarised, to 24, giving light about 90 per cent. polarised, should be placed in contact and fixed in a small tube, by means of corks, at an angle of about $32\frac{1}{2}°$ with the axis of the tube. Two such piles will serve as polariser and analyser.

Since reflected light is always more or less plane polarised, it has been suggested that the inconvenient glare from sunlight reflected from the surface of water might be reduced largely by the use of polarising spectacles made, for example, of thin plates of tourmaline. In this way, it is said, visibility into the water is improved, and also to some extent the visibility towards the horizon is increased.

5. Polarisation by Scattering

It has been explained above (see page 172) that the brightness and blue colour of the sky are due to light scattered by very small particles in and of the air. These particles may be regarded as being set in vibration relative to the aether by the light waves falling on them, and, in consequence, acting like separate sources of light.

When light from the sky is examined by an analyser, it is found to be partially plane polarised. The extent to which it is polarised depends upon the direction from which it is received, being a minimum when received from a direction facing or opposite the sun, and a maximum when received from directions at right angles to the sun. In the latter case it is not completely plane polarised, so that what is observed as the analyser is rotated is a distinct darkening of the field in one direction, but not perfect extinction. The same phenomenon can be observed if the light scattered by the suspended precipitate in a thiosulphate solution (see page 173) is examined. When viewed from the side, it will be found to show a very considerable degree of polarisation.

The occurrence of polarisation in scattered light is to be expected as a consequence of the fact that the vibrations in light waves are transverse, for thus a scattering particle can be set in vibration only in a plane at right angles to the direction of the incident light, and when the scattered light is viewed in a direction perpendicular to this incident light, the vibrations received are confined to one direction, and so the light received is plane polarised.

A very interesting experiment in illustration of this consists in passing a beam of plane polarised light through a tube containing the suspended precipitate of sulphur (see page 173), and then examining the scattered light in various directions in the plane at right angles to the incident light. It is found that when viewed in a direction parallel to that of the vibrations in the incident light, the track of the beam through the solution is almost invisible—that is, there is no scattered light, while in the direction at right angles its brightness is a maximum. In the first case the direction of view is end-on to that of the vibrations excited in the particles, and no light is propagated in that direction.

The haze which envelops distant objects is principally caused by scattered light, and a further application of the principles of polarisation has been proposed in order to improve the visibility in such cases. This might be accomplished by allowing the light to pass through polarisers which would cut off the bluish atmospheric haze, thus improving the detail and colour of the distant objects.

6. Polaroid

This is a modern commercial material which has been introduced recently for producing and detecting polarised light. It consists of a film of cellulose on which extremely small doubly refracting crystals of iodosulphate of quinine, an organic compound, are deposited uniformly at about 10^{12} to the square inch. These crystals are orientated with their axes all parallel to one another, so that the film acts as one large crystal. The material then acts like tourmaline, or as a Nicol prism, in that it transmits vibrations in one plane only. The great advantage of polaroid is that it can be prepared in large sheets and that it polarises by direct transmission. It is sold in the form of small discs and can be used thus for such purposes as the conversion of instruments—a projection lantern, for example, into polarising instruments.

Spectacles made of polaroid, suitably orientated, can be used to cut off or reduce the glare produced by light reflected from polished surfaces—very desirable, for example, in photography where

reflections may obscure details in the photograph. All that is necessary is to arrange that the light which is polarised by the reflections shall not be transmitted by the polaroid.

A further suggested use is the elimination of dazzle from motor car head lamps. If each lamp is arranged to project a beam of plane polarised light, this light is cut off when viewed through a sheet of polaroid, although the motor car itself remains visible.

Another suggested application is the production of stereoscopic effects (see page 239) in pictures and films.

7. Circularly and Elliptically Polarised Light

In general, when a beam of ordinary light passes through a doubly refracting crystal, the ordinary and the extraordinary beams travel with different velocities and a phase difference is thus produced between these beams.

If ordinary unpolarised light is examined with an analyser, no change of brightness of the field occurs as the analyser is rotated, since on the average the amount of vibration in all directions is constant, for, though the number of consecutive vibrations in any given direction is undoubtedly great, these occupy a very short time, and thus the direction of vibration remains constant only for a very small fraction of a second. If, however, the vibrations were circular in character, such as could be resolved into two equal simple harmonic motions of the same period, at right angles to one another, and having a phase difference of one quarter of a period (see Catchpool, *Sound*), the same effect would be observed on rotating the analyser.

Such light is said to be **circularly polarised**. It can be distinguished from unpolarised light by passing it through a thin plate of quartz, cut parallel to the optic axis and of such thickness as to slow down the propagation of one of the components over that of the other until they are in phase, and so combine to form plane polarised light, which can be detected by the use of the analyser in the usual way. Such a difference of phase of one-quarter of a period must be introduced between the components, the thickness of the crystal must be such as to contain one-quarter of a wavelength more in the extraordinary beam than in the ordinary beam, and such a plate is therefore called a *quarter-wave plate*. Conversely, if plane polarised light is incident on a quarter-wave plate at 45° with the principal plane, it will emerge as circularly polarised light.

If the vibrations in a beam of light are elliptical, then they can be resolved into two unequal simple harmonic vibrations, at right

angles to one another, and having a phase difference of one-quarter of a period. Thus, when viewed through an analyser, the light transmitted will be brighter for one direction of the analyser than for that at right angles. Such light is said to be **elliptically** polarised. It can be distinguished from a mixture of polarised and unpolarised light by passing it through a quarter-wave plate, when, for one position of the plate, it will be reduced to plane polarised light as in the preceding instance.

V. P. Singh

8. Colour Effects due to Polarisation

As has been stated above, the characteristic properties of doubly refracting crystals depend upon the fact that such crystals in general transmit light whose vibrations are in one given direction with a different velocity from light with vibrations in the direction at right angles. However, these velocities and their differences depend upon the wave-length—that is, the colour of the light. It follows that if plane polarised light is transmitted through such a crystal cut parallel to the optic axis, the character of the emergent light depends upon the colour of the light, and the thickness of the crystal. If white light falls on such a crystal, and the transmitted light be examined with an analyser, very brilliant colour effects are therefore seen.

Many *isotropic* bodies—that is, substances whose properties are the same in all directions, act like doubly refracting crystals when they are strained by bending or twisting, or by compression. If examined between crossed polariser and analyser, such a substance under the conditions of strain will exhibit characteristic colour fringes, the position of which will mark out the lines of strain. This fact is put to practical use in examining glass for imperfect annealing, and in examining the conditions of strain in small model structures built up of a suitable transparent material such as xylonite. In this way it is possible to determine what allowance for a safety margin is necessary in the construction of a building or other structure. The study of this effect is called *photo-elasticity*.

A similar effect was discovered by Kerr in 1875. He found that some dielectrics such as glass and nitrobenzene become doubly refracting in an electric field. This is believed to be due to the partial orientation of the molecules by the electric field so that a liquid or an amorphous glass assumes some of the properties of an *ordered crystal*. Thus, if plane polarised light is passed through nitrobenzene it becomes, in general, elliptically polarised when an electric field is applied, so that it cannot be extinguished by the

analysing Nicol. The Kerr cell, at one time used in television and still much used as a very rapid shutter, is an application of this effect. A cell of nitrobenzene between crossed Nicol prisms transmits nothing in the absence of an electric field, but will transmit some light if an electric field is applied perpendicularly to the common axis of the Nicol prisms. This type of shutter is not quite inertialess, because the molecules have to be oriented, but is much superior to any mechanical shutter, and is much used in laboratory experiments involving very short time-delays.

9. Rotation of the Plane of Polarisation

If a beam of unpolarised light is viewed through crossed Nicol prisms, or other polarising devices, the field of view is, of course, completely dark. If now a crystal of quartz, cut *perpendicular to the optic axis*, is interposed between the polariser and the analyser, it is found that the light is no longer completely cut off by the analyser, but that when the latter is rotated through a definite angle, depending upon the thickness of the quartz, the light is again extinguished. This indicates that the plane polarised light yielded by the first Nicol prism is still plane polarised after traversing the quartz, but the direction of vibration, and therefore the plane of polarisation, has been rotated through a certain angle, which is measured by that through which it is necessary to turn the second Nicol prism in order to extinguish the light. If, on looking towards the source of light, the rotation is clockwise, it is described as right-handed, and the substance producing it is said to be *dextro-rotatory*, and in the opposite direction, it is described as left-handed, and the substance is said to be *laevo-rotatory*. The amount of rotation varies directly as the thickness of the crystal, and approximately inversely as the square of the wave-length of the light. It follows from the latter that, if white light is used, chromatic effects will be seen as the analyser is rotated.

A substance which rotates the plane of polarisation in this way is said to be **optically active**. The property is possessed by substances other than quartz, particularly by certain organic compounds, such as sugar, quinine, turpentine, tartaric acid, camphor, etc. The rotation produced is sometimes right-handed and sometimes left-handed. Some such substances show markedly different rotatory powers in the solid state and in solution respectively.

The amount of rotation produced by an optically active substance for light of a given wave-length, and at a given temperature, depends upon the density and the thickness, or length, of the

substance through which the beam of light passes. This may be expressed by the relation,

$$S_{t\lambda} = \frac{10\,\theta}{l\rho},$$

in which θ is the actual rotation in degrees produced in a length, l, of the substance whose density is denoted by ρ, and $S_{t\lambda}$ is the specific rotation of the substance at a temperature, t, when the wave-length of the light is λ. Specific rotation is thus defined as the amount of rotation in degrees produced by traversing a path of 10 cm. length in a pure substance of unit density, at a given temperature and for light of a given wave-length. In the case of a solution of an optically active substance in an inactive solvent, the *specific rotation of the dissolved substance* is given by the rotation produced by 10 cm. of the solution, divided by the weight of the substance dissolved in 1 c.cm. of the solution. Since the specific rotation varies with the colour of the light used, it is usual to employ sodium yellow light.

A very important relationship between light and magnetism was discovered by Faraday in 1845, who showed that, when a beam of plane polarised light is passed through a transparent medium in a direction parallel to the lines of force of a strong magnetic field, its plane of polarisation is rotated by an amount depending upon the distance traversed and the strength of the field.

10. Measurement of Specific Rotation

By measuring the amount of rotation produced under standard conditions by a given length of a solution, it is possible to deduce the strength of the solution. This process is applied particularly to the estimation of sugar, when it is known as **saccharimetry**. Observations upon rotation of the plane of polarisation also afford other information of value in Chemistry.

An arrangement of crossed Nicol prisms, N_1, N_2 (Fig. 243), constitutes a simple **polarimeter** or **saccharimeter**, the prisms acting as polariser and analyser respectively The solution is placed in a tube, T, whose ends are closed with glass plates. Parallel light from a slit, S, and lens, L, is polarised by N_1 and then passes through the tube. By rotating the analyser, N_2, viewing the emergent light through a telescope, G, the amount of rotation produced by the solution is found by noting when extinction occurs. The amount of rotation is given by the pointer, P, attached to N_2, on the circular scale on the fixed plate, C. With this simple arrangement, however,

it is impossible to tell within several degrees when the field of view is at its minimum brightness. Hence, modifications have been introduced whereby the field is divided into two halves, and adjustment of N_2 has to be made until these appear equally bright, when the plane of polarisation will be in a definite position relative to the analyser. Such a comparison can be carried out much more accurately than the estimation of maximum darkness.

One method of effecting the modification is by means of a bi-quartz. A bi-quartz consists of two semi-circular plates of quartz, one left-handed and the other right-handed (see Art. 9), each 3·75 mm. in thickness, and cut so that their faces are at right angles to the optic axes. These two pieces of quartz are cemented together

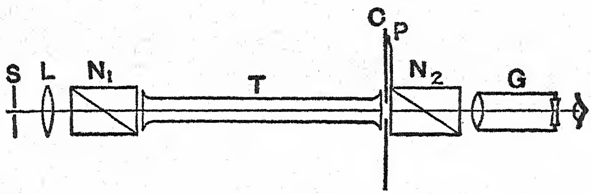


FIG. 243.

to form a circular plate, which is placed between the polariser, N_1, and the tube, T. As the plane polarised light passes through each half, it is rotated the same amount by each plate, but in opposite directions. The two halves of the field of view then appear unequally illuminated, if the light is monochromatic, unless the analyser, N_2, is in the correct position.

If white light is used, the two halves of the field of view are unequally coloured. The analyser is set to extinguish the yellow rays, when a grey violet tint, known as the *sensitive tint*, is seen. The quartz plates are cut of such a thickness that a slight rotation of the analyser turns one-half of the field of view red, and the other half blue. In this way the exact position of the plane of polarisation of the yellow rays can be obtained.

In determining the optical rotation of a substance the position of the analyser for uniform illumination of the field of view is read off when the tube is filled with the pure solvent. A known mass of the substance is then dissolved in the solvent, its total volume found, and the tube when filled with the solution is replaced in position. The analyser is then turned until uniform illumination is again found in the field of view. Observations are made with different lengths and different concentrations.

CHAPTER XVII

SPECTRA

IN the preceding chapters, the phenomena connected with the propagation of light, such as reflection, refraction, interference, etc., have been dealt with, and brief reference only has been made to the emission and absorption of light by matter. In this chapter more detailed consideration will be given to these questions. It is in this section of the study of light that the greatest advances have been made in recent years, principally by the investigation of spectra, leading to the establishment of a more definite theory as to the nature of light and its production, as well as its absorption by matter. Developments in the methods of producing and investigating spectra have played a large part in these advances. The basic principles of the production of a spectrum have been already discussed (Chapter IX.), and some of the information to be obtained from the study of spectra has been indicated.

1. The Complete Spectrum

It has been frequently stated that light—the rays forming the visible spectrum and which excite the sensation of vision—consists of a transverse wave motion in the aether, the violet rays being of shortest wave-length and highest frequency, the red rays being of longest wave-length and lowest frequency. Also the wave theory of light has been dealt with (Chapter XIV.).

The ultra-violet (see page 158)—that is, the invisible rays more or less immediately beyond the violet, and the infra-red—that is, the invisible rays more or less immediately below the red, also consist of a transverse wave motion in the aether, the ultra-violet being of shorter wave-length than the violet, and the infra-red of longer wave-length than the red.

There are, however, other aether waves, and the essential fact to be mentioned here is that the whole multitude of aether waves now known—cosmic rays, gamma rays, X-rays, ultra-violet rays, visible light rays of violet, blue, green, yellow, orange, and red, infra-red rays, Hertzian rays, wireless rays, are all identical in general character, differing only in wave-length and frequency.

The birth of the clue to this identity really dates back to **Faraday,** who discovered that some connexion existed between light and electricity. This in turn was translated by **Maxwell** into the

language of mathematics, giving rise to the famous electromagnetic theory, and ultimately to the modern idea that the above are all electromagnetic waves of varying wave-length and frequency. It is equally a characteristic of all, from the shortest to the longest wave-length, that their velocity is the same—186,000 miles per second, the well-known velocity of light (Chapter XII.).

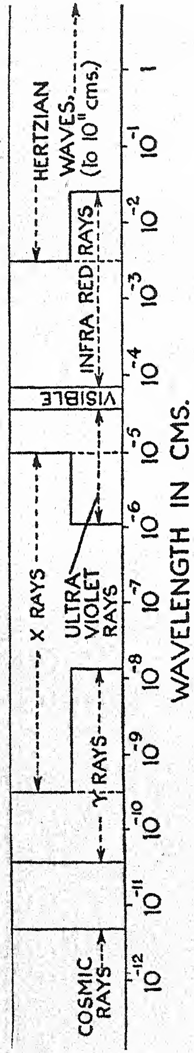


Fig. 244.

The cosmic rays, discovered and investigated by Millikan, and considered by him to proceed from the birth of atoms such as helium, oxygen, silicon, and iron in the heavenly bodies in space at extremely low temperature and pressure, are the shortest waves, their wave-length being about 2×10^{-12} cm. (see Fig. 244). The wave-length of the gamma rays from radium and other radioactive substances is about 4×10^{-10} cm., and of X-rays about $1{\cdot}5 \times 10^{-9}$ cm. The ultra-violet rays, which promote chemical action, have a wave-length of about 4×10^{-5} cm., while the visible spectrum extends from a wave-length of about 4×10^{-5} cm. for the violet rays to about 8×10^{-5} cm. for the red, and the near infra-red from about 8×10^{-5} cm. to $2{\cdot}3 \times 10^{-3}$ cm., the far infra-red to about 3×10^{-2} cm. Next come the Hertzian rays which may have a wave-length up to 10^4 cm., and finally wireless waves are the longest in wave-length which may be measured in kilometres. A wide range of wave-lengths is used in broadcasting, and still longer waves than those used in wireless exist, such as those from an alternator in an alternating current power station.

It will be seen from the diagram that overlapping between certain rays occurs in the range of wave-lengths, as in the case of Hertzian and infra-red rays. The important point to bear in mind is that all the above are electromagnetic waves differing only in wave-length and frequency. It will be seen, also, that the part occupied by the visible spectrum in the whole range of known waves is extremely small.

2. The Quantum Theory

The development of the electromagnetic theory of light led to complete explanations of the propagation of light and the phenomena of interference, polarisation, etc., but the processes by which light is emitted and absorbed by matter still remained unexplained. It was supposed reasonably that, since the long waves of wireless could be generated by the oscillations of electricity, light waves might have a similar origin, and the very short wave-length suggested that this origin might be within the atoms and molecules of matter. The discovery of the *electron* by **J. J. Thomson** in 1897 led to the belief that by applying the laws of electrical theory to the motion of the electrons in atoms the required explanation might be obtained. All attempts failed, however, until **Planck** in 1900 published the accounts of his work on the character of the radiation from a hot body.

Graphs showing the relation between the energy or intensity of the radiation emitted by a hot body and wave-length can be obtained experimentally by using fairly simple apparatus (see page 159). A typical curve showing the energy emitted by an electric arc is shown (Fig. 135). The exact shape of the curve varies with the nature of the body and the character of its surface—that is, different bodies heated to the same temperature differ considerably in the extent to which radiation of the various wave-lengths is emitted. Also, **Kirchhoff** in 1856 had showed that there was a simple relation between the power of a body to emit radiation of any wave-length and its power of absorbing radiation of the same wave-length—a body which shows little power of absorbing radiation of a certain wave-length has a correspondingly small power of emitting radiation of that wave-length. From this fundamental relation between the emissive and absorptive powers of a body, it follows that a body which has the property of absorbing all the radiation, of whatever wave-length, that falls on it has also the greatest possible emitting power for all wave-lengths. Such a body is known as a *perfectly black body*, or a *full radiator*. Although no such body is known, something almost equivalent to it can be obtained, and the radiation from such a body maintained at various temperatures can be measured. The problem was to explain why the radiation from such a body should have the character shown by the experimental curves.

Planck showed that it was possible to explain the experimental results only by assuming that, whenever an atom emitted or absorbed energy in the form of radiation, it did so only in definite minimum

amounts at a time. Each of these amounts of energy is called a quantum of energy. It was shown that the value of the energy quantum for radiation of any frequency is proportional to the frequency. This is expressed by the relation

$$E = hn,$$

in which E is the quantum of energy for radiation of frequency, n, and h is the universal constant, or Planck's constant.

The value of h is $6{\cdot}55 \times 10^{-27}$ erg. sec. Thus, for example, for the yellow light from a sodium flame, the frequency of which is about 509 billion per second, the value of the quantum of energy is about $3{\cdot}33 \times 10^{-12}$ erg. The atoms in the flame giving off this yellow light emit the radiation, not in a steady continuous stream, but in quanta of this magnitude at a time. Instead of expressing this energy in ergs, it is sometimes more conveniently expressed in terms of the *electron-volt*, which is defined as the energy transferred to an electron when it moves through a potential difference of one volt. The value of this unit is $1{\cdot}59 \times 10^{-12}$ erg, so that the energy quantum for sodium yellow light may be quoted as 2·1 electron-volts.

3. Photo-electric Effect

Planck's work led to the explanation of phenomena which had hitherto remained unexplained. Amongst these is the *photo-electric effect* on which the action of the modern photo-electric cell, used in talking films, television, etc., depends. The effect was discovered by **Hertz,** 1888, who found that if a metal plate is exposed to light, it acquires a small positive charge, due to the emission of electrons from the surface. It has been shown that this emission of these electrons depends not only on the intensity of the light but on its frequency. If the frequency falls below a certain value, known as the *threshold frequency,* depending on the particular metal used, no electrons are emitted however intense the light may be. The energy of the photo-electrons is given by the relation

$$E = k\,(n - n_0),$$

in which n is the frequency of the actual radiation used, n_0 the threshold frequency, and k is a constant.

Einstein, 1905, showed that the phenomenon could be explained by Planck's ideas, and suggested that the energy of the radiation itself existed in *quanta* (see Art. 2). According to this idea, the energy of the radiation emitted from any source is shot out from the source in showers of atoms of radiant energy, called *photons.* It was then predicted that the constant, k, in the relation above

should be equal to Planck's constant, h, and this was confirmed experimentally by Millikan, 1916.

Thus it seemed that the wave theory of light was being replaced by something like the abandoned corpuscular theory, although the photon of Einstein was very different from the corpuscle imagined by Newton. Later research work has shown that there are many other optical phenomena which can be explained by the photon theory, while the wave theory can give no explanation. However, these phenomena are nearly all concerned with the emission and absorption of light, and indeed the whole modern development of the theory of radiation and the structure of matter is based on the fundamental suggestions of Planck and Einstein.

The experimental study of spectra has played a very great part in the development of the theories outlined here, and some of the more important facts will now be dealt with.

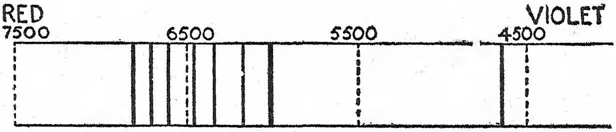


FIG. 245.

4. Line Spectra

If a small quantity of a substance such as strontium chloride or cupric chloride be brought into the non-luminous flame of a Bunsen burner, the salt will be vaporised owing to the high temperature, and the flame will be coloured, crimson in the case of strontium chloride and green in the case of cupric chloride. If now the light emitted be examined by means of a prism (see page 147), it will be found that the images on the screen do not show a *continuous* spectrum with every variety of colour, nor do they show merely the red or green parts respectively. Indeed, they are quite distinct from the spectrum described already, and consist of a number of bright lines, images of the slit through which the light is passed, whose positions are for the same substance invariable.

Such spectra were first studied experimentally by Bunsen and Kirchhoff about 1860. It is found that each element when vaporised in the flame gives one or more, often a large number, of such lines, each and all characteristic for the given element, so that no two lines occupy the same position. The positions of the chief lines for strontium chloride are shown (Fig. 245), the number supplying the wave-lengths in *tenth-metres* (see page 157). The position and

relative strength of the lines for the metals sodium, lithium, potassium, calcium, and thallium are also shown (Fig. 246), as observed when a volatile salt of these metals is introduced in the Bunsen flame. For most other metals higher temperatures are required, and in these cases sparks from an induction coil, or an electric arc, may be passed between poles of the metal in question.

Similar spectra can also be obtained from gases. To obtain the spectrum of a gas, the gas is introduced into a Geissler tube (Fig. 247) which is then partially exhausted and sealed. The discharge from an induction coil is then passed between two terminals, usually of aluminium, sealed into the ends of the tube, when the

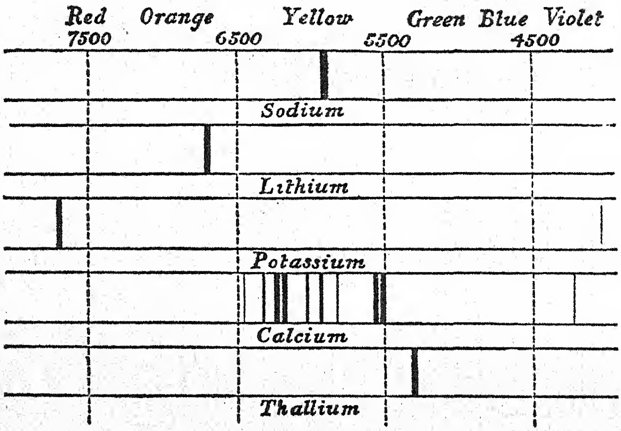


FIG. 246.

gas becomes incandescent. The light from the central portion is examined as before. If the pressure is high, the spectrum is continuous, but as the pressure is decreased, the spectrum becomes discontinuous, and finally narrows down to a series of bright bands or lines, whose position is characteristic for each gas.

In the case of liquids, the spectra are obtained from the light yielded by electric sparks between two platinum points, one of which is in the liquid and the other just above.

5. Spectroscopy

It is often possible to recognise the presence of a substance by means of the colour imparted to a flame, though when several substances are present together one may mask the other. Sodium

salts, for instance, give a bright yellow colour to the flame, which will completely mask the violet tinge given by potassium salts. However, if the light from a mixture of substances be passed through a prism, the spectrum lines of each appear in their proper places, and it is possible to recognise each and all of them by observing the positions of the lines which are visible.

Experiment. Place a little calcium chloride in a watch-glass, and moisten it with hydrochloric acid. Dip a clean platinum wire into the paste, and then hold the wire in the hot part of a Bunsen flame. Note the red coloration imparted to the flame. Repeat with salts of the following metals: sodium (yellow), lithium (rose), potassium (violet), barium (apple green), strontium (crimson), copper (bluish or emerald green), thallium (green).

If the light produced is made to illuminate the slit of a *spectroscope* (see page 230), or a *spectrometer* (see page 231), the spectra may be mapped. For this purpose the prism of the instrument is fixed in the position of minimum deviation (see page 87) for sodium yellow light, and, the slit being illuminated by the various lights in turn, the angle of deviation is read for each. Curves can then be plotted between the wave-length and angular deviation.

FIG. 247.

By removing the ordinary eye-piece of the telescope, and substituting a photographic objective, photographs of different spectra may be obtained. Unless special plates are used, however, these photographs confine themselves chiefly to the violet and ultra-violet portions of the spectrum, red light being photographically weak.

For purposes of measurement, the spectroscope is often fitted with an additional tube, T′ (Fig. 248), carrying a scale at one end and a collimating lens at the other. This tube is placed with the lens facing the prism, and its position is adjusted so that an image of the scale, illuminated by some convenient source of light, is reflected from one face of the prism into the telescope, T. When the scale tube is properly focused, the image of the scale will be seen in the telescope in coincidence with the spectrum, and the position of any line or band in the spectrum can be referred to its position on the scale. By means of this scale the lines of any spectrum can be mapped. A map constructed in this way, however, has no absolute value, and would be different for different instruments, or for a different adjustment of the same instrument.

By a graphical construction the map can be reduced to an absolute scale of wave-lengths. If the positions of a number of lines of *known* wave-length are observed, a graph can be constructed, having as abscissae the scale divisions and as ordinates the wave-lengths corresponding to given positions on the scale. Then, by noting the position of any line on the scale, and measuring the ordinate of the graph corresponding to that position, the wave-length of the line considered can be determined approximately.

Another method is to compare two spectra. This is best done by arranging to obtain both spectra together in the same field. For this purpose, a small right-angled total reflection prism (see page 178) is fitted over one half of the slit of the collimator, and a source of light, giving one of the required spectra, is placed on one side so that the rays from it are reflected into the upper half of the slit. The other half is illuminated directly by the source giving the other spectrum, and thus both spectra are seen together, the one in the upper half of the field and the other in the lower half.

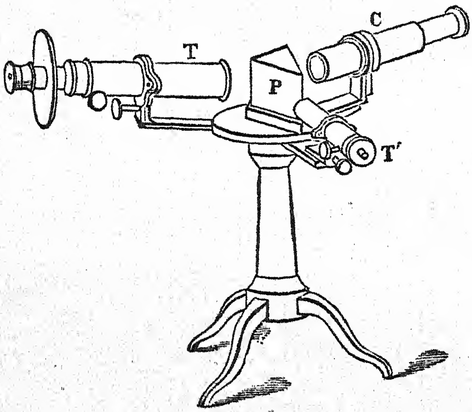


FIG. 248.

The spectroscope shown (Fig. 248) is a simple form. In the better forms of the instrument there are two or more prisms instead of one. This is necessary in order to obtain greater dispersion. With one prism, only a comparatively short spectrum can be obtained, and any peculiarities are not noticed readily. With a train of prisms, so that the light passes through them successively, the dispersion is increased by each prism, and a very long spectrum may be obtained. Such spectra can be readily photographed, showing great detail, with the modern form of *spectrograph*, which is based on the principles here described.

In one form of spectroscope a constant deviation prism (see page 180) is used. In this case, the collimator and telescope of the instrument are rigidly fixed at right angles to each other. In order

to make observations through the spectrum, the prism is rotated by means of a drum attached to the table carrying the prism. The drum is calibrated so that the wave-length of any line in the spectrum may be found directly on the scale.

6. Absorption Spectra

There are many solid bodies which show a characteristic colour by light reflected without penetration from their surfaces, whereas thin films or plates of the same substance viewed by transmitted light appear of a different colour. Such bodies are said to possess *surface colour*, and seem to have a preference for reflecting certain wave-lengths and transmitting others. Thus gold when burnished is yellow by reflected light, but if a thin film of gold, gold leaf, is examined by transmitted light, it is of a dull green colour. Many aniline dyes exhibit the same phenomena, and if an alcoholic solution of fuchsine, commonly called magenta dye, be allowed to evaporate to dryness on a glass plate, it will be found that it transmits red light, but reflects green.

Solutions of salts in many cases appear coloured by transmitted light. A solution of copper sulphate is blue, one of potassium permanganate is rose purple, of potassium chromate yellow. Such phenomena are in all cases due either to selective reflection, or to *selective absorption*. A body whose surface reflects indiscriminately all wave-lengths appears white. The yellow colour of gold is due to the fact that most of the red, green, blue, and violet rays are transmitted and gradually absorbed, the predominant reflected rays being yellow. Similarly, copper sulphate solution is of a blue colour because the rays of other colours contained in white light are absorbed in the solution, and the blue rays only transmitted.

Selective absorption can be illustrated readily by arranging a bright white light, prism, and lenses (see page 146), to throw a pure spectrum on a screen, and then interposing various substances in the path of the rays. The spectrum will be found in some cases to be crossed by definite dark bands or lines, while in some cases whole portions of the spectrum will be blotted out.

Another method is to throw an ordinary spectrum on a screen, and then insert, between the prism and the screen in the path of the light, a prism of the material under examination, the refracting edge of this prism being placed perpendicular to that of the former. The spectrum now obtained will be blotted out in places, and strongly curved on each side of these spaces. Observation will show that for rays on the red side of the absorption band the

refractive index is abnormally increased, whilst for rays on the blue side the refractive index is abnormally decreased. This is **Kundt's law.** (See also page 154.)

Experiment. Fit up a spectroscope with a bright source of white light, and place the following bodies in the path of the light, using small parallel-sided glass cells to contain the liquids. Observe the results enumerated below.

Ruby glass	Red light only transmitted.
Cobalt blue glass	Red and blue rays only transmitted.
Potassium bichromate solution	Red and orange rays only transmitted.
Cuprous ammonium sulphate solution	Blue and violet rays only transmitted.

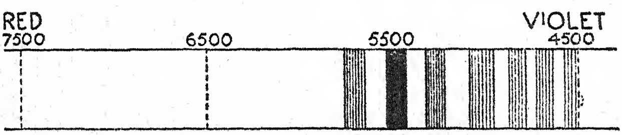


FIG. 249.

Potassium permanganate solution ..	Spectrum crossed by several dark bands in the yellow, green, and blue regions (see Fig. 249).
Blood, diluted	Two dark bands in orange and yellow regions, violet region blotted out.

Many other simple examples are presented by dyes and by fluids derived from organisms. The positions of the absorption bands in the spectra are just as definite and characteristic as the lines are in line spectra (Art. 4), and hence the spectroscope or spectrograph may be used for the purpose of recognising such bodies in solution.

Many vapours give absorption spectra. Thus iodine vapour gives a large number of narrow dark bands, and water vapour gives such a characteristic absorption spectrum that its appearance in the spectrum of sky-light is looked upon by meteorologists as an almost certain forecast of rain.

7. Band Spectra

A so-called band spectrum consists of a number of bright bands of light, one edge of the band being sharp, the other edge being more vaguely defined as the intensity falls off gradually towards this edge. Thus the spectrum has a fluted appearance. If examined with a good spectroscope, each band is found to consist of a large number of bright lines, very close together where the edge of the band is sharp, but getting further and further apart towards the other edge.

Band spectra are emitted by compounds or molecules, unless the substance is heated to such a high temperature that the substance breaks up into its constituent elements, when the line spectra of the elements are produced.

Most carbon compounds give spectra of this type, and the blue part of a candle flame, or of a Bunsen flame, shows a band spectrum.

It has been found that a substance can emit both a band spectrum and a line spectrum, but not simultaneously. This was first observed in the case of nitrogen, the band spectrum being produced when the gas is in the molecular state, and the line spectrum resulting when the gas is in the atomic state.

8. The Doppler Effect

If a source of light is moving towards an observer, the wave-length of the light emitted is decreased, and if moving away from the observer, the wave-length is increased. This effect is fully explained in most textbooks on Sound (see *Textbook of Sound*, Catchpool, § 25), and was first explained by Doppler. It was shown by Fizeau that the same effect occurs with light.

Thus, if the source of light is moving relative to the observer, the wave-length, λ', of the light observed will be slightly different from the value, λ, it would have if there were no such relative motion. The difference of wave-length, $(\lambda' - \lambda)$, is given approximately by the relation,

$$\frac{\lambda' - \lambda}{\lambda} = \frac{V}{c},$$

where V is the component of the velocity of the source along the line joining it to the observer, and c is the velocity of light. Since c is large (Chapter XII.), V must be very large if the change of wave-length is to be appreciable.

In consequence of this effect, the lines in the spectrum of the light are displaced, towards the violet if the motion is towards the

observer, and towards the red if the motion is away from the observer. Thus, if the spectrum of a moving body is compared with the spectrum of a stationary source giving some of the same lines as the moving body, the displacement of the lines can be measured easily, and from that the velocity of the body can be ascertained.

By such means, Huggins has shown that the bright star, *Sirius*, is receding from the earth at a speed of about 30 ml. per sec. Also, Keeler has shown by similar means that the *rings* of *Saturn* consist of a multitude of meteorites revolving around the parent body.

Again, it is known that the sun rotates, and hence it would be expected that, if the light from one edge of the equatorial zone, say, that which is moving towards the earth, is examined, the Fraunhofer lines (see page 157) would show a slight shift towards the violet, while with the light from the opposite extremity, there would be a slight shift towards the red. This is found to be the case, and by careful observations, the speed of rotation of the zone of the sun's surface can be found. This speed can also be measured by the observation of the motion of *sun-spots*, and it is found that the values obtained by the two methods agree, thus providing a check on the theory of the Doppler effect.

The Doppler effect taken in conjunction with the Kinetic Theory of Gases, also explains why the lines in the spectrum of a gas widen when the temperature of the gas is raised.

9. The Zeeman Effect

About a hundred years ago, Faraday suspected that there must be some relation between magnetism and light, and experimented with a sodium flame placed between the poles of a strong electro-magnet, observing the usual D line spectrum by means of a spectroscope. However, no modification could be found.

Some years later, in 1896, Zeeman, using a more powerful spectroscope, discovered that the magnetic field causes each line of the spectrum to be split up into a group of lines very close together. Further investigation showed that each line is polarised in a characteristic way, and that the effect was not the same when the direction of the light was parallel to the lines of force of the magnetic field as when the direction of the light was at right angles to the lines of force. In the latter case, each line produced a group of three lines, one in the position of the original line and one on either side, while in the former case only two lines are obtained, one on either side of the position of the original line, the original line being

missing. Later experiments showed that far more complex groups of lines are obtained, and by the application of the ideas of the quantum theory a complete explanation has been given.

It may be mentioned here that, by studying the Zeeman effect in the case of the solar spectrum, it has been found possible to obtain information concerning the magnetic field of the sun.

10. The Stark Effect

After the discovery of the Zeeman effect, experiments were carried out to determine whether the light emitted from a source placed in an electrostatic field would show a similar effect. By using very strong electric fields, **Stark in 1913** found that each spectrum line was split up into a group arranged symmetrically on either side of the original position.

11. Spectra and the Structure of Matter

As mentioned above (Art. 3), the study of spectra has led in recent years to a more precise knowledge of the structure of atoms and molecules, and to an explanation of the emission of light by such atoms and molecules. For many years spectroscopists have been engaged in the production and mapping of the various forms of spectrum obtained from elements and compounds under varying conditions, so that the positions and the wave-lengths of the lines and bands in the spectra of most elements and compounds are known with precision. Various forms of spectroscope and spectrograph have been devised and utilised, methods of technique have been perfected, so that an enormous store of data has been accumulated, from which not only elements and compounds can be detected by their spectrum lines and bands, but on which an almost complete theory as to the structure of matter has been built up in recent years.

Space will not permit a complete account of the work, and such a complete account is beyond the scope of this book, but some of the more important advances will now be dealt with briefly.

In 1913, **Bohr** put forward a theory of the structure of atoms based on Planck's quantum theory (Art. 2). According to this theory, the atom consists of a positively charged *nucleus* with *electrons* revolving around it. Almost the whole mass of the atom is in the nucleus which is comprised of a number of *protons*—hydrogen nuclei, each bearing a positive charge—and electrons. The number of protons is the same as the atomic weight, and the

number of electrons outside the nucleus is the same as the atomic number. A proton carries a unit positive charge and an electron a unit negative charge, so that in order that the atom may have no resultant electric charge, the number of electrons in the nucleus is equal to the atomic weight less the atomic number.

According to the still more modern view, since the discovery of the *neutron*, which has the same mass as the proton but carries no charge, the nucleus may be considered to consist of equal or nearly equal numbers of protons and neutrons. The neutron can be considered as a proton joined to an electron, the charges being neutralised.

The case of hydrogen is regarded as one of the simplest, the atom consisting of a single electron revolving round one proton. Bohr assumed that the electron moved in circular paths or orbits, and found that the radii of the different orbits were proportional to the squares of the natural numbers—that is, 1^2, 2^2, 3^2, . . . If the electron is removed from the inner orbit to an outer one its energy is increased, and now the electron may fall back to an orbit of smaller radius with emission of energy. This energy is radiated according to the wave theory, and may cause a spectrum line of definite wave-length in the visible spectrum. Now, according to the quantum theory, energy may only be emitted in *quanta*, and if n is the frequency of the light emitted, the energy emitted is given by hn, where h is Planck's constant. Every time the electron falls back to an orbit of smaller radius, a spectrum line is produced. By application of the quantum theory, Bohr showed that

$$n = \mathrm{R}\left(\frac{1}{p^2} - \frac{1}{q^2}\right)$$

in which R is a constant known as Rydberg's constant, and p and q are whole numbers. From observations of the wave-lengths of the lines in the visible spectrum of hydrogen, Balmer had previously shown in 1885 that the frequencies of the lines could be represented by the relation,

$$n = \mathrm{K}\left(\frac{1}{2^2} - \frac{1}{q^2}\right),$$

in which K is a constant, and q is a whole number greater than 2. Thus, Bohr's theory agrees with Balmer's results if $p = 2$, which means that the electron is falling from some higher orbit into the second one.

Now, by putting $p = 1$, and $q = 2, 3, \ldots$, another series of spectrum lines should result. Such a series has been observed by

Lyman, though these lines are not in the visible spectrum, but beyond the violet. They arise when the electron changes suddenly from some outer orbit to the innermost. Similarly, another series for which $p = 3$ and q is a whole number greater than 3, has been discovered by Paschen in the infra-red portion of the hydrogen spectrum. Thus, experiment gives results in close agreement with those obtained by Bohr's theory for the hydrogen atom.

Other atoms are more complex, owing to the presence of a number of electrons moving in orbits round the nucleus, and in addition the electrons exert forces on each other, so that at first some difficulty was experienced in applying Bohr's theory. With the advances in the knowledge of the movements of electrons in the atom in recent years, however, the difficulty has been removed, and now very complex spectra can be explained completely. The main principle that there are orbits—or *energy levels,* as they are sometimes called—still holds, and all spectrum lines are caused by the emission of quanta of energy by electrons in jumping from one orbit to another.

In recent years it has been shown that the same fundamental quantum principles can also be used to explain the production of absorption spectra (Art. 6) and the very complex band spectra (Art. 7) of molecules. The molecule can, of course, be very complex in structure, and the more complex it is, the greater is the number of orbits or energy levels and the number of possible transitions is accordingly much greater. Thus a more complicated spectrum with many more lines than in the case of an element is to be expected, and at the same time the difficulties of applying Bohr's theory also increase. In spite of these difficulties, however, some of the simpler cases have been completely analysed, and it appears that further work on the subject will eventually lead to a complete interpretation of the structure of matter in all its forms.

EXERCISES

Note. *Where, in a few of these exercises, the nature of a lens, convex or concave, has been specified by means of a sign convention, the convention used is that described on page* 40 *(convex lenses having negative focal lengths) and* not *the " Real is Positive " convention.*

EXERCISES II

1. A spherical uniform source of light, 10 cm. in diameter, is placed 1 m. in front of a spherical opaque body, 5 cm. in diameter. Find the shortest distance from the latter at which a screen may be placed so as to have no umbra in the shadow cast upon it. Find also the diameter of the penumbra in this position.

2. A luminous sphere, 5 cm. in diameter, is placed 150 cm. from a disc of wood, 2 cm. in diameter. Find the diameters of the umbra and penumbra cast on a screen 50 cm. behind the disc of wood. The line passing through the centres of the luminous sphere and the disc is perpendicular to the latter and to the screen.

3. In Fig. 10 (page 13), CO = 3 m., OC′ = 20 cm., and the diameter of the aperture at O is 1 mm. Find the area of the circular spot of light at C′ due to the pencil of light from C. If AB = 2 m., find also the length of A′B′.

4. A dark room of side 10 ft., with white walls, has a small hole at the centre of one wall. Outside this hole and 56 ft. distant is a stone cross 15 ft. high, and an image appears on the wall of the room. How high will the image be ?

5. Under the same circumstances as in Exercise 4, the image of a tree 50 ft. high appears to be 8 in. high. How far is the tree from the hole ?

6. Explain the appearance of the bright circular and elliptical spots seen on the ground in the shadows of trees when the sun or the moon is shining.

EXERCISES III

1. The sun is 30° above the horizon, and its image is observed in a tranquil pool. What, in this case, is the angle of incidence and the angle of reflection ?

2. A narrow strip of plane mirror is hung horizontally against the wall of a room on a level with the eye of an observer. Draw a diagram showing how much of a side wall of the room will be visible by reflection from the mirror.

3. A man, 6 ft. tall, sees his image in a plane mirror hung vertically. The top of the mirror being 6 ft. from the ground, determine its smallest length in order that the man may see his full-length image in it.

4. Find the deviation produced by reflection at a plane mirror, when the angle between the incident and reflected rays is 80°.

5. A mirror, scale and telescope are used to measure the deflection of a suspended system. The scale is distant 1 m. from the mirror, and during the movement of the mirror the scale reading alters from 14 cm. to 44 cm. Find approximately the angle of deflection of the system.

6. Show that, if a ray of light be incident at any angle on one of two mirrors inclined at right angles to each other, the ray is reflected from the second mirror in a direction parallel to its original direction.

7. Make a measured drawing showing the positions of all the images formed by two mirrors, inclined to each other at 45°, of an object placed between the mirrors.

8. Two mirrors, M_1 and M_2, are inclined to each other at 50°, and an object is placed between them. Make a measured drawing showing in *black* the situation of all the images formed where the rays from the object strike M_1 first, and in *red* the situation of all the images where the rays strike M_2 first.

9 The angle between two mirrors is 10°. At what angle should a ray of light, travelling towards the intersection of the mirrors, be incident on either mirror in order that it may at the fourth reflection be reversed and travel back along the same path?

10. An object is placed ½ in. from one plane mirror and 1 in. from another plane mirror parallel to the first, so that the object is between them. Make a measured drawing showing the positions of all the images formed up to the fourth order.

11. A small object is placed between two parallel mirrors. The distance between the mirrors is 6 in., and the object is placed 2 in. from one of them. Find the distances between the corresponding members of the two series of images formed, and also the distances between the odd members of each series, and between the even members of each series.

12. A plane mirror which is at first 1 ft. from an object is then moved back 1 ft. *parallel to itself*. How far does the image move? Give a diagram in illustration.

EXERCISES IV

1. A concave spherical mirror is so placed that a candle flame is situated on its principal axis at a distance of 18 in. from its surface. An inverted image, three times as long as the candle flame itself, is seen sharply defined on the wall. What is the focal length of the mirror?

2. Prove that if an object is placed at a distance of $3f$ in front of a concave spherical mirror, of focal length f, then the image is one-half the size of the object.

3. A small object on the axis of a concave spherical mirror, at a distance of 16 in. from it, produces a *real* image which is three times its own size. Find the focal length of the mirror.

4. A gas flame is placed at a distance of 10 ft. from the wall of a room. What must be the radius of curvature of a concave spherical mirror, and where must it be placed, in order that it may produce on the wall an image of the gas flame magnified fourfold linearly?

5. How far from a concave spherical mirror of radius 3 ft. should an object be placed to give an image magnified three times? Would the image be real or virtual?

6. An object, 6 cm. long, is placed 1 m. in front of a concave spherical mirror of 10 cm. focal length. Find the nature and size of the image.

7. Prove that when an object is placed midway between a concave spherical mirror and its principal focus, the image is twice as large as the object.

8. A gas jet is placed on the principal axis of a concave spherical mirror and 10 cm. in front of it. A real inverted image is produced on a screen held in front of the mirror. If the length of the image is three times that of the flame, find the focal length of the mirror and the position of the screen.

9. An object 1 in. high is placed on the axis of a concave spherical mirror of 1 ft. focal length at a distance of 2 ft. from the mirror. Draw a diagram showing the position and size of the image, explaining all necessary lines in the construction.

10. Describe what an observer sees when he looks at himself in a concave spherical mirror, and moves slowly backwards from the mirror.

11. A luminous point is situated 30 ft. in front of a concave spherical mirror, and on the principal axis. Show by *scale* drawings what will become of the rays after reflection from the mirror, and also what will become of the rays after reflection, when the luminous point is brought up, first to 7 in., then to 4 in., from the mirror.

12. A luminous object is 108 ft. in front of a concave spherical mirror of 10 in. focal length. Where is its image formed? Is the image real or virtual, erect or inverted, enlarged, reduced, or the same size as the object?

13. A luminous point is 80 ft. in front of a concave spherical mirror of 24 in. radius. Calculate the position of its conjugate focus.

14. A luminous point is 12 in. in front of a concave spherical mirror of 7 in. focal length. Calculate the position of the conjugate point. If the luminous point be brought up to 4 in. from the mirror, calculate the position of its conjugate focus.

15. An object 6 in. high is 10 ft. in front of a concave spherical mirror of 18 in. focal length. Calculate the position and size of the image, and state whether it is real or virtual, erect or inverted.

16. A candle flame is placed at a distance of 3 ft. from a concave mirror formed of a portion of a sphere, the diameter of which is 3 ft. Determine the nature and position of the image of the candle flame produced by the mirror, and state whether it is erect or inverted.

17. Show how to find the position of the image of an arrow placed in front of a concave spherical mirror. Explain when it is an erect, and when an inverted image.

18. A small object is placed in front of a concave spherical mirror of 6 in. radius at a distance of 4 in. from the surface of the mirror. Where will its image be situated, will it be erect or inverted, and what will its dimensions be compared with those of the object? Where must the object be so that the image may be of the same size?

19. A plane mirror is placed 6 ft. in front of a concave spherical mirror of 2 ft. focal length. Find where an object must be placed between the two mirrors in order that images and object may coincide.

20. Explain the formation of images by a concave cylindrical mirror. Find the relation between the distances of the two conjugate foci from the mirror. What is the position of the image of a point which is at the distance of the diameter from the reflecting surface of the cylinder?

21. A small object, 0·1 in. long, is placed at a distance of 3 ft. from a convex spherical mirror of 12 in. focal length. What is the length of the image and its distance from the mirror?

22. A penny is held 8 in. in front of a convex spherical mirror of 1 ft. radius. Where will its image be, and what will be its diameter compared with that of the penny?

23. An object is held in front of a convex spherical mirror, at a distance equal to the focal length of the mirror. Determine the size, nature, and position of the image.

24. An image produced by a convex spherical mirror of focal length f is $\frac{1}{r}$ th the size of the object. Show that the distance of the object from the mirror is $(r - 1) f$.

25. Trace the changes in the position of the image formed by a convex spherical mirror as the object is moved from a great distance up to the surface of the mirror.

26. Explain, giving a drawing, how it is that an observer sees himself as he does in a polished metal sphere.

27. A luminous point is successively 80 ft., 25 in., and 3 in. in front of a convex spherical mirror of 8 in. focal length. Calculate the corresponding position of the conjugate point.

28. An object 6 in. long is placed symmetrically on the axis of a convex spherical mirror, and at a distance of 12 in. from it. The image formed is found to be 2 in. long. What is the focal length of the mirror?

29. A luminous point is 9 in. in front of a spherical mirror of 6 in. focal length. Show, by *scale* drawings, the course of the rays after reflection, and the position of the principal focus.

EXERCISES V

(See Table I., page 353)

1. If a ray of light passes from one medium to a second, making an angle of incidence of 45°, and an angle of refraction of 30°, show that the refractive index for the media is $\sqrt{2}$.

2. A ray of light passes from alcohol to a parallel plate of Iceland spar, 1 in. thick, and then into air. The ray is incident on the Iceland spar at 45°. Make a *scale* drawing showing the exact path of the ray. The refractive index of Iceland spar is 1·75, and of alcohol 1·4.

3. In an experiment, as described on page 61, the following readings were taken:—

Values of i, 10° 30′, 24° 24′, 39°, 58°, and the corresponding
values of r, 6° 48′, 15° 48′, 25° 45′, 34° 12′.

Find the mean value of the refractive index.

4. A vessel, 6 in. deep, is filled with alcohol. What is the apparent depth of the liquid?

5. A piece of plate glass, of refractive index 1·6 and 5 in. thick, is placed between the eye and an object. Find what alteration will take place in the apparent distance of the object from the eye.

6. Find the relative refractive index from Canada balsam to air.

7. Find the absolute refractive index of carbon disulphide, given that the relative refractive index from carbon disulphide to glass is 0·9, and the absolute refractive index of glass is 1·512.

8. The refractive index of water is 1·33, and the velocity of light in air is 300,000,000 m. per sec. Find the velocity of light in water.

9. The critical angle of a given medium is 60°. What is its refractive index?

10. The sine of the critical angle for two media is 0·777. What is the refractive index from the rarer to the denser of the two?

11. In an experiment, as described on page 71, the following readings were taken:—

$$p_1 = 3{\cdot}05,\ p' = 2{\cdot}05;\ p_2 = 2{\cdot}90,\ p_2' = 1{\cdot}94.$$

Find the mean value of the refractive index.

12. If a glass of water with a spoon in it is held a little above the level of the eye of an observer, and the observer looks upwards at the under surface of the water, it is found that he cannot see that part of the spoon which is above the water. Explain this.

13. Describe Wollaston's method for the determination of the refractive index of a liquid by means of total reflection. In an experiment with a certain liquid, the angle between the two positions of extinction was 97°. Find the refractive index, and identify the liquid.

14. Explain, with the aid of a diagram, what occurs when light is incident on a glass plate. Explain why a transparent substance such as glass is opaque when finely powdered.

15. A piece of a colourless mineral is dropped into a colourless liquid, and the mineral is invisible in the liquid. How are the refractive indices of the mineral and of the liquid related?

16. Explain the quivery appearance seen above hot rocks or bricks, and the streaky appearance of water in which ice, sugar, or acid is being dissolved.

EXERCISES VI

1. Light incident at an angle of 60° on one face of an equilateral glass prism is deviated through an angle of 30° at the first face. Draw a diagram showing the path of the rays through and out of the prism, and find the refractive index of the glass.

2. A ray of light falls normally on the middle of one face of a glass prism whose section is an equilateral triangle. Show by a measured drawing the whole path of the ray.

3. Show by accurate measured drawings the path of a ray of light, incident at an angle of 30° from the normal, on (*a*) a thick plate of glass with parallel sides, (*b*) a glass prism with a refracting angle of 60°.

4. The angle of a prism is 60°, and the refractive index of its material is $\sqrt{2}$. Show that the angle of minimum deviation is 30°.

5. Show that when a ray of light is refracted through a prism, in the position of minimum deviation, the path of the ray in the prism is perpendicular to the line bisecting the angle of the prism.

6. The angle of minimum deviation produced by a hollow prism, filled with a certain liquid, is 30°. If the refracting angle of the prism is 60°, what is the refractive index of the liquid?

7. A hollow watertight prism containing air, with flat glass sides, is immersed in a glass tank full of water. Draw and explain a diagram showing the path of a ray of light passing through the water and the prism.

8. A glass prism, of refracting angle 5°, is immersed in water. Find the approximate deviation produced in a ray of light passing through the prism if the absolute refractive indices of glass and water are 1·5 and 1·33 respectively.

9 Show that if the refracting angle of a prism be greater than twice the critical angle for the medium of which it is composed, no ray of light can pass through it.

EXERCISES VII

1. A small air bubble at the centre of a glass sphere is seen from a point outside the sphere. What is the apparent position of the bubble? Explain with the aid of a diagram.

2. A small air bubble in a sphere of glass 4 in. in diameter appears, when looked at so that the bubble and the centre of the sphere are in a line with the eye, to be 1 in. from the surface. What is its true distance from the surface?

3. Draw three parallel straight lines 1 in. apart, in the plane of the paper, to represent rays of light incident upon a glass sphere of radius 2 in., with its centre upon the last of the series, and trace, by geometrical construction, the paths of the rays within and beyond the sphere.

4 A brass sphere of 2 cm. radius is surrounded by a glass shell of 6 cm. external radius. What is the apparent thickness of this shell?

5 A block of transparent jelly, of refractive index 1·33, is bounded on one side by part of the convex surface of a sphere of radius 8 mm. Find the position of the principal focus within the mass of the material.

6. In order to determine the refractive index of the glass of a double convex lens, the radii of curvature of its surfaces were measured and found to be 30 cm. and 31 cm. respectively. Its focal length was also determined and found to be 30·5 cm. Find the refractive index.

7. Find the focal length of a plano-convex lens, given that the radius of curvature of its convex surface is 50 cm., and that the refractive index of its material is 1·6.

8. Prove that the focal length of a plano-concave glass lens is equal to twice the radius of curvature of the concave surface.

9. Two large thin watch-glasses are cemented together, so as to form a double-convex lens filled with air, and are then immersed in water. Trace the paths of rays of light falling upon the lens from a luminous point on the axis of the lens and under water.

10. The focal length of a lens *in vacuo* is 2 ft. The refractive indices of glass and water being 1·5 and 1·33 respectively, find the focal length of the lens when immersed in water.

11. A gas flame is at a distance of 6 ft. from a wall. Where must a convex lens, of 1 ft. focal length, be placed in order to give a distinct image of the flame on the wall?

12. An object, 1 in. long, is placed at a distance of 1 ft. from a convex lens of 10 in. focal length. Find the nature, position, and size of the image.

13. If an object, 10 cm. from a convex lens, has its image magnified four times, what is the focal length of the lens?

14. An object is at a distance of 3 in. from a convex lens of 10 in. focal length. Find the nature, position, and size of the image.

15. An object is placed 6 in. from a lens, and an image, three times as large, is seen on the same side of the lens as the object. Find the focal length of the lens.

16. A convex lens, of 3 in. focal length, is used to read the graduations of a scale, and is placed so as to magnify them three times. Show how to find at what distance from the scale it is held, the eye being close up to the lens.

17. An object is placed at a distance of 6 in. from a converging lens of 1 ft. focal length. Find the position and size of the image.

18. The image formed by a convex lens is n times the size of the object. Show that the distance of the object from the lens is $-\frac{(n+1)f}{n}$, where f is the focal length.

19. On a sheet of paper placed vertically is drawn a capital L. If an observer stands 3 ft. in front of the paper and holds a double-convex lens, of 6 in. focal length, half-way between his eye and the lens he will see an image of the letter. Draw a diagram of the image as seen, and state whether it is larger or smaller than the object.

20. An object, 3 in. high, is placed successively at distances of 45, 20, 18, 8 in. from a convex lens of 10 in. focal length. Calculate, in each case, the position and height of the image, state whether it is real or virtual, erect or inverted.

21. Repeat Exercise 20 for the same object at the same distances from a concave lens of 9 in. focal length.

22. Show, by the application of suitable relationships, that the image formed by a concave lens is always less than the object.

23. A convex lens, of 10 in. focal length, is combined with a concave lens, of 6 in. focal length. Find the focal length of the combination.

24. Find the focal length of a lens which is equivalent to two thin convex lenses, of focal lengths 20 cm. and 30 cm., placed in contact.

25. A convex lens, of focal length $-$ 12 cm., is placed in contact with a concave lens, and the focal length of the combination is found to be $-$ 24 cm. Calculate the focal length of the concave lens.

26. A contact combination of a convex and a concave lens has a focal length of $-$ 19·3 cm. The focal length of the convex lens being $-$ 10·02 cm., find that of the concave lens.

27. A convex lens is focused on a mark on a sheet of paper, and a thick plate of glass is then put between the paper and the lens. It is found that the mark can no longer be seen distinctly. Explain this, and illustrate by a diagram the paths of the rays in the two cases.

28. The middle of a candle flame is placed on the axis of a convex lens, and, at a greater distance from the lens but on the same side of it, a plane mirror is arranged perpendicular to the axis. When a sheet of white paper is brought gradually near to the lens, on the side remote from the flame and mirror, images of the flame are seen in two positions. Explain this, and illustrate by means of a diagram.

29. A candle flame is placed 6 in. from a plane mirror, and a convex lens, of 3 in. focal length, is placed between the candle and the mirror, and 2 in. from the latter. Find the position of the image formed.

30. A candle flame is placed 20 cm. from a plane mirror. Find where a convex lens, of 5 cm. focal length, must be placed in order that the image of the flame may coincide with the flame itself.

EXERCISES VIII

1. In an experiment made to determine the focal length of a convex lens by the method of Art. 4, I., (4) (page 131), the following corresponding values of d_1, d_2, were observed:—

d_1. 52·5, 38·4, 32·5, 28 cm.
d_2. 30·5, 41·4, 51·9, 69·5 cm.

Find by a graphical method the focal length of the lens.

2. An object and screen were fixed on an optical bench at a distance apart of 94·1 cm. Between them a convex lens was moved about, and in two positions, 71·3 cm. apart, images were formed on the screen. Find the focal length of the lens.

3. A convex lens was used to form a real image of an object. Between this image and the lens, at a distance of 29·25 cm. in front of the image, a concave lens was placed, and it was found that the image was thrown back 12·25 cm. Find the focal length of the concave lens.

4. A convex lens yielded a real image of a piece of illuminated gauze at a distance of 150 cm. from itself. At 50 cm. behind the lens was placed a concave lens, and behind this a plane mirror, and it was found that an image of the gauze was thrown back on the gauze itself. Find the focal length of the concave lens.

5. A convex lens yielded a real image of a piece of illuminated gauze at a distance of 40·38 cm. from itself. An equi-convex lens was placed behind this lens, and it was found that an image of the gauze was formed alongside the gauze itself when the second lens was 29·68 cm. behind the first. Draw a diagram, and find the radii of curvature of the surfaces of the second lens. If the focal length of this lens is − 10·1 cm., find the refractive index of the glass of which it is composed.

6. A long focus equi-convex lens was fixed in a clip on an optical bench, and a plane mirror was also mounted on a stand. The bases of the two stands could be moved together. At the start they were placed close up to an illuminated gauze, with the mirror facing the gauze, and the lens between the gauze and the mirror. Then they were withdrawn together gradually. At distances from gauze to lens of 57·2 cm. and 112·1 cm., images of the gauze were thrown on the screen alongside the gauze itself. At the latter distance, when the mirror was withdrawn, the image disappeared. Find the focal length of the lens, the radii of curvature of its surfaces, and the refractive index of the glass.

7. The experiment in Exercise 6 was repeated with a convex meniscus. The concave surface was placed facing the gauze, and at distances from gauze to lens of 8·9 cm., 35·2 cm., and 35·3 cm., images were thrown back on the screen. At the last distance, the image disappeared when the mirror was withdrawn. Find the focal length of the lens, the radii of curvature of its surfaces, and the refractive index of the glass.

8. A hollow glass prism, of refracting angle 39° 33′, was filled with water and set on the table of a simple spectroscope. The angle of minimum deviation for sodium light was found to be 13° 57′. Find the refractive index of water.

9. The experiment in Exercise 8 was repeated with carbon disulphide, using a prism of refracting angle 40° 24′, and the angle of minimum deviation for sodium light was found to be 28°. Find the refractive index of the liquid.

EXERCISES IX

1. Draw the section of a prism. Draw also the section of a beam of sunlight passing through the prism, and by the diagram show how this light is acted upon by the prism.

2. A spectrum cast upon a white screen is observed through a piece of purple glass. What appearance does it present, and what is the cause of this appearance ?

3. How may it be disproved experimentally that white light passing through a piece of coloured glass acquires colour from the glass ? What is it that really happens ?

4. If one piece of glass is held up to the sun it appears dark red, and another appears dark blue. If the two pieces of glass are held together, the sun cannot be seen through them at all. How is this ?

5. A lamp-flame, observed through a glass prism, appears to be coloured blue on one side and red on the other. Draw a diagram tracing the rays from the lamp to the eye, and showing which side of the coloured image is red and which side is blue.

6. Given a powerful source of light, such as a lime-light or an electric arc, explain how a spectrum of it could be obtained on a screen.

7. Describe an experiment proving that white light is compound. How can it be shown that the constituents into which it is resolved are not likewise compound ?

8. If a shower of very small equilateral prisms of glass, of refractive index 1·65, fell between an observer and the sun, what would be the general effect ? Calculate the angular radius of the halo seen.

EXERCISES X

1. A total reflection prism is employed to deviate a ray of light through an angle of 60°. What is the shape of its section ?

2. Explain why it is possible to take a photograph with a small pinhole, pierced in an opaque screen, in place of a lens, and why it is not possible to do so when the hole is large.

3. A pinhole camera is made in the form of a cube, of edge 1 ft., with a hole in the centre of one side. It is placed opposite a building 60 ft. high at a distance of 100 yd. Find the size of the image.

4. A lantern slide is 3·25 in. square, and an enlarged image is to be formed, by the aid of a lens of 6 in. focal length, on a screen 20 ft. distant from the lens. What kind of lens should be used, at what distance from the slide must it be placed, and what will be the size of the image ?

5. Explain, with the aid of a diagram, the effect of a convex lens held close to the eye and employed as a simple microscope. Prove an approximate relation for the magnifying power of the lens, its focal length and the least distance of distinct vision by the naked eye being given.

6. A person whose nearest distance of distinct vision is 18 in. uses a reading lens of focal length 6 in. What magnification is obtained ?

7. A person can see objects distinctly only at a distance of about 4 in. from the eye. Calculate the focal length of lenses to be used for reading, walking, and for viewing distant objects. Assume that 10 in. is the normal nearest distance of distinct vision, and that in walking the average distance at which it is required to see clearly is 15 ft.

8. An aged person sees distinctly from infinity up to about 20 in. from the eye. What spectacles should be worn to remedy this defect?

9. A person is under water. Do objects in the water appear to be the same distance from him as they would in air? Why are near objects indistinct?

EXERCISES XI

1. Find the focal length of a sphere of glass of 10 cm. radius, the refractive index of the glass being 1·5.

2. Describe the astronomical telescope. Trace the course of a pencil of rays of light, from any point on a distant object, through it, and find the magnifying power.

3. What is the magnifying power of a telescope whose object glass is of 12 ft. focal length, and its eye-piece of 0·5 in. focal length?

4. The focal length of the object glass of a telescope is 3 ft., and that of the eye-piece is 3 in. Draw a curve showing how the magnifying power varies with the distance of the object.

5. A telescope is held with its object glass under the surface of the water of a pond. The water wets the outer surface of the glass, but does not enter the telescope. The telescope is focused so that objects at the bottom of the pond are seen clearly. Is the telescope now longer or shorter than when used for viewing objects at the same distance in air? Does it make any difference what kind of convex lens is used for the object glass?

6. Find the focal length of a lens equivalent to a combination of two lenses, each of focal length f, and placed at a distance apart equal to $2f$.

7. What is meant by the chromatic aberration of a lens, and how is it corrected in the object glass of a telescope? The mean refractive indices of two specimens of glass are 1·52 and 1·66 respectively, and the differences in the refractive indices for the same two lines of the spectrum are 0·018 and 0·022 respectively. Find the focal length of a lens of the second glass, which, when combined with a convex lens of the first glass of focal length 50 cm., will make an object glass achromatic for these two lines.

8. An achromatic lens of focal length 1 m. is to be constructed of lenses of crown glass and flint glass, whose refractive indices are 1·5 and 1·65 respectively, and whose dispersive powers are in the ratio of 5 : 8. The crown glass lens is to be equi-convex, and one surface of the flint glass lens is to fit it. Find the radii of curvature of the surfaces.

9. Describe the astronomical telescope fitted with Ramsden's eye-piece, and draw a diagram showing the path of a pencil of rays of light from a distant object through it. What advantage has Ramsden's eye-piece over Huyghens', and why is the latter usually employed in microscopes?

10. A concavo-convex lens, of focal length 30 cm. and radii of curvature 10 cm. and 30 cm., is silvered on the concave surface. Show that the lens acts as a plane mirror.

11. In a Newtonian reflector whose speculum is of 10 ft. focal length, what must be the focal length of the eye-piece to give a magnifying power of 250?

12. Compare the light-grasping power of two spherical mirrors whose diameters are 13 in. and 36 in., and of the human eye when the pupil is 0·25 in. in diameter.

13. Describe a simple form of microscope with two lenses, and trace pencils from different points of an object through it. If the rays emerge parallel, what change must be made in the relative positions of the lenses in order that the object may be seen clearly?

14. The images formed by the objective of a compound microscope are 8 in. from the lens. Find the magnifying power of the instrument, given that the focal length of the objective is 0·5 in., and that of the eye-piece 2 in.

15. Compare the dispersive powers of carbon disulphide and of water from the following experimental observations, the prisms being placed in the position of minimum deviation for the yellow light:—Deviations of the red, yellow, and violet light with carbon disulphide, $21° 45'$, $28°$, $30° 47'$, the refracting angle of the prism being $40° 24'$; with water, $13° 52'$, $13° 57'$, $14° 20'$, the refracting angle of the prism being $39° 33'$.

EXERCISES XII

1. How has the velocity of light been determined from observations of the eclipses of Jupiter's first satellite? Assuming that the satellite revolves round the planet in a constant period of 40 hr., that the velocity of the earth in its orbit is 18 ml. per sec., and that of light is 187,000 ml. per sec., find the greatest and the least apparent intervals between successive eclipses.

2. Describe the method by which Fizeau investigated the velocity of light.

3. Describe a method of measuring the velocity of light based upon the use of a revolving toothed wheel. The distance between the two stations being 9·3 ml., and the number of teeth being 100, the rotation is started and gradually increased in speed. Find the number of rotations per sec. of the wheel when the light reflected from the distant station has disappeared and reappeared 10 times. The velocity of light is 186,000 ml. per sec.

4. Describe Foucault's method of measuring the velocity of light by means of a rotating mirror. What is the effect of introducing a tube, with glass ends, containing water between the rotating and fixed mirrors, and what relation is there between the velocity of light in a medium and its refractive index?

5. Explain carefully some method of measuring the velocity of light. How has it been shown that lights of different colour travel through air at very nearly the same rate?

EXERCISES XIII

1. The intensities of two sources of light are in the ratio of 9 : 16. Find the ratio of the distances at which they must be placed from a screen, in order to produce on it the same illumination.

2. The lines joining the points A, B, C form an equilateral triangle. D is the middle point of the side BC. A screen is placed at A with its surface parallel to BC. Sources of light placed at B, C, and D are found to illuminate the screen at A equally. Compare their intensities.

3. If a 16 candle-power gas-flame at a distance of 10 ft. illuminates a surface to a particular degree, at what distance must a 20 candle-power electric glow lamp be placed from that surface to illuminate it to the same degree?

4. The distance between two incandescent lamps of 16 and 25 candle-power respectively is 6 ft. Show that there are two positions, on the line joining the lamps, at which a screen may be placed so as to receive equal illumination from each lamp, and determine these positions.

5. In Rumford's photometer (Fig. 208, page 264) L_1S_1 is found to be 115 cm., and L_2S_2 to be 201 cm. Compare the intensities of L_1 and L_2.

6. A rod is fixed vertically in front of a vertical white screen. Three sources of light, A, B, C, of 16, 18, and 48 candle-power, are placed at distances of 2, 3, and 4 ft. respectively from the screen, and are so arranged that the three shadows of the rod thrown by them are close together but do not overlap. Compare the relative degrees of illumination of the shadows.

7. In a Rumford photometer the shadows of the rod thrown by a bats-wing gas-flame and a Welsbach incandescent gas-light are equally bright when the gas-flame is 2 ft. from the screen and the Welsbach 4 ft. 3 in. How many times more light does the latter give than the former?

8. In Foucault's photometer (Fig. 209, page 265) $EL_1 : EL_2$ as $a : b$. Find the relative intensities of L_1 and L_2.

9. The intensities of two sources of light are in the ratio 4 : 9. If these sources are 200 cm. apart, where would a Bunsen's photometer be in accurate adjustment between them?

10. The grease spot of a Bunsen photometer disappears when a standard candle-flame is 10 in. from one side and an electric glow lamp is 36 in. from the other side. What is the candle-power of the lamp?

11. Three standard candles are placed 10 in. from one side of the screen of a Bunsen photometer. How far must a 5000 candle-power electric arc be placed from the other side in order to cause the disappearance of the grease spot?

EXERCISES XIV

1. Write a brief account of the corpuscular theory and the wave theory of light.

2. What are the reasons for believing that light is propagated in waves, and that the wave motion is transverse to the direction of propagation?

3. Point out the differences between a sound wave, a light wave, and a wave traversing a stretched string.

4. Explain how the rectilinear propagation of light has been explained on the basis of the wave theory.

5. Show how the laws of reflection and refraction may be deduced from the wave theory of light.

6. Explain the refraction of light on the wave theory, and show that the refractive index for two media is the ratio of the velocities of light in the media.

7. Prove by means of the wave theory of light that the focal length of a concave spherical mirror is equal to half its radius of curvature.

8. Apply the wave theory of light to find the relation between the focal length of a thin lens, the radii of curvature of its surfaces, and the refractive index of the material.

EXERCISES XV

1. Under what conditions can interference of light take place? Describe a simple method based on interference for the measurement of the wave-length of light.

2. Compare the interference bands obtained by using (*a*) a single mirror, (*b*) a double mirror.

3. A vertical slit is illuminated by sodium yellow light, and the light then passes through a bi-prism. Interference bands are observed in an eye-piece, and successive bands are found to be 0·201 mm. apart, the distance from the slit to the eye-piece being 73·5 cm. When a convex lens is placed between the bi-prism and the eye-piece, two positions of the lens are found in each of which two clear images of the slit are seen. The distances between the images are 3·732 mm. and 1·242 mm. in the two positions. Find the wave-length of the light.

4. A slit, illuminated by sodium light, is placed 2 mm. from the plane of a mirror. What will be the distance between successive interference bands formed on a screen 1 m. from the slit, if the wave-length of sodium light is 0·0000589 cm.?

5. Explain the colours of thin films. Compare the colours produced by reflected and transmitted light when thin films are used.

6. A convex lens is placed on a slab of plane glass and is illuminated by sodium yellow light. The diameter of the 10th dark ring is found to be 0·406 cm. If the wave-length of the light is 0·0000589 cm., find the radius of curvature of the lower surface of the lens. Using lithium red light, the diameter of the same 10th ring is found to be 0·433 cm. Find the wave-length of lithium red light.

7. Give a general explanation of the action of a diffraction grating, and explain how a sharp image of a slit is obtained when using monochromatic light.

8. A parallel beam of sodium yellow light falls normally on a diffraction grating having 14,500 lines per in. The first order spectrum is observed and two bright lines are seen at deviations of $19° 39'$ and $19° 40' 30''$ respectively. Find the wave-lengths of the light for these two lines.

EXERCISES XVI

1. What experimental evidence is there for the belief that light vibrations are transverse and not longitudinal?

2. Explain what is meant by polarisation of light. Indicate briefly the methods of producing plane polarised light, and describe how it may be shown experimentally that the light is polarised.

3. How may the polarisation of light be brought about by reflection and refraction?

4. Show how it is possible to account for the blue colour of the sky?

5. Explain what is meant by optical rotation. Define specific rotation and describe how it can be measured.

6. A sugar solution is placed in a tube 20 cm. long, and the tube is placed between the crossed nicols of a polarimeter which is illuminated by sodium light. The optical rotation caused by the solution is 10°. If the specific rotation is 67°, find the strength of the sugar solution.

7. Describe some practical purposes for which polarisation can be utilised.

EXERCISES XVII

1. Describe how you would proceed to examine the spectrum of a salt. What variation in the method would be necessary if you wished to examine the spectra of (*a*) metals, (*b*) gases?

2. What are the distinctions between (*a*) a continuous spectrum, (*b*) a line spectrum, (*c*) a band spectrum, (*d*) an absorption spectrum?

3. Write a brief account of the simple quantum theory. How is this theory supported by the photo-electric effect?

4. What reasons are there for supposing that visible light, X-rays, ultra-violet, and infra-red radiation are essentially similar in nature?

5. Explain what is meant by the Doppler effect, and discuss its application to light waves.

6. It is found that a certain line in the spectrum of a star is displaced by the Doppler effect, and the apparent wave-length is 0·000043074 cm. If the true wave-length is 0·000043079 cm., and the velocity of light is 300,000,000 m. per sec., find the relative velocity between the star and the earth.

EXAMINATION QUESTIONS

Reflection. Chapters I. to IV

1. A horizontal ray of light is reflected successively once at each of two plane vertical mirrors inclined at a fixed angle A. If the mirrors are rotated about a vertical axis coinciding with their line of intersection, show that the final direction of the ray is unchanged.

Describe the sextant and explain how it is used to determine the altitude of a star when the sea-sky horizon is not visible.

2. Derive the relation between the distances of object and image from a spherical reflecting surface in terms of the radius of curvature of the surface.

Explain two distinct optical methods of measuring the radius of curvature of a convex mirror.

3. Deduce an expression connecting the distances of an object and its image from a mirror with the focal length of the mirror.

An image of a very short object is produced by a mirror. Find the relation between the magnifications produced when the object is (*a*) along, (*b*) perpendicular to, the axis of the mirror.

4. Show that if a beam of rays diverging from a point near the axis strikes a small concave mirror, the rays after reflection will pass through another point. How is the position of the second point related to that of the first? If the first point is 17 cm. from the surface of the mirror and the second is 17 cm. from the centre of curvature, what is the radius of curvature of the mirror?

5. What is meant by the focal length of a convex mirror? Describe *one* method by which you would measure the focal length of such a mirror. Explain why such mirrors are used as driving mirrors on motor cars.

Find the position and size of the image formed of an object 10 cm. high and 5 m. distant from a convex mirror of 100 cm. radius of curvature.

6. Explain how the image of a small object situated on the axis of a concave mirror changes as the object moves along the axis from a great distance up to the pole of the mirror. Give diagrams.

A small object is placed on the axis of a spherical mirror at a distance of 20 cm. from it. A virtual image is formed which is 12 cm. from the mirror. Determine whether the mirror is concave or convex, and find its radius of curvature.

7. Discuss, and illustrate by a diagram, the reflection of a wide beam of parallel rays by a spherical concave mirror of large aperture, and explain what is meant by the focus of the mirror.

An object of length 5 cm. is placed transversely to the axis of a concave mirror of radius 30 cm., with its centre on the axis and at a distance from the mirror equal to half the focal length. Give a construction showing the formation of an image, and calculate its size and its distance from the mirror.

Refraction. Chapters V. to VII

8. Explain how the refractive index of a medium can be determined accurately by the method of apparent depth, and derive the formula required in the calculation.

A ray of light falls at an inclination i on a parallel-sided plate of glass of thickness d, and of refractive index μ. Find the perpendicular distance between the incident and emergent rays in terms of these quantities. Find a numerical value when $i = 30°$, $\mu = 1·5$, and $d = 1$ cm.

9. An object is viewed normally through a plate of glass of thickness t and refractive index μ. Find the apparent displacement.

A parallel-faced slab of glass 6 cm. thick of refractive index 1·6 rests on the horizontal base of a tank. On this rests a layer 8 cm. deep of a liquid of refractive index 1·5, and on the latter floats a layer of water 10 cm. deep of refractive index 4/3. A mark on the lower surface of the glass is viewed vertically from above. Find the apparent position of the mark.

10. Describe and explain a method for the determination of the critical angle of refraction for air and water.

If an eye, situated under water, looks towards the surface, external objects appear to lie in a circular window situated in the surface. Explain this, and find the diameter of the window if the eye is at a depth of 5 ft. and the refractive index of water is 4/3.

11. State the laws of reflection and refraction of light and explain the term *critical angle*.

Describe, with the aid of ray diagrams, how a right-angled isosceles glass prism may be used (a) to invert a beam of light without deviating it, (b) to turn a beam back in a direction parallel to its incident direction. In each case point out which, if any, faces of the prism will have to be silvered.

12. Under what conditions does total reflection of light occur? The refractive index of lead glass is 1·6 and that of soda glass is 1·5 for the yellow sodium line. Calculate the critical angle for the surface of separation between these two media.

13. A beam of parallel rays is refracted through a transparent prism in a principal plane. Show that at minimum deviation of the emergent beam a simple formula connects the index of refraction of the material of the prism with the angle of the prism and the angle of deviation.

14. What is meant by the minimum deviation of a ray of light passing through a prism? Obtain an expression for the refractive index of the material of the prism in terms of the angle of minimum deviation and the angle of the prism. Describe how you would measure the angle of minimum deviation with a spectrometer, using sodium light.

15. Establish the formula relating object and image distances in the case of refraction at a spherical surface.

A hemisphere is made of glass of refractive index 3/2 and is placed in air. Its radius is 4 cm. and a point source of light is situated on its axis at a distance of 12 cm. from the curved surface. Find, by calculation or construction, the position of the image formed by the hemisphere, and draw a diagram to show the path of a pencil of rays. (The source lies nearer to the curved surface.)

16. A pin is fixed vertically to the inside of a thin cylindrical glass vessel of radius 10 cm. and containing water of refractive index 4/3. Calculate the apparent position of the pin when viewed diametrically through the vessel. Give a careful diagram and prove any formula you use in your calculation.

17. Rays of light are refracted from a medium of refractive index n_1, through a spherical surface of radius r, separating it from a medium of refractive index n_2. Derive a formula relating the positions of object and image.

A point source of light is situated at a distance of 50 cm. from the surface of a sphere of glass of radius 10 cm. and of refractive index 1·5. A narrow

pencil of rays with its axis normal to the sphere is refracted by it. Find the position of the final image.

Lenses. Chapter VII

18. Deduce the relation existing between the distances of object, image, and principal focus from a thin lens. Explain the convention of signs you employ.

A source of light is 40 cm. in front of a screen, and a lens between them produces a sharply focused image of the source on the screen. On moving the lens 10 cm. towards the screen the latter has to be moved back 5 cm. to bring the image of the source again into focus. What is the focal length of the lens?

19. Find an expression for the linear magnification produced by a simple lens.

An object is placed in front of a screen and a convex lens is placed between them for the purpose of throwing an image of the object on the screen. If the focal length of the lens be f, find an equation relating to the distance, d, between the screen and object when the image is formed and the distance, u, of the lens from the object. Show that the formation of an image on the screen is impossible if $d < 4f$.

20. Establish a formula connecting the focal length of a lens in terms of the refractive index of its material and the radii of curvature of its surfaces.

A convex lens of focal length 20 cm. and having equal spherical surfaces of 24 cm. radius is placed in water of refractive index 4/3. Calculate the distance from the lens at which parallel rays would be brought to a focus.

21. Two positions of a thin convex lens, separated by d cm., give images of illuminated cross-wires on a screen for a fixed distance D cm. between the cross-wires and the screen. Calculate the focal length of the lens in terms of d and D. Show also that the size of the object is a geometric mean between the sizes of the two images.

22. Light is divergent from a point on the axis of a simple thin lens and is converged to a point on the other side of the lens. Show that the sum of the reciprocals of the distances of the two points from the lens is constant. Deduce a formula connecting the value of this constant with data giving the shape and material of the lens.

23. A convex lens placed 20 cm. from an object forms a real image 30 cm. from the lens. If a concave lens is inserted between the image and the convex lens 18 cm. from the latter, the image moves through a distance of 16 cm. still remaining real. Draw a diagram showing how this last image is formed, and calculate the focal lengths of the two lenses. If the length of the object perpendicular to the axis of the lens is 4 mm., what is the length of each of the two images?

24. Two convex lenses of focal lengths 10 cm. and 20 cm. are placed at a distance of 5 cm. apart. An object is placed at a distance of 12 cm. in front of the former. Find the position of the final image and the value of the magnification.

Find the focal length of a single lens which produces an image in the same position when placed in the position of the first lens, and determine the magnification in this case.

25. Draw diagrams to show the formation of (*a*) a virtual, and (*b*) a real image by a thin convex lens.

A thin convex lens of focal length 5 cm. is placed at 6 cm. from a screen. It is desired to focus on the screen the image of an object placed at 20 cm. from the lens. Find the focal length of the thin lens which must be placed in contact with the convex lens to do this.

Experiments. Chapter VIII

26. Define *principal focus* of a convex lens. Describe a good method of measuring the focal length of such a lens.

A source of light is at a fixed distance of 80 cm. from a screen. It is found that there are two positions, 20 cm. apart, in which a convex lens will throw a real image of the source on the screen. Find the focal length of the lens and the linear magnification of the image in each case.

27. A plane mirror is placed behind a converging lens and a pin in front of it. The pin is adjusted until an image coincident with the pin is produced and (*a*) erect, (*b*) inverted. Draw diagrams to illustrate the formation of the images and explain how the focal length may be deduced in each case.

28. A concave mirror of radius of curvature 60 cm. is placed on a table and holds a shallow pool of a liquid. When a parallel beam of light falls vertically on it the rays are brought to a focus at a point 21·1 cm. above the mirror. Draw a careful diagram showing the course of the rays and deduce the refractive index of the liquid.

29. Describe how you would determine experimentally, using optical methods throughout, the radii of curvature of the surfaces and the focal length of a thin diverging meniscus (convexo-concave) lens.

If the values are 10, 20, and 40 cm. respectively, what is the refractive index of the material of the lens?

30. Describe a spectrometer and explain how you would use it to measure the refractive index of the material of a prism for sodium yellow light.

31. Explain how you would determine the refractive index of a liquid by means of a spectrometer and deduce the formula for its determination.

With the collimator slit vertical explain how you would level the table of the instrument with the aid of the prism, and state what other adjustments are necessary in preparing the apparatus for use.

32. Explain how you would use a spectrometer to measure the angle of a prism.

Two rays of light, making an angle B with one another, are incident one on each of the adjacent faces of a prism including an angle A. If the angle between the two reflected rays is C, find the value of A in terms of B and C.

33. Describe *two* accurate methods for the determination of the refractive index of a liquid.

34. What is meant by the minimum deviation of a ray of light passing through a prism? Obtain an expression for the refractive index of the material of a prism in terms of the angle of minimum deviation and the angle of the prism. Describe how you would measure the angle of minimum deviation with a spectrometer, using sodium light.

Dispersion. Chapter IX

35. Explain what is meant by a pure spectrum and show how it is obtained with an ordinary spectrometer.

How can an approximately pure spectrum be seen through a prism without using lenses?

Show how it is possible using two thin prisms to obtain (*a*) dispersion without deviation, (*b*) achromatism.

36. Explain the meaning of the terms "spectrum," "dispersion," "dispersive power." Show that for a prism of small angle the dispersive power of the material of the prism is given by the angle of dispersion divided by the mean deviation.

37. What is meant by chromatic aberration? How is it corrected in (*a*) prisms, (*b*) lenses?

The difference between the refractive indices for blue and red light in flint glass may be taken as 0·33, and in crown glass as 0·10. It is desired to make a converging lens of 30 cm. focal length which will be achromatic for these two colours. Find the focal lengths of the two lenses, given that the mean refractive index of flint glass is 1·65 and of crown glass 1·50.

38. Compare and contrast the properties of ultra-violet, visible, and infra-red radiations. Describe experiments to illustrate the properties of these radiations.

39. Describe the solar spectrum and explain its characteristics as far as you can. What other types of spectra are known?

Explain how you would form a pure spectrum, giving reasons for the arrangements necessary.

40. What is meant by the terms "infra-red" and "ultra-violet" radiation? Describe one method only by which you could produce a beam of ultra-violet radiation free from light. Mention some of the most important physical properties of ultra-violet radiation and compare the wave-lengths of such rays with those of visible light.

41. What is meant by the term "achromatism"?

Explain in what sense a convex and concave lens in contact can be made achromatic. How is this condition brought about in this case? Two thin prisms of different kinds of glass are arranged so that when a parallel bundle of rays of white light is refracted through them successively the emergent bundle of red rays is parallel to the emergent bundle of blue rays. If the appropriate refractive indices are 1·523 and 1·514 for one prism and 1·664 and 1·645 for the other, find the ratio of the refracting angles.

42. Explain what is meant by deviation, dispersion, and dispersive power for light passing through a transparent prism. Obtain expressions for these quantities for a prism of small angle in terms of the refractive indices. Show how two prisms may be combined to produce deviation without dispersion. Why is such a combination not strictly achromatic for white light in practice?

43. Write a short essay on colour, and explain clearly why two objects which appear exactly the same colour by gas-light may appear of different tints by daylight.

44. Give an account of the phenomena of fluorescence and phosphorescence, and describe how they may be demonstrated.

45. Explain what is meant in optical theory by the terms *dispersion* and *dispersive power of a transparent substance*. Describe how you would measure the dispersive power of the glass of a prism, explaining what sources of light you would use.

Optical Instruments. Chapters X. and XI

46. Give an account of the pin-hole camera. Discuss the effect on the image of changing (*a*) the shape, (*b*) the size, of the aperture.

47. Distinguish between a real and a virtual image, and discuss what is meant by depth of focus.

Draw a diagram of a pin-hole camera, pointing out what factors determine the definition and the brilliancy of the resulting image. Compare the action of such a camera with that of a fixed-focus box camera fitted with one convex lens.

48. Describe and explain the method of use of *two* of the following instruments: (*a*) range finder, (*b*) sextant, (*c*) optical lever, (*d*) projection lantern.

49. Explain the method used in projecting lantern slides.

If you were required to project a 3-inch square slide to give a 30-inch square reproduction on a screen at a distance of 11 ft. from the slide, what would be the focal length of the projector lens you would use and where would you put it? If there were also supplied a convex lens of focal length 6 inches and an approximate point source of light, how would you arrange these to get a well-illuminated picture?

50. Draw a diagram showing the formation of the image of a small object by a lens used as a magnifying glass.

Deduce a formula for the magnifying power of a lens and indicate the method you would use to find the magnifying power directly in the laboratory.

51. Explain, with a diagram, the use of a convex lens as a magnifying glass. What do you mean by the magnifying power of such an instrument? Find its value for a lens of 5 cm. focal length used by a person whose least distance of distinct vision is 25 cm.

52. Describe the optical system of the eye. Explain how the defects of short sight and astigmatism may be remedied by the use of lenses.

A short-sighted person can only see objects whose distance is between 5 inches and 25 inches from him. What spectacles must he use to see very distant objects, and what will be the distance of the nearest object he can see while wearing them?

53. Explain, giving diagrams, the defects in vision known as "long sight" and "short sight" and the method of correcting them. Find the power of the lens which must be used to enable an eye, which cannot see clearly an object closer to it than 5 metres, to read a book placed at 25 cm.

54. Write a short essay on the eye, its defects and their correction.

55. Describe the action of an astronomical telescope and give a diagram illustrating the passage of a non-axial pencil of light through the system to an eye.

The magnifying power of an astronomical telescope when used to view an object at infinity is 10 and the distance apart of the lenses is 11 in. On viewing an object nearer to the telescope the eye-piece has to be moved 0·25 inch. Find the distance of the object from the telescope. In each case the final image is viewed at infinity.

56. Show how you would use a converging lens of 1 m. focal length and a second similar lens of 20 cm. focal length to form a telescope. Draw one or more diagrams to show how the rays from a distant object pass through the instrument. What would be the magnifying power of the telescope when used to view distant objects?

Show that if the second lens were not available, the first could still be usefully employed to examine distant objects. Assuming 20 cm. to be the least distance of distinct vision, what is the magnifying power given by the single lens?

57. Define the magnifying power of a telescope by means of angles subtended at the eye-piece.

Deduce an expression for the magnifying power (*a*) when the eye is focused at infinity, and (*b*) when the eye is focused at the distance, D, of distinct vision.

Draw a careful diagram to illustrate the passage of a pencil of rays through the telescope of a spectrometer when it is in the condition in which it is used in the experiment on the refractive index of a prism.

58. How can a concave and a convex lens be used to form a simple telescope? Give a practical application of the principles illustrated and discuss the modifications necessary in order to avoid chromatic effects.

Define the magnifying power of such an instrument and deduce an expression for it.

59. Explain with the help of diagrams the action of (*a*) an astronomical telescope, (*b*) an opera glass.

Calculate the magnifying power of an astronomical telescope when both the object and image are at infinity. How would you determine the magnifying power experimentally?

60. Describe, with the aid of a diagram, the construction of *either* (*a*) a microscope, *or* (*b*) a telescope. What is meant by the magnifying power of the instrument you describe and how may it be measured?

61. Describe a compound microscope and a telescope and draw diagrams illustrating the passage of a non-axial pencil of light through the systems from a point on the object to the image.

A converging lens and a diverging lens each of focal length 12 cm. are placed 18 cm. apart with their axes coincident. An object is placed 24 cm. from the converging lens on the side away from the diverging lens. Find the position of the final image.

62. Draw a diagram to show how two lenses can be arranged to form a microscope. What are the principal defects in the image produced by such a microscope? Indicate how *one* of these defects is overcome in practice.

Why are oil-immersion objectives used for high power work?

63. What is meant by the term *magnifying power* when applied to an optical instrument?

Draw a careful ray diagram to illustrate the action of a compound microscope.

How would you determine experimentally the magnifying power of such an instrument?

64. Compare and contrast the optical system of a compound microscope and an astronomical telescope, each system being formed of two thin lenses, and deal in particular with the magnification produced by each instrument.

Velocity and Wave Theory. Chapters XII. and XIV

65. How does the velocity of light vary with the nature of the medium and the colour of the light?

Describe a method of measuring the velocity of light in air or in free space.

66. Contrast briefly the main methods of measuring the velocity of light and describe in detail the method which you consider the most accurate.

67. Describe an accurate method of measuring the velocity of light, and show how the result would be deduced from the observations.

Show that the refractive index for the surface of separations of two media is equal to the ratio of the velocities of light in the two media.

68. Show that the wave theory of light leads to the result that light travels more slowly in water than in air.

How has the velocity of light in water been measured?

69. A plane wave of light is incident obliquely on a plane glass surface. How is the refraction of the light explained on the wave theory?

If the refractive indices of water and glass are 4/3 and 3/2 respectively, find the critical angle for a surface of separation of water and glass,

70. How has the velocity of light been measured? How did measurements of the velocity of light support the wave theory of light propagation?

71. Describe how the velocity of light may be determined by a rotating mirror method. Explain the significance of the results obtained when a vessel containing water is appropriately placed in the path of the light.

72. Write a short essay on any *one* of the following theories as to the nature of light: (*a*) corpuscular, (*b*) wave, (*c*) electro-magnetic.

Photometry. Chapter XIII

73. Describe an accurate form of photometer and explain its mode of action.

Two lamps of the same candle-power, which may be assumed to give out light uniformly in all directions, are fixed to the horizontal ceiling of a room at a height of 10 ft. above the floor and 20 ft. apart. Compare the illuminations at two points on the floor, one vertically below one lamp, the other vertically below the point midway between the lamps.

74. Explain the terms "candle-power" and "foot-candle," and describe some instrument which makes measurements in one of these units.

Find the ratio of the illuminations produced by the sun on the ground, assumed horizontal, when the altitude of the sun is (*a*) 90°, (*b*) 60°.

75. Explain how you would use a photometer to determine the proportion of light cut off by a plate of imperfectly transparent material.

Describe carefully the type of photometer you would use and explain how you would make the necessary calculation.

76. How does the illumination of a surface depend on its distance from the source of light, considered as a point source, and also upon the inclination of the rays? How would you test your statements experimentally?

A small bright source of light is situated centrally at the top of a wall 10 ft. square. A small vertical surface is held perpendicular to the wall at one of its vertical sides. Find the distance between the surface and the source if it has the same illumination as it would have on the floor directly below the source.

77. Calculate the illumination on the ground midway between two lamp-posts 100 yd. apart and 15 ft. high, if the lamps are each of 400 candle-power, defining the unit in which illumination is measured. How would you check your result by observation?

78. Describe concisely the Lummer-Brodhun photometer and some form of flicker photometer, stating the particular advantages in each case.

79. Define *illumination*, *candle-power*, and *lumen*.

Describe how you would find the mean horizontal candle-power of an electric lamp, using an accurate form of photometer, explaining (*a*) how you would eliminate the effects of stray light, and (*b*) how you would compare lights of different colours.

Miscellaneous. Chapters XV. to XVII

80. Explain the difference between ordinary light and plane polarised light.

Give a very brief account of one use made of polarised light.

81. Describe the construction of some form of polarimeter and explain the action of the instrument.

82. What is meant by the term "plane polarised light"? Describe and explain two methods of producing it, and state how you would distinguish such light from ordinary light.

83. What features of a light wave correspond to intensity, colour, and polarisation?

Explain how the refractive index for light of different colours may be determined.

84. How may a beam of plane polarised light be produced, and how is it distinguished from unpolarised light?

Describe how you would observe and measure the effect of passing a beam of plane polarised light through a column of sugar solution.

85. Explain the terms *dispersion* and *pure spectrum*. Describe how you would produce a pure spectrum and write a short account of the various types of spectra.

86. Explain how a pure spectrum may be formed.

What information can be obtained from the examination of the spectra of the sun and stars? Explain the principles on which the deductions depend.

87. Describe the dispersion of light by optical substances and the best way of making use of dispersion to form the spectrum of a source of light. Draw a diagram of the arrangement of apparatus for this purpose. What are the chief types of emission spectra and by what different means are they produced?

88. What do you understand by a *continuous spectrum*, a *line spectrum*, and a *pure spectrum*?

Describe, with a clear diagram, the arrangements you would make to project a pure spectrum on a screen, and indicate how you would detect the existence of ultra-violet rays beyond the visible region.

89. Give some account of the Doppler effect, quoting one example from sound and one from light.

90. Describe an apparatus suitable for the measurement of intervals of time of the order of that taken by a light wave to travel a distance of a few miles. Explain how it is used to measure these intervals.

A plane mirror recedes from a source of yellow light at a velocity of v cm. per sec. How fast does the image appear to move and what effect has this motion on the wave-length of the light? Calculate the difference in wave-length between the light of the incident and reflected beams, if the yellow light is of wave-length λ and the velocity of propagation is c. The light is incident normally on the mirror.

TABLES OF OPTICAL CONSTANTS

I

REFRACTIVE INDICES

Mean values, Sodium yellow light

Solids

Diamond	2·417	Rock salt	1·544
Flint glass (heavy)	1·963	Quartz	1·544
Iron	1·73	Jena glass (crown)	1·517
Iceland spar	1·658	Alum (potash)	1·456
Jena glass (flint)	1·65	Ice	1·31
Crown glass (heavy)	1·613	Copper	0·65
Mica	1·60	Sodium	0·12

Liquids

Carbon bisulphide	1·632	Sulphuric acid	1·43
Canada balsam	1·53	Hydrochloric acid	1·41
Turpentine	1·47	Alcohol (ethyl)	1·362
Olive oil	1·46	Ether (ethyl)	1·354
Paraffin oil	1·44	Water	1·333

Gases and Vapours

Carbon bisulphide	1·00148	Air	1·00029
Chlorine	1·00077	Oxygen	1·00027
Carbon dioxide	1·00045	Water	1·00026
Ammonia	1·00038	Hydrogen	1·00014
Nitrogen	1·00030	Helium	1·000035

II

DISPERSIVE POWERS

Calculated from the relation $\omega = \dfrac{\mu_H - \mu_A}{\mu_D - 1}$, *where* μ_H, μ_D, *and* μ_A *are the refractive indices of the substances given for the violet* (H), *greenish-yellow* (D), *and red light* (A) *corresponding to the Fraunhofer lines* (*see page* 157).

	μ_H	μ_D	μ_A	ω
Carbon bisulphide	1·700	1·628	1·609	0·143
Flint glass	1·653	1·619	1·609	0·073
Rock salt	1·569	1·544	1·537	0·059
Water	1·344	1·333	1·329	0·045
Crown glass	1·551	1·534	1·528	0·043

III

WAVE-LENGTHS

Values in tenth-metres (10^{-8} *cm.*)

(*a*) CHIEF FRAUNHOFER LINES (see page 157)

LINE	WAVE-LENGTH	SUBSTANCE	LINE	WAVE-LENGTH	SUBSTANCE
A	7661	Oxygen	G	4308	Iron
B	6867	Oxygen	H	3968	Calcium
C	6563	Hydrogen	K	3934	Calcium
D	5893	Sodium	L	3820	Iron
E	5270	Iron	M	3720	Iron
F	4861	Hydrogen	N	3581	Iron

(*b*) ELECTROMAGNETIC SPECTRUM (see page 316)

Hertzian waves ..	10^{15}–10^{6}	Green	4920–5500
Infra-red	10^{4}–10^{7}	Blue	4550–4920
Red	6470–7700	Violet	3600–4550
Orange	5880–6470	Ultra-violet	600
Yellow	5500–5880	X-rays	10–150

IV

CANDLE-POWER

Ordinary gas jet	10–18	Electric arc	1,000–2,000
Welsbach mantle	45	Eddystone lighthouse	80,000
Argand burner	11–17	Belle Isle lighthouse ..	30,000,000
Electric bulb	10–150		

V

BRIGHTNESS

Values in candle-power per sq. cm.

Candle	0·5	Arc crater	17,200
Sun's disc	165,000	Opal bulb surface ..	1·7
Moon's disc	0·5	Clear blue sky ..	0·4
Electric lamp filament ..	600	Paper for reading ..	0·0015

VI

REFLECTION OF LIGHT

(*a*) *Percentage of light*, R, *reflected by a surface of glass of refractive index* 1·55 *for varying angles of incidence*, *i*

i	R	*i*	R
0	4·65	50	6·50
10	4·65	60	9·73
20	4·68	70	18·00
30	4·81	80	39·54
40	5·26	90	100·00

(*b*) *Percentage of normally incident light reflected by the polished surface of various metals*

Copper	48	Steel	55
Gold	74	Speculum	64
Nickel	63	Silver-backed glass	88
Platinum	61	Mercury-backed glass	71
Silver	93		

VII

OPTICAL ROTATION

Mean values for sodium yellow light.

Cane sugar solution	$+ 66{\cdot}45°$ per dm.
Turpentine	$- 37°$,, ,,
Tartaric acid solution	$+ 15{\cdot}06°$,, ,,
Nicotine	$- 162°$,, ,,
Quartz	$+ 21{\cdot}72°$ per mm.

ANSWERS TO EXERCISES

Note. *Numerical values given below for the focal lengths of lenses are expressed in terms of the Sign Convention given on p. 40. Readers using the "Real is Positive" convention, should substitute "+" for "–," and vice versa, in such answers.*

EXERCISES II. (*Page* 330)

1. 100 cm.; 20 cm.
2. 1 cm.; 4·33 cm.
3. 0·000894 cm.^{2}; 13·33 cm.
4. 2·68 ft.
5. 750 ft.

EXERCISES III. (*Page* 330)

1. 60°.
3. 3 ft.
4. 100°.
5. 8° 36′.
9. 30°.
11. 12 in., 24 in., 36 in.; 12 in., 12 in.
12. 2 ft.

EXERCISES IV. (*Page* 331)

1. 13·5 in.
3. 12 in.
4. 5·33 ft.; 13·33 ft. from wall.
5. 2 ft., real; 1 ft., virtual.
6. Real; 0·66 cm. long.
8. 7·5 cm.; 30 cm. from mirror.
12. 10·08 in.; real, inverted, reduced.
13. 12·15 in.
14. 16·8 in.; 9·33 in. behind.
15. 21·18 in.; 1·06 in. high; real, inverted.
16. Real; 1 ft. from mirror; inverted.
18. 1 ft. from mirror, inverted, three times as large; at the centre of curvature.
19. $2\sqrt{3}$ ft. from plane mirror.
20. One-third of diameter from pole.
21. 0·025 in.; 9 in. behind.
22. 3·43 in. behind; 0·43.
23. $\frac{1}{2}$, virtual, $\frac{f}{2}$ behind.
27. 7·9 in., 0·61 in., 2·2 in. behind.
28. 6 in.

EXERCISES V. (*Page* 333)

2. Relative refractive index, 1·25; angle of refraction, 34°·5.
3. 1·50.
4. 4·38 in.
5. 1·875 in. nearer.
6. 0·654.
7. 1·68.
8. 225,000,000 m. per sec.
9. 1·15.
10. 1·29.
11. 1·495.
13. 1·336; water.
15. Equal.

EXERCISES VI. (*Page* 334)

1. 1·73.
6. 1·41.
8. 0° 36′.

EXERCISES VII. (*Page* 335)

2. 1·2 in.
4. Front, 3·43 cm.; side, 3 cm.
5. 32·2 mm. from surface.
6. 1·5.
7. – 83·33 cm.
10. 8 ft.
11. $(3 \pm \sqrt{3})$ ft. from wall.
12. Real, 5 ft. from lens, 5 in. long.
13. Real image, – 8 cm.; virtual image, – 13·33 cm.

14. Virtual, 4·29 in. from lens, 1·43 times object. 15. — 9 in.
16. 2 in from scale. 17. 12 in. from lens on same side; twice as large.
19. 9 in. from eye; half linear dimensions.
20. (*a*) 12·86 in. behind, 0·86 in., real, inverted; (*b*) 20 in. behind, 3 in., real inverted; (*c*) 22·5 in. behind, 3·75 in., real, inverted; (*d*) 40 in. in front, 15 in., virtual, erect.
21. (*a*) 7·5 in., 0·5 in.; (*b*) 6·21 in., 0·93 in.; (*c*) 6 in., 1 in.; (*d*) 4·24 in., 1·59 in.; in front, virtual, erect in each case.
23. 15 in. 24. — 12 cm.
25. 24 cm. 26. 20·84 cm.
29. 2·18 in. in front of lens, or 10 in. in front of mirror.
30. 15 cm. or 10 cm. from mirror.

EXERCISES VIII. (*Page* 337)

1. — 20 cm. 2. — 10·02 cm.
3. 99·1 cm. 4. 100 cm.
5. 10·7 cm.; 1·53. 6. — 112·1 cm.; 121·9 cm.; 1·54.
7. — 35·3 cm.; 11·9 cm., 35·2 cm.; 1·51.
8. 1·331. 9. 1·628.

EXERCISES IX. (*Page* 338)

8. 51° 10′.

EXERCISES X. (*Page* 338)

1. Equilateral triangle. 3. 2·4 in.
4. Convex; 6·15 in.; 126·75 in. square. 6. 4.
7. 6·67 in., 4·09 in., 4 in., concave. 8. Convex; 20 in. focal length.

EXERCISES XI. (*Page* 339)

1. 15 cm., measured from centre. 2. 228.
5. Front surface convex, telescope longer; front surface concave, telescope shorter.
6. *f*/4. 7. 66·67 cm.
8. Focal lengths, — 37·5 cm., 60 cm.; radii, convex, 37·5 cm.; concave, 37·5 cm., 975 cm.
11. 0·408 in. 12. 2704 : 20736 : 1.
14. 138. 15. 0·105 : 0·033, or 3·16 : 1.

EXERCISES XII. (*Page* 340)

1. Greatest, 40 hr. 13·9 sec.; least, 39 hr. 59 min. 46·1 sec. 3. 1000.

EXERCISES XIII. (*Page* 340)

1. 3 : 4. 2. $3\sqrt{3}$: 8 : 8. 3. 11·2 ft.
4. Screen between lamps, 2·67 ft. from 16 c.p.; screen outside, 24 ft. beyond 10 c.p. lamp.

5. $116^2 : 201^2$. 6. 5 : 7 : 6. 7. 4·52.
8. $a^2 : b^2$. 9. 80 cm. from less intense.
10. 12·96 c.p. 11. 408·25 in.

EXERCISES XV. (*Page* 342)

3. 0·0000589 cm. 4. 0·0147 cm.
6. 69·95 cm.; 0·000067 cm. 8. 0·00005889 cm., 0·00005896 cm.

EXERCISES XVI. (*Page* 342)

6. 7·46 gm. per 100 c.cm. of solution.

EXERCISES XVII. (*Page* 343)

6. 34,810 m. per sec. towards the earth.

EXAMINATION QUESTIONS. (*Page* 344)

4. $17\sqrt{2}$ cm. 5. 45·45 cm. behind mirror; 0·91 cm. high.
6. Convex, 60 cm. 7. 10 cm. long; 15 cm. behind mirror.
8. 0·194 cm. 9. 7·42 cm. from base of tank.
10. 6·61 ft. 12. 69° 38′.
15. 36 cm. from curved surface. 16. 2·5 cm. from wall.
17. 10 cm. beyond sphere. 18. 10 cm.
20. 60 cm. 23. − 12 cm., 21 cm., 6 mm., 14 mm.
24. 14·67 cm. beyond second lens, 1·33; − 7·45 cm., 1·64.
25. − 60 cm. 26. − 18·75 cm.; $\frac{3}{5}$, $\frac{5}{3}$.
28. 1·42. 29. 1·5.
32. $A = \dfrac{C - B}{2}$. 37. − 20 cm., + 60 cm.
41. 1·26 : 1. 49. − $\frac{10}{11}$ ft.; 1 ft. from slide.
51. 6. 52. Concave, 25 in. focal length, 6·25 in.
53. Convex, 26·3 cm. focal length. 55. 410 in.
56. 5; 6/5. 61. 8 cm. beyond diverging lens.
69. 62° 43′. 73. 1·024 : 1.
74. 0·866 : 1. 76. 19·37 ft.
77. 0·354 ft.-candle. 90. $\dfrac{2v\lambda}{c}$.

INDEX

The numbers refer to pages.

teal not this book, for fear and shame,
ok up and see the owner's name,
when you die the god will say,
here is the book you stole ~~that~~ day,
d if you say I cannot tell;

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